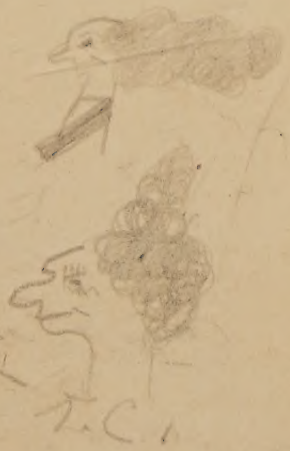
# INTERMEDIATE ARITHMETIC

Kenneth Brooke



# INTERMEDIATE ARITHMETIC

BY

DAVID EUGENE SMITH, LL.D.

PROFESSOR OF MATHEMATICS IN TEACHERS COLLEGE
COLUMBIA UNIVERSITY, NEW YORK

GINN & COMPANY

BOSTON · NEW YORK · CHICAGO · LONDON

ENTERED AT STATIONERS' HALL

COPYRIGHT, 1904, 1905

BY DAVID EUGENE SMITH

ALL RIGHTS RESERVED

910.7

The Athenæum Press

GINN & COMPANY · PROPRIETORS · BOSTON · U.S.A.

# PREFACE

The following have been controlling ideas in the preparation of this book:

1. In sequence of topics, to follow the plan adopted in the author's Primary Arithmetic, that of recognizing the value of the various courses of study in use in different parts of the country. Whatever originality may be demanded and legitimately shown in the preparation of a text-book, an author is bound to recognize the consensus of opinion as to topics and sequence. For example, modern courses invariably suggest the repetition of the most important portions of arithmetic from time to time, but they favor a somewhat exhaustive treatment of each subject whenever it is under discussion. The extreme spiral system, in which no topic is ever thoroughly treated at one time, but each is repeated until the pupil wearies of it, is psychologically too unwarranted to be considered seriously. On the other hand, the old-time plan of presenting important chapters but once is equally unscientific. Between these extremes lies the mean of the modern courses of study.

2. In arrangement by grades, to recognize the prevailing courses of study in the country, and to outline the work usually covered in the fifth and sixth school years, reserving for the author's Advanced Arithmetic the work of the seventh and eighth years.

3. In the selection of problems, to touch the actual life of this country at this time; to give correct ideas of the business customs of to-day; to embody the mathematical principles in interesting and instructive groups of problems;

to touch the genuine interests of pupils in the story of our national resources and industries rather than to dwell upon the technicalities of minor trades in which they have no immediate or prospective concern; and to come in contact with human life rather than with those phases of science which are quite as foreign to the interests of boys and girls as are the mere abstract problems of numbers.

4. In the matter of abstract drill work, to recognize the fact that a large number of "problems without content" are necessary to concentrate the attention on the operations and to acquire the computing habit. The numbers selected have been those demanded by the conditions of the present day, the fractions and compound numbers being those in common use rather than those never met in business, and the integers being the ordinary ones of daily life. Very large numbers have generally been used only in such applied problems as represent the real conditions that the children meet in their geography, their elementary science, and their newspaper reading.

5. In the matter of illustrations, to recognize that pictures aid in the understanding of certain number relations, that they are often helpful in suggesting simple material for the teacher's use, and that they may be legitimately used in rendering more interesting and real various groups of problems.

In fine, the book is written for the use of those teachers who wish to preserve the best that was in the old-style arithmetic, with its topical system and its abundant drill, while giving to it a modern arrangement and securing "mental discipline" through problems of to-day rather than through the tiresome, meaningless, unreal inheritances of the past.

DAVID EUGENE SMITH

# CONTENTS

## CHAPTER I

### I. REVIEW OF PRIMARY ARITHMETIC. COMMON FRACTIONS


| | PAGE |
|---|---|
| Introduction | 1 |
| Notation and Numeration | 2 |
| Addition | 6 |
| Subtraction | 10 |
| Multiplication | 14 |
| Division | 20 |
| Factors and Multiples | 30 |
| Common Fractions | 36 |
| Decimal Fractions | 76 |
| Denominate Numbers | 90 |


### II. DECIMAL FRACTIONS AND PERCENTAGE


| | |
|---|---|
| Addition and Subtraction of Decimals | 105 |
| Multiplication of Decimals | 106 |
| Division of Decimals | 110 |
| How to solve Problems | 115 |
| Pictures of Magnitudes | 120 |
| Bills and Receipts | 125 |
| Percentage | 129 |
| Discount | 147 |
| Review of Denominate Numbers | 149 |
| Measurements and Comparison | 151 |


## CHAPTER II

### I. PERCENTAGE AND ITS APPLICATIONS. SIMPLE INTEREST. MEASUREMENTS


| | |
|---|---|
| The Formal Solution of Problems | 159 |
| Percentage | 181 |
| Profit and Loss on Purchases | 194 |
| Commission | 199 |

PAGE

INTEREST . . . . . 202
CHECKS AND RECEIPTS . . . . . 204
DENOMINATE NUMBERS REVIEWED . . . . . 206

## II. SIMPLE INTEREST. RATIO AND PROPORTION

SIMPLE INTEREST . . . . . 211
RATIO . . . . . 217
PROPORTION . . . . . 223
MEASURES . . . . . 228
GENERAL BUSINESS APPLICATIONS . . . . . 242
MISCELLANEOUS PROBLEMS . . . . . 254
TABLES FOR REFERENCE . . . . . 265
DEFINITIONS . . . . . 268

INDEX . . . . . 277

# INTERMEDIATE ARITHMETIC

## CHAPTER I

### I. REVIEW OF PRIMARY ARITHMETIC: COMMON FRACTIONS

### INTRODUCTION

**1. To the pupil.** You have already learned much about Arithmetic. You have learned to count and to write numbers, at least to a million, to add, subtract, multiply, and divide integers, and to do some work with fractions and mixed numbers.

You have also learned how to write United States money and decimal fractions, how to measure things, and the ordinary tables of denominate numbers. Besides all this, you have seen that you have need for Arithmetic at the store, in your games, in reading, and in many other ways.

There is, however, much more to learn about the uses of Arithmetic, and some more to learn about how to work with numbers. Every one of us is interested in what the world is doing, and in our country and its products. We all like to know about our farms, our mines, our forests, our great industries, and the accounts that we must sometime keep, and we find much of interest about these matters in Arithmetic.

## NOTATION AND NUMERATION

### ORAL EXERCISE

1. What name do you give to 3 tens? to 10 tens?

2. What name do you give to 1000 thousands?

3. Looking at your fingers, do you see why people came to count as they do, by tens? Why was it?

4. Read aloud these numbers:

| | | | | |
|---|---|---|---|---|
| 1,427 | 2,341 | 4,004 | 99,999 | 10,010 |
| 234,567 | 381,426 | 500,500 | 203,203 | 990,990 |
| 3,505,246 | 8,421,205 | 6,006,006 | 17,243,321 | 82,603,627 |

**2. Notation.** The writing of numbers by means of symbols is called *notation.*

**3. Numeration.** The naming of numbers is called *numeration.*

Our common notation started in India about 2000 years ago. The Arabs took it to Europe. Therefore these symbols are called *Arabic numerals.* You know that there are nine of these numerals besides the *zero,* which is also called *naught* or *cipher.*

**4. Periods.** When more than three figures are used in an integer they are separated into groups of three so as to be read more easily. The groups are called *periods.*

For example, 1,397,352, sometimes printed as 1 397 352.

**5. Separatrix.** The comma used in separating periods is often called a *separatrix.*

**6. Place value.** The value of a period and of a figure depends upon the place it occupies. Therefore each has a *place value.*

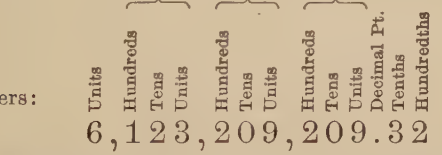
Such definitions as the above need not be memorized. The teacher should, however, be sure that the terms and the ideas which they suggest are understood.

### ORAL EXERCISE

1. In the number 234, what place is occupied by the 4? by the 3? by the 2?

2. In the number 123,456,789, what name is given to the period 789? to the period 456? to the period 123? to the whole number?

7. **Reading numbers.** In a number like 6,123,209,209.32 we speak of the three places in each period as orders.




Teachers should require pupils to read numbers accurately.

100.023 is read one hundred and twenty-three thousandths,

.123 " " one hundred twenty-three thousandths,

123 " " one hundred twenty-three.

Number names beyond billions are too rarely used to be taught.

Omit the following exercise with classes not ready for the multiplications.

### WRITTEN EXERCISE

*Let us see what a large number like a million means:*

1. It is 1000 mi. from Chicago to New York. How many feet does this equal?

2. How many seconds in 1 min.? in 1 hr.? in 1 da.? in 11 da.? Have you a million seconds yet?

3. How many hours in 1 yr.? in 100 yr.? Have you a million hours yet?

4. If there are 25 pupils in your room, how many rooms would be needed for 100 pupils? for 1000? for 1,000,000?

## Some Uses for Large Numbers

### ORAL EXERCISE

*Read the following aloud:*

1. The number of different books published in a certain year in the United States was 8141, in England 6044, in France 13,053, and in Germany 25,331.

2. In the same year the value of the wheat produced in the United States was $467,350,156; of corn, $921,555,768; of oats, $293,658,777.

3. In a certain year the United States coined $99,272,943 worth of gold, Great Britain $63,769,609, and Russia $83,221,525.

4. In the common schools of our country there were in a certain year 15,341,220 pupils, and the average number present each day was 10,513,518. To teach them required 421,288 teachers.

5. From 1800 to 1900 the population of New York City increased from 60,489 to 3,437,202; that of Philadelphia, from 41,220 to 1,293,697; that of Boston, from 24,937 to 560,892; and that of Chicago, from nothing to 1,698,575.

6. Suppose Germany keeps an army of 576,666 men; France, 545,962; Great Britain and India, 347,042; Russia, 1,049,003, while the United States has only about 50,000. Can you explain this great difference?

7. Our country mined in one year 228,717,579 tons of coal. Great Britain mined 223,616,279 tons; Germany, 135,844,419 tons; France, 32,862,712 tons; Russia, 12,800,000 tons; Canada, 4,142,242 tons; Mexico, 409,125 tons; and the rest of the world mined enough to bring the total to 723,617,836 tons.

### WRITTEN EXERCISE

*Copy the following, writing the numbers in Arabic figures:*

1. In Alabama there were at one time five thousand, six hundred two factories, employing fifty-two thousand, nine hundred two persons.

2. In California there were two hundred five million, three hundred ninety-five thousand, twenty-five dollars invested in manufacturing, and the value of the product was three hundred two million, eight hundred seventy-four thousand, seven hundred sixty-one dollars a year.

3. New York has the largest amount of manufacturing of any of our states. It employed recently eight hundred forty-nine thousand wage earners and produced two billion, one hundred seventy-five million, seven hundred sixty-six thousand, nine hundred dollars' worth of goods a year.

4. The expenses of our government are very great. In eighteen ninety-seven they were three hundred two million, seven hundred eighty-six thousand, three hundred eighty-six dollars. In eighteen ninety-nine they were six hundred seventy-three million, fifty thousand, two hundred ninety-three dollars. They are now probably between four and five hundred million dollars a year.

5. One of the great industries in the United States is supplying the people with meat. The value of the products was recently seven hundred eighty-six million, six hundred three thousand, six hundred seventy dollars in one year. Next to that comes the iron industry. It recently produced five hundred ninety-six million, five hundred eighty-eight thousand, thirty-four dollars' worth of goods in one year.

## ADDITION

### ORAL EXERCISE

1. State rapidly two one-figure numbers whose sum is 11; also two whose sum is 12; 13; 14; 15; 16; 17; 18.

2. Write upon the blackboard the series 7, 0, 8, 6, 9, 5, 4, 2, 1, 3, and read the numbers, each increased by 2; by 7.

Much drill work of this kind should be given. A short time daily devoted to such oral work is wisely spent.

**8. Addition.** The process of finding a number that equals two or more other numbers is called *addition.*

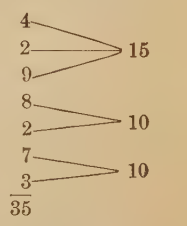
**9. Addends.** Numbers to be added are called *addends.*

**10. Sum.** The result in addition is called the *sum.*

**11.** *Only numbers considered as like numbers can be added.*

For example, \$3 + \$5 = \$8. But 3 turnips + 5 potatoes are neither turnips nor potatoes, although if we consider them as vegetables we may say, "3 vegetables + 5 vegetables are 8 vegetables."

**12.** Remember to read a column like a word. Try to see the 10's, or the 5's and 15's. In this column you should see the two 10's and the 15, and write the answer, 35. It is, however, not wise to skip numbers in order to group by 10's.




**13. Checks.** To *check* or *prove* your work, read the column from the top down, if you first read it upwards.

### WRITTEN EXERCISE

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| 3,288 | 2,287 | 33,042 | 317,675 | 768,265 |
| 13,590 | 9,954 | 128,600 | 1,240,443 | 1,838,921 |
| 4,057 | 5,320 | 41,584 | 292,248 | 1,159,356 |
| 5,841 | 1,386 | 65,936 | 533,502 | 662,973 |

## Our Country's Extent and Resources

### WRITTEN EXERCISE

1. The census at the beginning of this century showed 65,843,302 native and 10,460,085 foreign-born persons in the United States. What was the total population?

2. At that time the population of our three largest cities was as follows: New York, 3,437,202; Chicago, 1,698,575; Philadelphia, 1,293,697. What was their total population?

3. We sold $1,487,764,991 worth of goods and produce to foreign countries in 1901; $1,381,719,401 in 1902; and $1,420,141,679 in 1903. What was the total?

4. The amount of gold in the United States in a certain year was $1,174,600,000; of silver, $660,000,000; and of paper money, $437,800,000. What is the total amount of money in the country?

5. The area of the United States is 3,602,990 sq. mi.; of the Philippine Islands, 143,000 sq. mi.; of Porto Rico, 3600 sq. mi.; of Hawaii, 6740 sq. mi.; and of other islands, 554 sq. mi. What is the total area?

6. The four countries producing the most wheat at the opening of this century were as follows: United States, 756,269,573 bushels a year; Russia, 427,780,477 bushels; France, 307,388,463 bushels; and India, 248,593,946 bushels. How much did all four produce?

7. The number of children in our elementary public schools in a certain year was 14,662,488, and in private schools 1,193,882; the number of pupils in public high schools was 488,549, and in private ones 166,679. There were also 103,251 in colleges and universities, and 123,514 in professional schools. Required the total.

8. Our country produced 302,502,328 lb. of wool in 1901, 316,341,032 lb. in 1902, 287,450,000 lb. in 1903. How many pounds did it produce in these three years?

9. The amount of coal mined in this country in 1900 was 225,759,980 tons of soft coal and 67,538,536 tons of hard coal. How many tons were there of both?

10. Of the world's cotton supply in 1903, America produced 10,877,000 bales, the East Indies 1,050,000 bales, and other countries 1,230,000 bales. What was the total production for that year?

11. The amount of money deposited in our savings banks in a certain year was as follows: New England states, $1,000,175,255; other eastern states, $1,308,940,-078; southern states, $16,530,046; central states, $257,-491,072; other states, $180,438,675. Required the total amount.

12. Of our 200,000 miles of railroad, the greatest mileage for any state is in Illinois, which had not long ago 11,119 mi. Next in amount of mileage was Pennsylvania with 10,538 mi., and then came Texas with 10,189 mi. What was the total mileage of these three states?

13. The number of bushels of grain grown in the United States in a recent year was as follows: corn, 2,105,102,516 bu.; wheat, 756,269,573 bu.; oats, 800,125,989 bu.; barley, 58,925,833 bu.; rye, 23,995,927 bu.; and buckwheat, 9,566,966 bu., each year. Required the total.

14. The annual income of the colleges and universities in various parts of the country was recently as follows: North Atlantic division, $9,187,016; South Atlantic division, $2,015,910; South Central division, $2,061,615; North Central division, $7,865,045; Western division, $1,659,468. What was the total for all these divisions?

**14.** If the column is very long, business men often write on a separate slip of paper the partial sums as here shown, so as to check mistakes more easily. Whether you do this or not, always check your work by adding again in the opposite direction.

| |
|---|
| $2.35 |
| 3.46 |
| 9.87 |
| 7.03 |
| 21 |
| 15 |
| 21 |
| $22.71 |

The following exercise is intended for rapid drill work. To save the time of copying, teachers may prefer to have the pupils place a slip of paper below the line and write the sum on that.

### WRITTEN EXERCISE

*Add the following:*

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| 124,483 | 69,869 | 316,499 | 15,755 | 241,400 |
| 438,432 | 22,410 | 202,991 | 22,328 | 162,393 |
| 179,136 | 61,830 | 412,279 | 44,151 | 332,823 |
| 158,912 | 50,675 | 773,703 | 85,564 | 620,923 |
| 173,188 | 76,953 | 3,524,102 | 335,653 | 3,188,448 |
| 98,251 | 18,263 | 2,452,542 | 267,405 | 1,952,034 |
| 27,341 | 62,483 | 73,192 | 134,207 | 412,375 |
| 41,273 | 21,276 | 147,628 | 29,632 | 208,607 |

| 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|
| 257 | 1,094 | 40,936 | 12,665 | 221,295 |
| 1,102 | 10,048 | 419,216 | 114,760 | 226,821 |
| 184 | 1,831 | 70,206 | 24,079 | 418,496 |
| 334 | 2,546 | 87,261 | 27,388 | 507,254 |
| 355 | 1,431 | 65,563 | 17,380 | 493,263 |
| 604 | 1,433 | 52,647 | 15,037 | 257,770 |
| 253 | 2,508 | 71,038 | 28,446 | 201,619 |
| 429 | 2,479 | 76,987 | 36,461 | 578,298 |
| 415 | 1,195 | 64,883 | 11,270 | 424,853 |
| 315 | 1,503 | 65,109 | 20,483 | 434,017 |
| 459 | 3,488 | 111,778 | 29,918 | 573,200 |

## SUBTRACTION

### ORAL EXERCISE

1. What number added to the lower of these numbers makes the upper one?

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 | 8 | 10 | 20 | 25 | 47 | 92 |
| 4 | 3 | 4 | 16 | 16 | 28 | 76 |

2. If you hand a merchant \$1, how much change should you get if you owe him 75 ct.? 60 ct.? 25 ct.? 88 ct.?

Throughout the course there should be much oral drill on such problems as the above. It is expeditious, in the case of $75-48$, to think: "48 and 2 are 50, and 25 more are 75; $2 + 25 = 27$."

$$\begin{array}{r} 75 \\ 48 \\ \hline 27 \end{array}$$

**15. Difference.** The number which added to one number makes another is called the *difference* between them.

That is, the difference between 75 and 48 is 27, because $48 + 27 = 75$.

**16. Subtraction.** The operation of finding the difference between two numbers is called *subtraction.*

Subtraction may also be defined as the operation of taking one number from another, and the result may be defined as the remainder or difference. But since a great deal of subtraction is "making change," where the work is based on addition, the definitions first given are better.

**17. Subtrahend.** The number to which the difference is added, or the number which is subtracted, is called the *subtrahend.*

$$\begin{array}{rl} 563 & \text{minuend} \\ 127 & \text{subtrahend} \\ \hline 436 & \text{difference or remainder} \end{array}$$

**18. Minuend.** The sum of the difference and subtrahend, or the number from which we subtract, is called the *minuend.*

**19. The process.** In subtracting 756 from 1632,

| *We write only this:* | *But this assists in explaining:* |
|---|---|
| 1632 | $1632 = 1500 + 120 + 12$ |
| 756 | $756 = 700 + 50 + 6$ |
| 876 | $800 + 70 + 6 = 876$ |

We cannot subtract 6 from 2, but $12 - 6 = 6$. We have now used 10 of the 30, leaving 20. We cannot subtract 50 from 20, but $120 - 50 = 70$. We have now used 100 of the 1600, and $1500 - 700 = 800$.

The above is one of the most common methods of subtracting, but the class should follow the method learned in the earlier grades. It is confusing to learn new methods that are little better than the old ones. The teacher is advised to give but little time to asking for such definitions as those on page 10, for the terms are rarely used in business. The business man says, "*Deduct* \$10 and pay me the *balance*," instead of "*Subtract* \$10 and pay me the *remainder*."

**20. Making change.** If you owe 70 cents and give the merchant \$1, he says: "70 and 5 are 75, and 25 more makes a dollar," at the same time laying down 5 cents and 25 cents. He thus sees that $\$1 - \$0.70 = \$0.30$.

### ORAL EXERCISE

1. Make change for \$1, when you owe 55 ct.; 37 ct.
2. Also for \$2, when you owe \$1.49; \$1.62, \$1.16.
3. Also for \$5, when you owe \$2.75; \$3.10; \$1.48.

### WRITTEN EXERCISE

1. \$143.92 — \$127.65.
2. \$237.50 — \$148.75.
3. \$926.50 — \$735.75.
4. \$432.75 — \$246.50.
5. \$329.50 — \$137.75.
6. \$823.45 — \$696.75.
7. 437,625 — 129,735.
8. 1,237,635 — 982,987.

## Interesting Facts in Our History

*A few important dates*

1492. Columbus discovered America.
1609. The Hudson River first explored.
1620. Landing of the Pilgrims at Plymouth.
1664. New York City captured from the Dutch.
1776. Signing of the Declaration of Independence.
1789. George Washington inaugurated President.

### WRITTEN EXERCISE

1. How many years since Columbus discovered America?

2. How long after that was the Hudson explored?

Landing of the Pilgrims

3. How many years since Washington was inaugurated?

4. How long ago did the *Mayflower* reach Plymouth?

5. How many years after that did the English capture New York from the Dutch?

6. How many years from the Declaration of Independence to the next Fourth of July after to-day?

7. How long after Columbus discovered America was the landing of the Pilgrims?

8. It lacked only 2 years of being a century and a quarter from the battle of Lexington to that of Manila, 1898. When was the battle of Lexington?

### ORAL EXERCISE

1. The subtrahend is 7, the minuend 21; what is the difference? How much is 21 minus 7?

2. The difference is 11, the minuend 16; what is the subtrahend? What number minus 16 equals 11?

3. The subtrahend is 27, the remainder 14; what is the minuend? 27 minus what number equals 14?

4. The minuend is 25 more than the subtrahend; what is the difference?

5. The subtrahend is 47 less than the minuend; what is the difference?

6. The subtrahend and difference together equal 39; what is the minuend?

7. Mr. Roberts owed Mr. Lambert $75 yesterday. To-day he has paid him $25. What balance is due?

### WRITTEN EXERCISE

*Perform these subtractions, and check every answer by adding the remainder and subtrahend. See how many you can perform in five minutes.*

| | |
|---|---|
| 1. 20,265 – 5927. | 2. 32,502 – 27,635. |
| 3. 28,765 – 21,976. | 4. 41,232 – 36,525. |
| 5. 81,463 – 29,658. | 6. 75,000 – 29,127. |
| 7. 234,684 – 102,793. | 8. 648,329 – 409,681. |
| 9. 642,306 – 609,008. | 10. 920,102 – 735,008. |
| 11. $435.27 – $107.62. | 12. $308.08 – $127.09. |
| 13. $2087.50 – $1296.49. | 14. $3042.50 – $1209.87. |
| 15. $62,341.75 – $12,986.50. | 16. $42,350.50 – $19,625.08. |
| 17. $48,002.75 – $31,609.50. | 18. $29,875.25 – $20,975.26. |

## MULTIPLICATION

### ORAL EXERCISE

1. Read the products by 2: 4, 7, 1, 9, 6, 2, 8, 3, 11, 5, 10.

This or a similar set of numbers should be written on the board.

2. In the same way, read rapidly the products by the various numbers from 3 to 10. Also by 20, 30, 100.

3. Count by 2's from 2 to 20, and give the multiplication table of 2's. Give the multiplication tables from 3 to 10.

4. How much is $4 + 4 + 4 + 4 + 4$? 5 times 4? Multiplication is a short form of what other process?

**21. Abstract numbers.** A number that does not refer to any particular kind of object or measure is called an *abstract number.*

For example, 10, 42, $3\frac{1}{2}$, are abstract numbers.

**22. Concrete numbers.** A number that refers to some particular kind of object or measure is called a *concrete number.*

For example, \$10, 32 ft., 6 mo., are concrete numbers.

**23. Multiplication.** The process of taking one number as many times as there are units in another is called *multiplication.*

This means multiplication by an abstract integer. Thus, to multiply \$3 by 2 means that \$3 is taken 2 times.

**24. Multiplicand.** The number multiplied is called the *multiplicand.*

**25. Multiplier.** The number by which we multiply is called the *multiplier.*

**26. Product.** The result of multiplying is called the *product.*

\$325 multiplicand
3 multiplier
\$975 product

**27. Nature of the numbers.** It is therefore seen that

1. *The multiplier must be thought of as abstract.*
2. *The product is like the multiplicand.*

That is, if we have 3 times 17 ft., the 3 is abstract, and the product is feet, like the multiplicand.

**28. Multiple.** The product of two abstract integers is called a *multiple* of either.

For example, 35 is a multiple of 7 and of 5.

**29. Factors.** The numbers which multiplied together make another number are called its *factors*.

For example, the factors of 30 are 2, 3, and 5.

Teachers are advised not to require much memorizing of definitions. The important thing is that the words shall be used correctly. Those just defined are rarely heard in business. In reading $2 \times \$6$ and $\$6 \times 2$, teachers should follow the custom of the school. The former is preferably read "two times \$6," and the latter "\$6 multiplied by 2." In order not to confuse children who have learned other ways of reading, this book uses the word *times* where there might be any misunderstanding.

**30. Power.** The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $2 \times 2 = 4$, and 4 is called the second power, or square, of 2. $2 \times 2$ is written $2^2$. So $2 \times 2 \times 2$, or $2^3$, is the third power of 2. It is also called the *cube* of 2.

### ORAL EXERCISE

1. State four multiples of 7; of 3; of 9; of 11; of 15.
2. State the factors of 35; of 77; of 21; of 49; of 121.
3. State the squares of 7, 6, 9, 8, 10, 11, 20, 100, 1000.
4. What is the square of 5? the cube of 3? the fourth power of 2? the cube of 5? of 2? of 10?

**31. The process.** You have already learned that in multiplying, for example, \$635.50 by 215,

| *This is the complete operation:* | | | | *But we write only this:* |
|---|---|---|---|---|
| \$635.50 | | | | \$635.50 |
| 215 | | | | 215 |
| \$3177.50 | 5 | times | \$635.50 | 3177 50 |
| 6355.00 | 10 | " | " | 6355 0 |
| 127100.00 | 200 | " | " | 127100 |
| \$136632.50 | 215 | " | " | \$136632.50 |

**32. Zero in the multiplier.** If a zero appears in the multiplier,

| *This long process might be taken:* | | *But we need only this:* | |
|---|---|---|---|
| 247 | 247 | 247 | 247 |
| 305 | 350 | 305 | 350 |
| 1235 | 000 | 1235 | 12350 |
| 000 | 1235 | 741 | 741 |
| 741 | 741 | 75335 | 86450 |
| 75335 | 86450 | | |

WRITTEN EXERCISE

*Multiply:*

1. $923 \times 936$.
2. $237 \times 243$.
3. $406 \times 572$.
4. $560 \times 437$.
5. $206 \times 302$.
6. $725 \times 725$.
7. \$41.27 by 237.
8. \$68.95 by 419.
9. \$309.25 by 406.
10. \$723.30 by 507.
11. \$234.56 by 129.
12. \$507.75 by 625.
13. \$802.75 by 308.
14. \$7235 by 5200.
15. \$1245.25 by 209.
16. \$3005.75 by 2002.
17. \$8126.05 by 3030.
18. \$4040.40 by 4040.

## Multiplying by Powers of 10

### ORAL EXERCISE

1. Multiply by 10: 10 ct., 50 ct., $1.20, $3.50.

2. Multiply by 100: 3, 42, 565, $1.50, $21.75.

3. Multiply by 1000: 4, 7, 10, 40, 52, 100, 250.

4. State a short way of multiplying an integer by 10; by 100; by 1000.

5. State a short way of multiplying a decimal by 10; by 100; by 1000.

**33. Multiplying by powers of 10.** *To multiply an integer by 10, annex a 0; by 100, two 0's; by 1000, three 0's.*

**34.** *To multiply a decimal by 10, move the decimal point 1 place to the right; by 100, 2 places; by 1000, 3 places.*

Hence to multiply 27 by 40, multiply by 4 and annex a 0.

$$\begin{array}{r} 27 \\ 40 \\ \hline 1080 \end{array}$$

Also to multiply $25.16 by 300, multiply by 3 and move the decimal point 2 places to the right.

$$\begin{array}{r} \$25.16 \\ 300 \\ \hline \$7548 \end{array}$$

### WRITTEN EXERCISE

1. 210 times $346.
2. 320 times $421.
3. 800 times 932 ft.
4. 700 times 864 rd.
5. 9600 times $4300.
6. 4300 times $8250.
7. 7000 times $15.42.
8. 9000 times $21.75.
9. 6520 times $12.50.
10. 2750 times $31.02.

11. If a train goes 48.2 mi. an hour, how far will it go in 20 hr., at the same rate?

12. If it costs $14.75 a year to educate you, how much does it cost to educate 300 children, at the same rate?

### ORAL EXERCISE

1. On the blackboard, multiply 68 by 5. Divide 680 by 2.

2. On the blackboard, multiply \$12.25 by 5. Also divide \$122.50 by 2. How do the results compare? Tell a short way of multiplying by 5.

3. Because $5 = 10 \div 2$, to multiply an integer by 5 is the same as to annex 0 and divide by what number?

4. On the blackboard, multiply \$42.74 by 50. Also divide \$4274 by 2. Compare the results.

5. Because $50 = 100 \div 2$, to multiply by 50 we may first multiply by what number, and then divide by what one?

6. On the blackboard, multiply \$32.56 by 25. Also divide \$3256 by 4. Compare the results. Because $25 = 100 \div 4$, to multiply by 25 we may first multiply by what number, and then divide by what other number?

**35. Aliquot part.** A number that will exactly divide another is called an *aliquot part* of that number.

For example, 5 is an aliquot part of 10, and both 25 and 50 are aliquot parts of 100.

**36. Multiplying by aliquot parts.** *To multiply by 5, multiply by 10 and divide by 2.*

*To multiply by 50, multiply by 100 and divide by 2.*

*To multiply by 25, multiply by 100 and divide by 4.*

### WRITTEN EXERCISE

1. Multiply by 5: 723, \$2500, \$47.40, \$666.60.
2. Multiply by 50: 237, 425 ft., \$68.50, \$987.40.
3. Multiply by 25: 134, 802 ft., \$49.60, \$384.40
4. Multiply by 500: 16, 64, 726, \$42.80, \$124.40.

## The American City

### WRITTEN EXERCISE

1. There were in a certain year 19,075,847 of our people living in cities having a population of 25,000 or more. If this was $\frac{1}{4}$ of our total population, what was the total?

2. In the United States there were recently 161 places of more than 25,000 inhabitants, 356 between 8000 and 25,000, and 532 between 4000 and 8000. How many were there having more than 4000?

3. If the valuation of the land and buildings in Buffalo is \$221,405,290; in Indianapolis, \$94,935,180; in New Orleans, \$103,753,915; and in St. Louis, \$316,041,190, by how much does the St. Louis valuation exceed that of each of these other cities?

4. Including the suburbs, the population of Chicago at the beginning of the century was 1,939,327; of Philadelphia, 1,738,192; of Baltimore, 656,047; and of Louisville, 292,502. If the population of New York City and its suburbs was as much as that of all these four cities, how much was it?

## DIVISION

### ORAL EXERCISE

1. How much is 10 times \$17? How many times is \$17 contained in \$170? How much is $\frac{1}{10}$ of \$170?

2. How many times is \$3 contained in \$12? in \$14, with what remainder? in \$20? in \$42? in \$61?

3. If the product of two numbers is 35 ft., and one of the numbers is 7 ft., what is the other? If one of the numbers is 5, what is the other?

**37. Division.** The operation by which, given the product of two numbers and one of them, the other is found, is called *division.*

**38. Dividend and divisor.** The given product is called the *dividend* and the other given number is called the *divisor.*

**39. Quotient and remainder.** The number found by division is called the *quotient.* The part of the dividend remaining when the division is not exact is called the *remainder.*

The teacher should recognize that it is possible to give too much attention to such terms. In business the word *dividend* is commonly used in another way, and the word *quotient* is rarely employed.

**40. Two kinds of division.** Because 10 times \$17 = \$170, therefore \$170 ÷ \$17 = 10, and \$170 ÷ 10 = \$17.

**41.** *If the dividend and divisor are concrete, they must be alike, and the quotient is abstract.*

For example, \$170 ÷ \$17 = 10. This is sometimes called *measuring,* because the \$170 is measured by \$17.

**42.** *If the dividend is concrete and the divisor is abstract, the quotient is like the dividend.*

For example, \$170 ÷ 10 = \$17. This is sometimes called *partition,* because the \$170 is separated into 10 equal parts.

### ORAL EXERCISE

1. Divide by 2: 16, 24, 480, 248, 408, 418.
2. Divide by 3: 27, 36, 630, 963, 906, 615.
3. Divide by 4: 40, 84, 480, 844, 408, 816.
4. Divide by 5: 55, 60, 550, 555, 600, 655.
5. Divide by 6: 30, 66, 360, 606, 612, 726, 636.
6. Divide by 7: 63, 77, 350, 560, 497, 714, 775.
7. Divide by 8: 72, 56, 640, 648, 816, 968, 641.
8. Divide by 9: 63, 99, 720, 279, 819, 558, 633.
9. If 6 sheep cost $33, how much will 1 cost? 2?
10. If 4 dictionaries cost $36, how much will 1 cost?
11. If 1 dictionary costs $9, how many can be bought for $36? for $63? for $72? for $99? for 180?
12. A school paid $96 for school desks at $3 each. How many were bought? How many at $4 each?
13. If 4 school desks cost $13, how much will 1 cost? How much, if 5 cost $16? if 8 cost $25? if 6 cost $21?

### WRITTEN EXERCISE

1. If 11 acres of land are worth $1485, what is the average price per acre? How much are 4 acres worth?
2. If a dealer pays $336 a dozen for suits of ready-made clothes, how much does he pay per suit? for 7 suits?
3. A man pays $1276 for the rent of a city house for 11 months. At this rate, how much does he pay for 5 months?
4. A wagon dealer pays $1215 for 9 wagons. At this rate, how much do 4 wagons cost? 7 wagons?
5. At the rate, of $788 for 8 acres of land, how much must a man pay for 3 acres? for 5 acres? for 100 acres?

**43. The process.** You have already learned how to divide. For example, to divide \$1375.44 by 521,

| *This is the complete operation:* | | *But we write only this:* |
|---|---|---|
| \$2.64 | | \$2.64 |
| 521)\$1375.44 | | 521)\$1375.44 |
| 1042 | = 521 times \$2 | 1042 |
| 333.44 | still to be divided | 3334 |
| 312.60 | = 521 times \$0.60 | 3126 |
| 20.84 | still to be divided | 2084 |
| 20.84 | = 521 times \$0.04 | 2084 |
| | Total quotient = \$2.64 | |

We see that \$1000 ÷ 521 equals no thousands of dollars; \$1300 ÷ 521 equals no hundreds; \$1370 ÷ 521 equals no tens; but \$1375 ÷ 521 equals about \$2. (We really notice that 13 ÷ 5 = 2 + something.) Subtracting 521 times \$2, there is \$333.44 still to be divided. \$333.40 ÷ 521 equals about \$0.60 (for 33 ÷ 5 = 6 + something). Subtracting 521 times \$0.60, there remains \$20.84, and \$20.84 ÷ 521 = \$0.04. Therefore the quotient is \$2.64.

**44. To check the result.** We have seen that the dividend is the product of the quotient and divisor, plus the remainder, if any.

In the above example, \$1375.44 = 521 times \$2.64.

**45. Remainders.** In 109 ÷ 33 = 3, with a remainder 10 still to be divided, the quotient is written $3\frac{10}{33}$.

$$\begin{array}{r} 3\frac{10}{33} \\ 33\overline{)109} \\ \underline{99} \\ 10 \end{array}$$

### WRITTEN EXERCISE

| | | |
|---|---|---|
| 1. \$863.24 ÷ 321. | 2. \$326.34 ÷ 333. | 3. \$626 ÷ 25. |
| 4. \$260.02 ÷ 302. | 5. \$1000.01 ÷ 201. | 6. \$975 ÷ 18. |
| 7. \$923.58 ÷ 233. | 8. \$9000.90 ÷ 107. | 9. \$456 ÷ 33. |
| 10. \$425.30 ÷ 120. | 11. \$1235.40 ÷ 225. | 12. \$221 ÷ 16. |

ORAL EXERCISE

1. $16 \div 8$; $160 \div 80$; $16{,}000 \div 8000$.

2. $96 \div 12$; $9600 \div 1200$; $96{,}000 \div 12{,}000$.

3. $132 \div 11$; $13{,}200 \div 1100$; $132{,}000 \div 11{,}000$.

4. $200\overline{)8600}$; $200\overline{)8600 + 100}$; $200\overline{)8700}$; $200\overline{)8750}$.

5. Instead of dividing 17,280 by 90, we will get the same result by dividing what number by 9?

6. Instead of dividing 172,800 by 1200, we will get the same result by dividing what number by 12?

**46. Dividing by multiples of 10.** We have seen that just as 86 ft. $\div$ 2 ft. $= 86 \div 2$, so $8600 \div 200 = 86 \div 2$. In the same way, in dividing 8637 by 200 we have

$$200\overline{)8600 + 37} \qquad 43 \quad + \tfrac{37}{200}$$

or

$$2\text{ø}\text{ø}\overline{)86|37} \qquad 43 \ \tfrac{37}{200}$$

WRITTEN EXERCISE

1. $46{,}400 \div 800$.
2. $46{,}800 \div 900$.
3. $42{,}000 \div 3000$.
4. $160{,}000 \div 5000$.
5. $102{,}000 \div 3400$.
6. $294{,}000 \div 4200$.
7. $276{,}000 \div 7000$.
8. $814{,}200 \div 3000$.
9. $354{,}200 \div 75{,}000$.
10. $489{,}100 \div 63{,}000$.

11. At 260 mi. a day, how long would it take a steamer to go from San Francisco to Guam, 5200 miles?

12. At a certain temperature sound travels 1132 ft. a second. How long will it take it to travel 23,772 ft.?

13. In one year the receipts of the Chicago post office were \$7,713,800. How much did they average per day?

14. When the annual expenses of our government were \$479,250,000, how much were the average daily expenses?

### ORAL EXERCISE

**47. Rapid oral work.** In adding 26 and 45 begin with the tens, saying: 26, 66, 71. In subtracting follow the first plan on page 10, or in the case of $75 - 48$ think: $75 - 40 - 8 = 35 - 8 = 27$. In multiplying by a two-figure multiplier it is better to begin with the tens, as in 12 times 24, where we should think: $240 + 48 = 288$.

| | | |
|---|---|---|
| 1. $29 + 67$. | 2. $48 + 96$. | 3. $63 + 78$. |
| 4. $78 + 96$. | 5. $89 + 48$. | 6. $67 + 93$. |
| 7. $69 + 92$. | 8. $83 + 49$. | 9. $78 + 87$. |
| 10. $160 + 75$. | 11. $230 + 93$. | 12. $420 + 88$. |
| 13. $125 + 86$. | 14. $137 + 68$. | 15. $142 + 81$. |
| 16. $562 + 120$. | 17. $287 + 130$. | 18. $329 + 120$. |
| 19. $63 - 21$. | 20. $52 - 22$. | 21. $67 - 38$. |
| 22. $81 - 57$. | 23. $92 - 39$. | 24. $83 - 68$. |
| 25. $121 - 30$. | 26. $146 - 60$. | 27. $153 - 73$. |
| 28. $260 - 140$. | 29. $320 - 110$. | 30. $430 - 160$. |
| 31. $275 - 185$. | 32. $263 - 143$. | 33. $328 - 108$. |

*Multiply in Exs. 34–45:*

| | | |
|---|---|---|
| 34. 241 by 2. | 35. 323 by 3. | 36. 420 by 2. |
| 37. 210 by 3. | 38. 410 by 5. | 39. 320 by 4. |
| 40. 430 by 3. | 41. 530 by 3. | 42. 610 by 6. |
| 43. 810 by 9. | 44. 800 by 11. | 45. 500 by 12. |
| 46. $819 \div 9$. | 47. $784 \div 8$. | 48. $455 \div 7$. |
| 49. $715 \div 5$. | 50. $312 \div 4$. | 51. $273 \div 3$. |
| 52. $612 \div 12$. | 53. $781 \div 11$. | 54. $340 \div 20$. |
| 55. $720 \div 12$. | 56. $891 \div 11$. | 57. $620 \div 20$. |
| 58. $840 \div 40$. | 59. $990 \div 30$. | 60. $840 \div 60$. |

### ORAL EXERCISE

*Find the cost in each example:*

1. Tea @ 98¢, and coffee @ 49¢.
2. 4 yd. silk @ $1.20 a yard.
3. Flour @ 39¢, and cheese @ 28¢.
4. Biscuits @ 24¢, and cake @ 37¢.
5. A book @ 65¢, and a hat @ $1.75.

*Find the price per pound, ounce, or piece, in Exs. 6–11:*

6. 6 lb. of figs cost $1.08.
7. 9 cans of soup cost $1.17.
8. 20 oz. of olive oil cost 64¢.
9. 60 cakes of soap cost $2.70.
10. 50 lb. of dried apples cost $5.25.
11. 8 lb. of English walnuts cost $1.28.

### WRITTEN EXERCISE

1. $273 + 421 + 68 + 9026 + 34{,}379 + 75 + 286$.
2. $3972 + 8298 + 4000 + 17{,}625 + 31 + 9 + 407$.
3. $4269 + 193{,}481 + 17{,}286 + 81{,}298 + 4876 + 923$.
4. $876{,}481 - 298{,}327$.
5. $298{,}008 - 43{,}661$.
6. $173{,}209 - 106{,}492$.
7. $381{,}200 - 179{,}699$.
8. $400{,}100 - 199{,}298$.
9. $673{,}100 - 194{,}287$.
10. $65{,}664 \div 456$.
11. $77{,}112 \div 189$.
12. $221{,}336 \div 102$.
13. $328{,}320 \div 360$.
14. $7{,}605{,}920 \div 112$.
15. $7{,}913{,}576 \div 209$.
16. $628 \times 325$.
17. $872 \times 896$.
18. $469 \times 832$.
19. $1268 \times 3472$.
20. $2081 \times 6235$.
21. $4287 \times 3009$.

## Our Country's Food Supply

**48. Wheat.** This great food product brings to our farmers three or four hundred million dollars a year.

### WRITTEN EXERCISE

1. If 43,525,000 acres of wheat in this country produce on an average 13 bu. to the acre, what is the total yield?

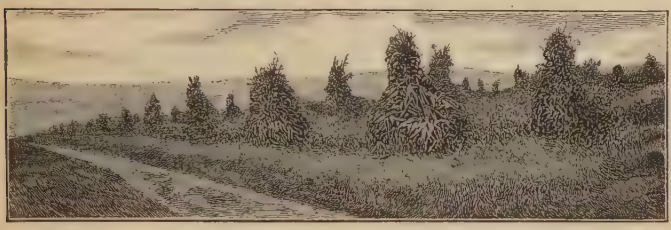
2. If Kansas produces 82,368,000 bu. a year, which is 12,000 times as much as Connecticut, how much does Connecticut produce?

3. If the wheat crop of the world is 2,586,000,000 bu. a year, and if the United States produces $\frac{1}{5}$ of it, how many bushels does it produce?

4. If the United States produces 547,303,000 bu. of wheat in one year, worth 70¢ a bushel, what is the total value of the crop?

5. If the value of the Minnesota yield of wheat averages $32,560,000 a year, and this is 4400 times that of New Hampshire, what is the value of the yield of the latter?

**49. Corn.** Another of the great food products of the country is corn. The greatest corn-producing states are Iowa, Illinois, Nebraska, Missouri, Kansas, and Indiana.



WRITTEN EXERCISE

1. When this country produced 2,105,102,400 bu. of corn a year, averaging 25 bu. to the acre, how many acres had we in corn?

2. If 3 bu. of corn could then be bought for $1, what was the total value of this yield of 2,105,102,400 bu.?

3. When Iowa's annual product amounted to 305,800,000 bu., this was how many times the 440,000 bu. produced by Maine?

4. To transport 1000 lb. of corn from St. Louis to New Orleans by river costs $1. How much will it cost to transport 1750 tons?

5. If the average value of corn for each of the 46,610 acres given to it in Connecticut in a certain year was $21, and for each of the 4,031,600 acres in Indiana $13, what was the entire value of the corn crop of each state?

6. If the average annual corn crop per acre is 40 bu. in Wisconsin, 36 bu. in Maine, 37 bu. in New Hampshire, 38 bu. in Massachusetts, 38 bu. in Indiana, and 38 bu. in Iowa, find the average by adding and dividing by 6.

**50. Cattle.** The greatest beef-producing states are Texas, Iowa, and Kansas. The greatest stock yards and packing houses in the world are in Chicago.

A Texas Cattle Ranch

**WRITTEN EXERCISE**

1. If Texas has 4,533,897 head of cattle, valued at $17 a head, how much are they all worth?

2. If Kansas has 2,076,489 head of cattle, valued at $27 a head, how much are they all worth?

3. If Iowa has 2,163,584 head of cattle, valued at $31 a head, how much are they all worth?

4. If this country sold 55,553,750 lb. of canned beef in a certain year, at 9 ct. a pound, what was the total value?

5. In a year when the returns showed 69,335 cattle in a certain county, their value was estimated at $1,525,370. What was the average value per head?

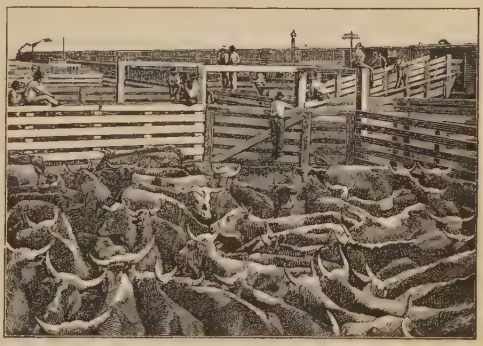
6. In a year when the returns showed 18,108 cows kept for milk in a certain county, their value was estimated at $543,240. What was the average value per head?

7. The number of cattle being raised for food in the United States in 1880 was 21,231,000, and our population was 50,155,783. In 1900 the number of cattle was 27,610,054, and our population was 76,303,387. What was the increase in cattle and in population in the 20 years?

**51. The stock yards.** The Chicago stock yards are the center of the great beef industry. From the packing houses the meat is shipped in refrigerator cars, and goes to various parts of the world.

### WRITTEN EXERCISE

1. In one year the receipts from the beef industry of the country were \$800,000,000, of which $\frac{1}{16}$ was profit. How much was profit?



2. If, out of every 100 lb. of an animal's weight, 56 lb. is salable as meat, how much salable meat is there in a 1150-lb. animal? What is it worth at \$8.00 a hundred, adding \$1.50 for preparing the meat for shipment?

3. If 4,489,500 cattle were shipped to some stock yards in one year, what was the average per day?

4. An eastern dealer buys 10,000 lb. of beef a month, at 14 ct. a pound, and sells it on an average of 19 ct. a pound. How much are his profits a month?

5. In 50 years the value of the beef produced annually in this country rose from \$11,981,642 to \$785,562,442. How much was the average annual increase in value?

6. If 700 lb. of meat are sold by a packing house for \$63, and it costs \$1 a hundred to ship it to New York, how much does it cost a pound delivered in New York?

## FACTORS AND MULTIPLES

### ORAL EXERCISE

1. What do you mean by the factors of a number?

2. Name two factors of 35; of 77; of 26; of 34.

3. Name three factors of 30; of 70; of 66; of 110.

4. Name a factor that is common to both 35 and 45.

5. Name two factors common to both 30 and 48.

6. In reducing $\frac{2}{4}$ to $\frac{1}{2}$ we found in primary arithmetic that we could divide both terms by 2, or *cancel* the 2. What factor do we cancel in reducing $\frac{6}{8}$ to its *lowest terms*, $\frac{3}{4}$?

7. Name the factors to be canceled in reducing to lowest terms $\frac{4}{8}$, $\frac{2}{8}$, $\frac{4}{10}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{15}{20}$. What need have we found for factors in arithmetic?

**52. Common divisor.** A factor that is common to two or more numbers is called a *common divisor.*

For example, 5 is a common divisor of 35 and 45, as we found in Ex. 4. It might be called a common factor, but common divisor is the name usually given it.

**53. Greatest common divisor.** The greatest factor common to two or more numbers is called their *greatest common divisor* (g.c.d.).

For example, although 3 is a common divisor of 12, 18, and 48, the number 6 is the *greatest* common divisor.

**54. Finding the g.c.d.** The greatest common divisor of such numbers as we shall meet in fractions is easily found.

For example, consider 12 and 30. We see that

$$12 = 2 \times 3 \times 2, \qquad 30 = 2 \times 3 \times 5,$$

and that 2 and 3 are the only common factors. Therefore $2 \times 3$, or 6, is the greatest common divisor.

### ORAL EXERCISE

1. Name a common divisor of 20 and 22; 16 and 36.

2. Name the greatest common divisor of 75 and 100.

3. Which of these numbers have no factors excepting one and themselves: 4, 5, 6, 7, 10, 11, 12, 13?

4. In reducing $\frac{12}{16}$ to lowest terms is it sufficient to cancel the common divisor 2? What kind of common divisor must be canceled in reducing a fraction to lowest terms?

**55. Prime number.** A number that has no factors except itself and 1 is called a *prime number.*

For example, 7, 11, 17 are prime numbers.

**56. Composite number.** A number not prime is called *composite.*

**57. Even numbers.** A number which contains the factor 2 is called an *even number.*

**58. Odd numbers.** Numbers that are not even are called *odd numbers.*

**59. Prime factors.** Factors that are prime numbers are called *prime factors.*

For example, the prime factors of 68 are 2, 2, and 17.

**60. Mutually prime numbers.** Numbers that have no common factors are said to be *prime to each other.*

For example, 12 and 35 are prime to each other.

### WRITTEN EXERCISE

1. Make a list of prime numbers to 100; of odd numbers.

2. Find the prime factors of 54; of 48; of 64; of 77.

3. Find the greatest common divisor of 96, 60, and 36.

### ORAL EXERCISE

1. Is 2 a factor of 14? of 27? of 50? of 45? of 600? How do you tell whether 2 is a factor of a number?

2. Is 2 a factor of 10? of any number of 10's? Then is it a factor of any number if it is a factor of the units?

3. Is 5 a factor of 20? of 78? of 45? of 72? of 800? How do you tell whether 5 is a factor of a number?

4. Is 5 a factor of 10? of any number of 10's? Then is it a factor of any number if it is a factor of the units?

**61. Divisible numbers.** When we speak of one number being divisible by another we mean exactly divisible.

**62. Divisibility by 2.** *A number is divisible by 2 if the units are so divisible.*

For example, 64, or $60 + 4$, must be divisible by 2 if 4 is so divisible, because 60 is divisible by 2.

**63. Divisibility by 5.** *A number is divisible by 5 if it ends in 0 or 5.*

**64. Divisibility by 3.** *A number is divisible by 3 if the sum of its digits is so divisible.*

For example, 411 is divisible by 3 because $4 + 1 + 1$ is.

### WRITTEN EXERCISE

1. Which of these are divisible by 2: 660, 4907, 6255, 3027, 1356, 5790, 2371, 4196, 2005, 37,268, 125,474?

2. Which of these are divisible by 5: 660, 7620, 4867, 7075, 3200, 4035, 9636, 8124, 3672, 12,475, 374,465?

3. Which of these are divisible by 3: 660, 1236, 5778, 9102, 8328, 3444, 1239, 9876, 4004, 7117, 31,476, 307,983, 206,340, 475,633, 2,124,816, 4,216,872, 15,681,123?

### ORAL EXERCISE

1. State the prime factors of 15, 18, 27, 35, 42, 75.
2. State one factor of 395, 123, 777, 692, 1275, 1263.
3. Which of these are divisible by 3: 77, 609, 1203?
4. Which of these are divisible by 2: 68, 4973, 2870?

**65. Other tests.** There are various tests of divisibility besides those on page 32. They are easily illustrated or explained, and may be given or not as the teacher prefers. The more important are the following.

**66. Divisibility by 4.** A number is divisible by 4 if the number represented by the two right-hand figures is so divisible.

**67. Divisibility by 6.** A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3.

**68. Divisibility by 8.** A number is divisible by 8 if the number represented by the three right-hand figures is so divisible.

**69. Divisibility by 9.** A number is divisible by 9 if the sum of its digits is so divisible.

**70. Divisibility by 11.** A number is divisible by 11 if the difference between the sums of the digits in its even and odd places is so divisible.

For example, 430,507 is divisible by 11, for $7 + 5 + 3 = 15$, and $0 + 0 + 4 = 4$, and $15 - 4 = 11$.

**71. Finding prime factors.** Find the prime factors of 2310.

$$\begin{array}{r} 2\overline{)2310} \\ 5\overline{)1155} \\ 3\overline{)231} \\ 7\overline{)77} \\ 11 \end{array}$$

By § 62, 2 is a factor. By § 63, 5 is a factor of the other factor, 1155. By § 64, 3 is a factor of the other factor, 231. It is easy to see that 7 and 11 are factors of 77. Hence the prime factors are 2, 5, 3, 7, 11.

**72. Finding the g.c.d.** In the same way it is easy to find the greatest common divisor of numbers.

For example, to find the greatest common divisor of 231 and 660. Factoring 660 and 231 as here shown, we have

$$231 = 3 \times 7 \times 11,$$

and

$$660 = 2 \times 2 \times 3 \times 5 \times 11,$$

in which 3 and 11 are the only common factors.

Therefore $3 \times 11$, or 33, is the greatest common divisor.

$$\begin{array}{r|l} 3 & 231 \\ 7 & 77 \\ 11 & \end{array} \qquad \begin{array}{r|l} 2 & 660 \\ 2 & 330 \\ 3 & 165 \\ 5 & 55 \\ & 11 \end{array}$$

WRITTEN EXERCISE

*Find the prime factors of the numbers in Exs. 1–30:*

| | | | | |
|---|---|---|---|---|
| 1. 775. | 2. 420. | 3. 385. | 4. 352. | 5. 320. |
| 6. 891. | 7. 147. | 8. 297. | 9. 176. | 10. 336. |
| 11. 130. | 12. 462. | 13. 390. | 14. 285. | 15. 375. |
| 16. 8910. | 17. 1250. | 18. 2430. | 19. 1280. | 20. 1024. |
| 21. 2970. | 22. 1375. | 23. 5390. | 24. 1568. | 25. 1331. |
| 26. 6300. | 27. 1875. | 28. 1215. | 29. 3645. | 30. 10,935. |

*Find the greatest common divisor in Exs. 31–48:*

| | |
|---|---|
| 31. 147 and 350. | 32. 128, 352, and 160. |
| 33. 130 and 195. | 34. 125, 175, and 275. |
| 35. 444 and 888. | 36. 165, 385, and 220. |
| 37. 720 and 1728. | 38. 1024, 1280, and 1792. |
| 39. 1536 and 3584. | 40. 1001, 5390, and 7700. |
| 41. 3125 and 1000. | 42. 1650, 1155, and 1485. |
| 43. 1440 and 1728. | 44. 1250, 1875, and 2500. |
| 45. 1875 and 2500. | 46. 1010, 2121, and 2727. |
| 47. 1275 and 1775. | 48. 2575, 2060, and 9270. |

### ORAL EXERCISE

1. Name two multiples of 5; of 7; of 9; of 11.

2. Name a multiple that is common to 3 and 5.

3. The multiples 35 and 70 are common to 5 and 7. Which is the least multiple that is common to 5 and 7?

4. In the same way name the least common multiple of 2 and 3, and then name two other multiples.

**73. Common multiple.** A multiple of each of two or more numbers is called a *common multiple* of the numbers.

**74. Least common multiple.** Of all the common multiples of two or more numbers the least is called the *least common multiple* (l.c.m.).

For example, 24 is a common multiple of 4 and 6, but 12 is the least common multiple.

**75. Finding the l.c.m.** The least common multiple of two numbers is easily found if their factors are known.

Thus, to find the least common multiple of 16 and 40. We see that $16 = 2 \times 2 \times 2 \times 2$, and $40 = 2 \times 2 \times 2 \times 5$.

To be a multiple of 16, the least common multiple must contain $2 \times 2 \times 2 \times 2$, and to contain 40 it must also contain 5. Therefore $2 \times 2 \times 2 \times 2 \times 5$, or 80, is the least common multiple.

### WRITTEN EXERCISE

*Find the least common multiple of:*

| | | |
|---|---|---|
| 1. 30 and 70. | 2. 77 and 110. | 3. 28 and 32. |
| 4. 64 and 96. | 5. 9, 12, and 50. | 6. 3, 8, and 96. |
| 7. 5, 15, and 75. | 8. 6, 33, and 96. | 9. 10, 25, and 50. |
| 10. 8, 16, and 64. | 11. 4, 63, and 135. | 12. 16, 72, and 96. |
| 13. 7, 15, and 51. | 14. 27, 54, and 84. | 15. 72, 66, and 111. |

## COMMON FRACTIONS

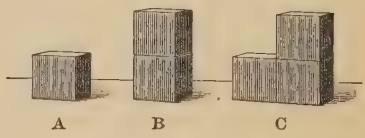
### ORAL EXERCISE

1. When anything is divided into 3 equal parts, what is each part called? into 5 equal parts? into 12 equal parts?

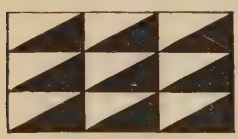
2. What part of A is shaded? of B? of C? From A we see that $\frac{1}{2}$ equals how many sixths? from B, that $\frac{1}{3}$ equals how many sixths?

3. How many sixths of the rectangle make the whole rectangle? How many thirds? How many halves?

4. Cube A is what part as large as B? as C? B is what part as large as C?




5. If C is called 1, what is A? B? If B is called 1, what is A? C? If A is called 1, what is B? C?



6. Each small rectangle is what part of the whole rectangle? Each black triangle is what part of the whole rectangle? What does this tell you about $\frac{1}{2}$ of $\frac{1}{3}$?

7. From the picture show that $\frac{3}{18}$ of the rectangle is $\frac{1}{2}$ of $\frac{1}{3}$ of it; also that $\frac{3}{18} = \frac{1}{6}$; also that $\frac{6}{18} = \frac{1}{3}$; also that $\frac{9}{18} = \frac{1}{2}$; also that $\frac{10}{18} = \frac{5}{9}$; also that $\frac{12}{18} = \frac{2}{3}$.

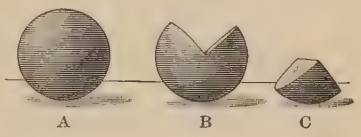
8. Name other fractions to which the following are equal: $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{9}$, $\frac{7}{21}$, $\frac{5}{25}$, $\frac{8}{40}$, $\frac{16}{24}$, $\frac{14}{21}$, $\frac{15}{25}$, $\frac{18}{27}$, $\frac{12}{28}$.

**76. Unit.** Any one thing is called a *unit*.

**77. Fraction.** One or more of the equal parts of a unit is called a *fraction*.

**78. Denominator.** The number which shows into how many equal parts a unit has been divided is called the *denominator.*

For example, a sphere has been divided into 4 equal parts, and B is $\frac{3}{4}$ of it. Then 4 is the denominator.




We read $\frac{1}{3}$ ft. "one third *of a* foot."

**79. Numerator.** The number which shows how many parts have been taken to make a fraction is called the *numerator.*

In the fraction $\frac{3}{4}$, 3 shows the number of fourths that have been taken, and is, therefore, the numerator.

**80. Terms.** The numerator and denominator together are called the *terms of the fraction.*

**81. Common fraction.** A fraction which has both terms expressed is called a *common fraction.*

For example, $\frac{2}{3}$, $\frac{5}{5}$, $\frac{7}{6}$, $\frac{6}{10}$, but not a decimal fraction like 0.6.

**82. Proper fraction.** A fraction whose numerator is less than the denominator is called a *proper fraction.*

For example, $\frac{2}{3}$, $\frac{1}{27}$, $\frac{3}{100}$, $\frac{1}{275}$.

**83. Unit fraction.** If the numerator is 1, the fraction is called a *unit fraction.*

For example, C is $\frac{1}{4}$ of a sphere.

**84. Improper fraction.** A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

For example, A is $\frac{4}{4}$ of a sphere, and A + C is $\frac{5}{4}$.

**85. Mixed number.** The sum of a whole number and a fraction is called a *mixed number.*

For example, if sphere A is 1, A + B is $1\frac{3}{4}$, A + C is $1\frac{1}{4}$.

### ORAL EXERCISE

*Read aloud the following:*

1. $\frac{2}{13}$. 2. $\frac{5}{16}$. 3. $\frac{17}{32}$ in. 4. $\frac{9}{20}$. 5. $\frac{7}{160}$ A.
6. $\frac{7}{32}$. 7. $\frac{7}{12}$. 8. $\frac{19}{320}$ mi. 9. $\frac{4}{33}$ rd. 10. $\frac{7}{144}$ in.
11. $2\frac{4}{5}$. 12. $3\frac{7}{8}$. 13. $9\frac{2}{3}$. 14. $8\frac{3}{16}$. 15. $25\frac{2}{3}$ in.
16. $\$12\frac{1}{2}$. 17. $33\frac{1}{3}$. 18. $15\frac{4}{21}$. 19. $9\frac{3}{32}$. 20. $157\frac{7}{8}$.
21. A cubic inch is $\frac{1}{1728}$ cu. ft., or $\frac{1}{46656}$ cu. yd.
22. An inch is $\frac{1}{12}$ ft., $\frac{1}{36}$ yd., $\frac{1}{198}$ rd., or $\frac{1}{63360}$ mi.
23. An ounce is $\frac{1}{16}$ lb., or $\frac{1}{32000}$ T., and 1 lb. is $\frac{1}{2000}$ T.

### WRITTEN EXERCISE

*Write the fractions or mixed numbers in Exs. 1–10:*

1. Sixty eighty-firsts.
2. Twenty-seven fiftieths.
3. Thirty-five fortieths.
4. Seventeen eighteenths.
5. Nineteen thousandths.
6. Seventy-seven sixtieths.
7. Eight two-hundredths.
8. Five one-hundred-tenths.
9. Two hundred twenty-seven one-hundred-sixteenths.
10. Five hundred one two-thousand-eight-hundred-sevenths.

*Write in words the following:*

11. $\frac{5}{7}$, $\frac{7}{8}$, $\frac{6}{11}$, $\frac{5}{19}$.
12. $\frac{7}{10}$, $\frac{3}{100}$, $\frac{5}{1000}$, $\frac{201}{101}$.
13. $\frac{15}{17}$, $\frac{27}{39}$, $\frac{41}{61}$, $\frac{75}{91}$.
14. $\frac{8}{15}$, $\frac{121}{230}$, $\frac{625}{144}$, $\frac{729}{2000}$.
15. $1\frac{2}{3}$, $25\frac{4}{5}$, $62\frac{5}{8}$, $93\frac{1}{3}$.
16. $1\frac{9}{11}$, $66\frac{2}{3}$, $125\frac{5}{8}$, $137\frac{7}{8}$.
17. $2\frac{3}{7}$, $13\frac{5}{27}$, $68\frac{19}{21}$, $71\frac{13}{17}$.
18. $4\frac{6}{19}$, $73\frac{2}{11}$, $234\frac{5}{18}$, $341\frac{9}{16}$.
19. $\$125.12\frac{1}{2}$, $\$6.66\frac{2}{3}$.
20. $\$333.33\frac{1}{3}$, $\$275.16\frac{2}{3}$.
21. $\$135.87\frac{1}{2}$, $\$14.37\frac{1}{2}$.
22. $\$112.16\frac{7}{8}$, $\$235.45\frac{1}{3}$.

In practical business, $\frac{1}{2}$ ct. and over, in the final result, is called 1 ct.; less than $\frac{1}{2}$ ct. is neglected in most cases.

## Reduction of Fractions

### ORAL EXERCISE

1. If A is 1, B shows there are how many halves in 1?

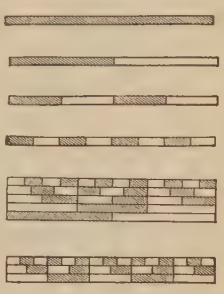
2. C shows how many fourths in 1? in $\frac{1}{2}$?

3. D shows that $\frac{1}{4}$ equals how many eighths? $\frac{1}{2} = \frac{?}{8}$.

4. E shows that $\frac{1}{3} = \frac{?}{6}$. $\frac{2}{3} = \frac{?}{6}$. $\frac{1}{3} = \frac{?}{9}$. $\frac{2}{3} = \frac{?}{9}$. $\frac{3}{6} = \frac{?}{2}$.

5. In E point to the twelfths. Tell how many twelfths equal a certain number of ninths, sixths, thirds, and halves.

6. In F tell as you did in E the relations between the fifths and tenths; the fifths and fifteenths.




**86. Multiplying or dividing terms.** We have found, as in primary arithmetic, that

*Multiplying or dividing both terms of a fraction by the same number does not change its value.*

For example, $\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$, and $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$.

**87. Canceling.** Dividing both terms of a fraction by the same factor is called *canceling* the factor.

**88. Reduction.** Changing the form of a fraction without changing the value is called *reducing the fraction.*

### WRITTEN EXERCISE

*Draw pictures like those above to show the following:*

1. $\frac{1}{7} = \frac{2}{14}$. 2. $\frac{8}{16} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$. 3. $\frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{24}{32}$.

### ORAL EXERCISE

**1.** From the picture $\frac{1}{2}$ equals how many eighths? how many fourths?

**89. Reduction to higher terms.** A fraction is said to be *reduced to higher terms* when the terms are made larger.

**2.** From the picture $\frac{6}{8}$ equals how many fourths?

**90. Reduction to lower terms.** A fraction is said to be *reduced to lower terms* when the terms are made smaller.

**91.** Reduce $\frac{30}{45}$ to lower terms.

By § 86 we may divide both terms by any number without changing the value of the fraction. By § 63, 5 is a factor, and, by § 64, 3 is a factor. Dividing by 5 and 3, $\frac{30}{45} = \frac{2}{3}$.

**92.** Reduce $\frac{3}{7}$ to twenty-firsts.

By § 86 we may multiply both terms by any number without changing the value of the fraction. Since $21 \div 7 = 3$, we multiply both terms by 3, and $\frac{3}{7} = \frac{9}{21}$.

### WRITTEN EXERCISE

1. Reduce to lower terms: $\frac{6}{16}$, $\frac{10}{22}$, $\frac{12}{14}$, $\frac{16}{32}$, $\frac{18}{27}$, $\frac{48}{60}$.
2. Reduce to fifteenths: $\frac{1}{5}$, $\frac{2}{3}$, $\frac{4}{30}$, $\frac{27}{45}$, $\frac{16}{60}$, $\frac{30}{75}$, $\frac{42}{30}$, $\frac{21}{105}$.
3. Reduce to sixteenths: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{10}{32}$, $\frac{28}{64}$, $\frac{48}{96}$, $\frac{30}{160}$, $\frac{25}{80}$.
4. Reduce to fiftieths: $\frac{1}{2}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{7}{25}$, $\frac{4}{100}$, $\frac{27}{150}$, $\frac{36}{200}$, $\frac{35}{250}$.
5. Reduce to thirty-seconds: $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{66}{64}$, $\frac{42}{96}$, $\frac{45}{160}$, $\frac{54}{192}$.
6. Reduce to hundredths: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{7}{20}$, $\frac{15}{20}$, $\frac{4}{25}$, $\frac{30}{200}$, $\frac{402}{600}$.
7. Reduce to one-hundred-forty-fourths: $\frac{7}{12}$, $\frac{2}{3}$, $\frac{16}{288}$, $\frac{11}{12}$.
8. Reduce $\frac{7}{8}$ and $\frac{7}{12}$ to fractions having the denominator 24; the denominator 72; the denominator 216.

## Integers and Mixed Numbers to Improper Fractions

### ORAL EXERCISE

1. How many half hours in an hour? in 3 hours?
2. How many quarter hours in an hour? in 2 hours?
3. How many halves in 1? in 3? How many fourths?
4. How many quarters in \$1? in \$$1\frac{1}{2}$? in \$$2\frac{1}{4}$? in \$$3\frac{3}{4}$?
5. Express as fourths: 1, $1\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{3}{4}$, 5, 7, $8\frac{1}{4}$, 10, 50, 80.
6. Express as halves: 1, $2\frac{1}{2}$, 3, 4, $4\frac{1}{2}$, 6, 15, $37\frac{1}{2}$, 70, 100.

**93. Reduction to an improper fraction.** Reduce $5\frac{7}{8}$ to an improper fraction.

Since $1 = \frac{8}{8}$, $5 = \frac{5 \times 8}{8} = \frac{40}{8}$; $\frac{40}{8} + \frac{7}{8} = \frac{47}{8}$.

7. How do you reduce a mixed number to an improper fraction?

### WRITTEN EXERCISE

*Reduce as directed in Exs. 1–9:*

1. 7 to sixths.
2. 9 to sevenths.
3. 6 to tenths.
4. 8 to elevenths.
5. 11 to sixths.
6. 13 to fourths.
7. 9 to halves.
8. 9 to twentieths.
9. 15 to thirds.

*Reduce to improper fractions:*

10. $5\frac{2}{3}$.
11. $6\frac{7}{8}$.
12. $7\frac{2}{3}$.
13. $8\frac{9}{10}$.
14. $6\frac{2}{3}$.
15. $3\frac{7}{8}$.
16. $4\frac{8}{9}$.
17. $6\frac{4}{9}$.
18. $9\frac{3}{16}$.
19. $7\frac{4}{11}$.
20. $8\frac{5}{8}$.
21. $6\frac{3}{7}$.
22. $8\frac{4}{15}$.
23. $7\frac{3}{14}$.
24. $8\frac{5}{12}$.
25. $16\frac{2}{3}$.
26. $12\frac{1}{3}$.
27. $33\frac{1}{3}$.
28. $66\frac{2}{3}$.
29. $2\frac{13}{15}$.
30. $333\frac{1}{3}$.
31. $166\frac{2}{3}$.
32. $666\frac{2}{3}$.
33. $125\frac{1}{2}$.
34. $43\frac{27}{32}$.
35. $146\frac{3}{8}$.
36. $235\frac{5}{9}$.
37. $825\frac{6}{7}$.
38. $999\frac{1}{9}$.
39. $675\frac{3}{5}$.

## Improper Fractions to Integers or Mixed Numbers

### ORAL EXERCISE

1. Express $2\frac{1}{2}$ as halves; $\frac{5}{2}$ as a mixed number.

2. In \$5 how many half dollars? Express 5 as halves. Express $\frac{10}{2}$ as an integer; also $\frac{15}{5}$, $\frac{20}{4}$, $\frac{30}{3}$, $\frac{40}{10}$.

3. Express as integers: $\frac{12}{6}$, $\frac{15}{3}$, $\frac{16}{2}$, $\frac{81}{9}$, $\frac{25}{5}$, $\frac{125}{25}$.

4. Express as mixed numbers: $\frac{15}{7}$, $\frac{17}{8}$, $\frac{20}{3}$, $\frac{32}{10}$, $\frac{37}{7}$, $\frac{73}{12}$.

**94. Reduction to an integer or a mixed number.** Reduce $\frac{124}{5}$ to an integer or a mixed number.

Since $\frac{1}{5}$ means that 1 has been divided into 5 equal parts, $\frac{124}{5}$ means that $124 \times 1$ has been divided into 5 equal parts. That is, $\frac{124}{5}$ means the same as $124 \div 5$.

Therefore $\frac{124}{5} = 124 \div 5 = 24\frac{4}{5}$.

**95.** Therefore *a fraction is an indicated division.*

*To reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.*

### WRITTEN EXERCISE

*Reduce to integers or mixed numbers:*

| | | | | |
|---|---|---|---|---|
| 1. $\frac{33}{3}$. | 2. $\frac{42}{7}$. | 3. $\frac{65}{6}$. | 4. $\frac{29}{3}$. | 5. $\frac{81}{8}$. |
| 6. $\frac{47}{6}$. | 7. $\frac{78}{5}$. | 8. $\frac{69}{7}$. | 9. $\frac{87}{11}$. | 10. $\frac{92}{12}$. |
| 11. $\frac{125}{25}$. | 12. $\frac{236}{33}$. | 13. $\frac{425}{30}$. | 14. $\frac{93}{92}$. | 15. $\frac{76}{76}$. |
| 16. $\frac{298}{16}$. | 17. $\frac{349}{32}$. | 18. $\frac{276}{18}$. | 19. $\frac{198}{16}$. | 20. $\frac{725}{42}$. |
| 21. $\frac{632}{32}$. | 22. $\frac{427}{16}$. | 23. $\frac{629}{24}$. | 24. $\frac{332}{64}$. | 25. $\frac{342}{12}$. |
| 26. $\frac{190}{95}$. | 27. $\frac{625}{50}$. | 28. $\frac{221}{11}$. | 29. $\frac{625}{25}$. | 30. $\frac{896}{128}$. |
| 31. $\frac{127}{25}$. | 32. $\frac{175}{119}$. | 33. $\frac{113}{104}$. | 34. $\frac{125}{75}$. | 35. $\frac{441}{21}$. |
| 36. $\frac{265}{53}$. | 37. $\frac{1221}{121}$. | 38. $\frac{1001}{77}$. | 39. $\frac{2002}{11}$. | 40. $\frac{1728}{144}$. |

### ORAL EXERCISE

1. Reduce $\frac{48}{80}$ to fortieths. Reduce the result to twentieths. Continue reducing as far as possible.

2. Reduce $\frac{42}{63}$ to twenty-firsts. Reduce the result to thirds. Can you reduce to any lower terms?

3. Write the fraction $\frac{144}{216}$ on the blackboard. Reduce it to as low terms as possible.

**96. Lowest terms.** A fraction is reduced to its *lowest terms* when the terms are prime to each other (§ 60).

For example, $\frac{44}{48} = \frac{22}{24} = \frac{11}{12}$, and $\frac{44}{48}$ is reduced to its lowest terms when reduced to $\frac{11}{12}$, for 11 and 12 are prime to each other (§ 60).

**97. Reduction to lowest terms.** *To reduce a fraction to lowest terms, cancel all common factors from both terms.*

We might cancel the greatest common divisor, but to find this we usually factor the terms, and it is easier to cancel the factors as we find them. Cancel as large factors as possible.

**98.** Practically we do not meet fractions whose terms are large numbers, like $\frac{3456}{5904}$, decimal fractions having taken their place. Even such easy fractions as are given below are rarely met.

### WRITTEN EXERCISE

*Reduce to lowest terms:*

| | | | | |
|---|---|---|---|---|
| 1. $\frac{28}{77}$. | 2. $\frac{35}{45}$. | 3. $\frac{15}{55}$. | 4. $\frac{24}{32}$. | 5. $\frac{56}{63}$. |
| 6. $\frac{63}{77}$. | 7. $\frac{45}{55}$. | 8. $\frac{26}{39}$. | 9. $\frac{56}{91}$. | 10. $\frac{66}{78}$. |
| 11. $\frac{45}{75}$. | 12. $\frac{21}{91}$. | 13. $\frac{51}{93}$. | 14. $\frac{27}{57}$. | 15. $\frac{63}{69}$. |
| 16. $\frac{85}{95}$. | 17. $\frac{44}{52}$. | 18. $\frac{69}{93}$. | 19. $\frac{40}{56}$. | 20. $\frac{60}{65}$. |
| 21. $\frac{72}{104}$. | 22. $\frac{60}{105}$. | 23. $\frac{72}{144}$. | 24. $\frac{111}{123}$. | 25. $\frac{65}{70}$. |
| 26. $\frac{12}{144}$. | 27. $\frac{75}{125}$. | 28. $\frac{35}{630}$. | 29. $\frac{130}{195}$. | 30. $\frac{147}{350}$. |
| 31. $\frac{125}{175}$. | 32. $\frac{1440}{1728}$. | 33. $\frac{1875}{2500}$. | 34. $\frac{385}{700}$. | 35. $\frac{231}{308}$. |

## Least Common Denominator

### ORAL EXERCISE

1. Express $\frac{3}{4}$ as eighths. Add $\frac{3}{4}$ and $\frac{1}{8}$; $\frac{3}{4}$ and $\frac{7}{8}$.

2. In adding $\frac{3}{4}$ and $\frac{1}{8}$ we first reduce $\frac{3}{4}$ to a fraction with what denominator?

3. In subtracting $\frac{3}{8}$ from $\frac{1}{2}$ we first reduce $\frac{1}{2}$ to a fraction with what denominator?

4. Before we add or subtract fractions, what must be the nature of the denominators?

*Express as fractions with the same denominator:*

5. $\frac{1}{2}$, $\frac{1}{4}$. 6. $\frac{1}{4}$, $\frac{1}{8}$. 7. $\frac{1}{2}$, $\frac{1}{8}$. 8. $\frac{1}{2}$, $\frac{5}{8}$.

**99. Similar fractions.** If several fractions have the same denominator, they are said to be *similar fractions* and to have a *common denominator.*

**100. Least common denominator.** If the common denominator is the smallest possible, the fractions are said to have the *least common denominator* (l.c.d.).

For example, $\frac{4}{8}$ and $\frac{3}{8}$ have a common denominator, but they may be reduced to $\frac{2}{4}$ and $\frac{1}{4}$, 4 being the least common denominator.

**101. Reduction to l.c.d.** To reduce $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$ to fractions having the least common denominator.

Since the required denominator must contain 4, 8, and 12, it must be their least common multiple.

We see that 4, 8, and 12 all contain 4, and that 8 also contains 2, and 12 contains 3. Therefore the least common multiple is $4 \times 2 \times 3$, or 24.

$$4)\underline{4,\ 8,\ 12}$$
$$1,\ 2,\ 3$$

Since $24 \div 4 = 6$, the terms of $\frac{1}{4}$ must be multiplied by 6. In the same way the other multipliers are found.

$$\frac{6 \times 1}{6 \times 4} = \frac{6}{24}$$
$$\frac{3 \times 3}{3 \times 8} = \frac{9}{24}$$
$$\frac{2 \times 5}{2 \times 12} = \frac{10}{24}$$

### ORAL EXERCISE

*Reduce to similar fractions, in Exs. 1–12:*

1. $\frac{2}{3}, \frac{1}{2}$.
2. $\frac{3}{5}, \frac{2}{3}$.
3. $\frac{4}{16}, \frac{2}{8}$.
4. $\frac{2}{5}, \frac{7}{10}$.
5. $\frac{5}{15}, \frac{2}{30}$.
6. $\frac{1}{7}, \frac{2}{14}$.
7. $\frac{3}{5}, \frac{4}{10}$.
8. $\frac{3}{8}, \frac{3}{4}$.
9. $\frac{10}{20}, \frac{15}{30}$.
10. $\frac{6}{8}, \frac{9}{10}$.
11. $\frac{2}{3}, \frac{3}{30}$.
12. $\frac{5}{7}, \frac{6}{14}$.

*Reduce to fractions having the least common denominator:*

13. $\frac{1}{2}, \frac{2}{3}$.
14. $\frac{3}{5}, \frac{4}{7}$.
15. $\frac{4}{10}, \frac{3}{5}$.
16. $\frac{3}{7}, \frac{2}{3}$.
17. $\frac{7}{8}, \frac{1}{16}$.
18. $\frac{3}{16}, \frac{5}{32}$.
19. $\frac{7}{12}, \frac{1}{4}$.
20. $\frac{2}{7}, \frac{4}{5}$.
21. $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$.
22. $\frac{2}{9}, \frac{2}{3}, \frac{5}{6}$.
23. $\frac{3}{8}, \frac{3}{5}, \frac{3}{10}$.
24. $\frac{7}{8}, \frac{1}{5}, \frac{1}{40}$.

### WRITTEN EXERCISE

*Reduce to fractions having the least common denominator, in Exs. 1–30:*

1. $\frac{3}{25}, \frac{6}{35}$.
2. $\frac{2}{49}, \frac{5}{63}$.
3. $\frac{8}{27}, \frac{5}{36}$.
4. $\frac{6}{25}, \frac{7}{55}$.
5. $\frac{3}{62}, \frac{4}{93}$.
6. $\frac{5}{36}, \frac{7}{72}$.
7. $\frac{8}{121}, \frac{13}{143}$.
8. $\frac{2}{7}, \frac{8}{9}, \frac{3}{8}$.
9. $\frac{7}{150}, \frac{11}{180}$.
10. $\frac{4}{625}, \frac{6}{125}$.
11. $\frac{12}{144}, \frac{11}{132}$.
12. $\frac{3}{130}, \frac{4}{143}$.
13. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.
14. $\frac{5}{222}, \frac{11}{420}$.
15. $\frac{5}{8}, \frac{3}{16}, \frac{4}{32}$.
16. $\frac{3}{25}, \frac{4}{15}, \frac{3}{10}$.
17. $\frac{4}{9}, \frac{8}{15}, \frac{6}{25}$.
18. $\frac{5}{32}, \frac{3}{16}, \frac{7}{8}$.
19. $\frac{3}{20}, \frac{7}{30}, \frac{2}{15}$.
20. $\frac{3}{50}, \frac{2}{25}, \frac{4}{5}$.
21. $\frac{2}{27}, \frac{4}{9}, \frac{5}{81}$.
22. $\frac{3}{5}, \frac{2}{13}, \frac{7}{130}$.
23. $\frac{2}{3}, \frac{3}{7}, \frac{4}{5}, \frac{1}{10}$.
24. $\frac{1}{16}, \frac{5}{8}, \frac{9}{112}$.
25. $\frac{4}{37}, \frac{7}{74}, \frac{5}{111}$.
26. $\frac{3}{26}, \frac{7}{39}, \frac{5}{52}$.
27. $\frac{2}{21}, \frac{5}{49}, \frac{4}{63}$.
28. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.
29. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{5}{16}, \frac{3}{32}$.
30. $\frac{1}{9}, \frac{2}{3}, \frac{5}{6}, \frac{7}{12}, \frac{5}{36}$.

*Reduce the fractional parts as directed above:*

31. $31\frac{4}{27}, 42\frac{7}{99}$.
32. $29\frac{5}{11}, 36\frac{13}{77}$.
33. $42\frac{17}{18}, 61\frac{25}{81}$.
34. $23\frac{4}{21}, 67\frac{9}{28}$.
35. $32\frac{4}{63}, 77\frac{6}{77}$.
36. $81\frac{7}{32}, 27\frac{7}{48}$.
37. $71\frac{7}{15}, 62\frac{11}{50}$.
38. $68\frac{2}{37}, 41\frac{11}{74}$.
39. $18\frac{7}{65}, 21\frac{11}{52}$.

## Addition

### ORAL EXERCISE

1. How many ninths are $\frac{2}{3} + \frac{4}{9}$? How many thirds?
2. How many elevenths are $\frac{7}{11} + \frac{4}{11}$? How many ones?
3. How many tenths are $\frac{9}{10} + \frac{9}{10} + \frac{2}{10}$? How many ones?
4. How much is $\frac{1}{2} + \frac{1}{4}$? $\frac{1}{2} + \frac{3}{4}$? $\frac{1}{2} + \frac{1}{8}$? $\frac{1}{2} + \frac{7}{8}$? $\frac{2}{3} + \frac{5}{6}$?

**102. Adding fractions.** Therefore,

*To add fractions, reduce them to fractions having the least common denominator, and add the numerators for the numerator of the sum. Then reduce to lowest terms.*

**103.** For example, add $\frac{3}{5}$, $\frac{9}{10}$, $\frac{7}{12}$.

The l.c.d. is evidently $5 \times 2 \times 2 \times 3 = 60$.

$$60 \div 5 = 12, \text{ and } \frac{12 \times 3}{12 \times 5} = \frac{36}{60}.$$

$$60 \div 10 = 6, \text{ and } \frac{6 \times 9}{6 \times 10} = \frac{54}{60}.$$

$$60 \div 12 = 5, \text{ and } \frac{5 \times 7}{5 \times 12} = \frac{35}{60}.$$

$$\tfrac{36}{60} + \tfrac{54}{60} + \tfrac{35}{60} = \tfrac{125}{60} = 2\tfrac{5}{60} = 2\tfrac{1}{12}.$$

**104. Adding mixed numbers.** To add $2\frac{3}{4}$ and $6\frac{7}{8}$.

$$2\tfrac{3}{4} + 6\tfrac{7}{8} = 2\tfrac{6}{8} + 6\tfrac{7}{8}$$
$$= 8\tfrac{13}{8} \text{ or } 9\tfrac{5}{8}.$$

$$\begin{array}{r} 2\frac{6}{8} \\ 6\frac{7}{8} \\ \hline 9\frac{5}{8} \end{array}$$

### WRITTEN EXERCISE

1. $\frac{1}{2} + \frac{2}{3}$.
2. $\frac{3}{4} + \frac{1}{8}$.
3. $\frac{4}{5} + \frac{1}{2}$.
4. $\frac{1}{8} + \frac{3}{4} + \frac{1}{2}$.
5. $\frac{1}{10} + \frac{2}{5} + \frac{1}{5}$.
6. $\frac{1}{16} + \frac{3}{8} + \frac{1}{4}$.
7. $\frac{3}{16} + \frac{3}{8} + \frac{3}{4}$.
8. $\frac{1}{6} + \frac{1}{3} + \frac{1}{12}$.
9. $\frac{5}{6} + \frac{7}{12} + \frac{5}{24}$.
10. $2\frac{1}{2} + 7\frac{1}{4} + 9\frac{3}{4}$.
11. $6\frac{7}{8} + 2\frac{1}{8} + 9\frac{1}{2}$.
12. $2\frac{3}{4} + 1\frac{7}{8} + 3\frac{5}{56}$.

## Subtraction

### ORAL EXERCISE

1. $\frac{1}{2} + \frac{1}{4}$. 2. $\frac{3}{4} - \frac{1}{2}$. 3. $\frac{3}{4} - \frac{1}{8}$. 4. $1\frac{3}{4} - \frac{1}{8}$.
5. $\frac{5}{6} + \frac{1}{3}$. 6. $1\frac{1}{6} - \frac{5}{6}$. 7. $1\frac{1}{3} - \frac{5}{6}$. 8. $5\frac{1}{3} - \frac{5}{6}$.
9. $\frac{2}{5} - \frac{1}{10}$. 10. $\frac{1}{3} - \frac{1}{6}$. 11. $\frac{1}{4} - \frac{1}{8}$. 12. $4\frac{1}{4} - 1\frac{1}{8}$.
13. $\frac{1}{6} - \frac{1}{12}$. 14. $\frac{5}{6} - \frac{1}{12}$. 15. $\frac{5}{6} - \frac{5}{12}$. 16. $3\frac{5}{6} - 1\frac{5}{12}$.

*Tell the number that* n *stands for in the following:*

17. $\frac{1}{2} + n = \frac{3}{4}$. 18. $\frac{3}{4} - n = \frac{1}{2}$. 19. $\frac{3}{4} - \frac{1}{2} = n$.
20. $\frac{2}{3} + n = 1\frac{1}{3}$. 21. $1\frac{1}{3} - n = \frac{2}{3}$. 22. $1\frac{1}{3} - \frac{2}{3} = n$.

**105. Subtracting fractions.** From our work in adding fractions we learn how to subtract.

*To subtract fractions, reduce them to fractions having the least common denominator, and subtract the numerators for the numerator of the difference. Then reduce to lowest terms.*

For example, $\frac{1}{2} - \frac{3}{16} = \frac{8}{16} - \frac{3}{16} = \frac{5}{16}$.

### WRITTEN EXERCISE

1. $\frac{1}{2} - \frac{5}{12}$. 2. $\frac{7}{8} - \frac{1}{16}$. 3. $\frac{3}{8} - \frac{1}{24}$. 4. $\frac{2}{3} - \frac{5}{12}$.
5. $\frac{3}{4} - \frac{3}{16}$. 6. $\frac{5}{8} - \frac{5}{24}$. 7. $\frac{7}{8} - \frac{5}{24}$. 8. $\frac{3}{4} - \frac{7}{12}$.
9. $\frac{5}{12} - \frac{1}{36}$. 10. $\frac{5}{24} - \frac{1}{12}$. 11. $\frac{7}{16} - \frac{7}{32}$. 12. $\frac{8}{9} - \frac{4}{21}$.
13. $\frac{6}{7} - \frac{2}{3}$. 14. $\frac{8}{9} - \frac{1}{6}$. 15. $\frac{7}{11} - \frac{1}{2}$. 16. $\frac{5}{12} - \frac{3}{8}$.
17. $\frac{9}{10} - \frac{3}{5}$. 18. $\frac{5}{9} - \frac{1}{12}$. 19. $\frac{8}{21} - \frac{3}{14}$. 20. $\frac{7}{9} - \frac{2}{21}$.
21. $\frac{2}{3} - \frac{4}{27}$. 22. $\frac{6}{7} - \frac{2}{35}$. 23. $\frac{5}{8} - \frac{3}{32}$. 24. $\frac{7}{11} - \frac{5}{33}$.
25. $\frac{11}{14} - \frac{4}{21}$. 26. $\frac{15}{16} - \frac{3}{64}$. 27. $\frac{9}{25} - \frac{9}{75}$. 28. $\frac{11}{12} - \frac{11}{84}$.
29. $\frac{15}{17} - \frac{2}{51}$. 30. $\frac{12}{13} - \frac{7}{39}$. 31. $\frac{8}{9} - \frac{7}{15}$. 32. $\frac{6}{7} - \frac{4}{5}$.

33. If you live $\frac{7}{8}$ mi. from school, how far have you still to go when you have gone $\frac{3}{4}$ mi.?

**106. Subtracting mixed numbers.** From $17\frac{1}{3}$ to subtract $8\frac{5}{6}$.

$17\frac{1}{3} - 8\frac{5}{6} = 17\frac{2}{6} - 8\frac{5}{6}$.

We may think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - 8\frac{5}{6} = 8\frac{3}{6} = 8\frac{1}{2}$.

$$\begin{array}{r} 17\frac{2}{6} \\ 8\frac{5}{6} \\ \hline 8\frac{1}{2} \end{array}$$

WRITTEN EXERCISE

*Add or subtract as indicated:*

1. $493\frac{1}{3} - 127\frac{5}{9}$.
2. $200\frac{1}{9} - 77\frac{5}{6}$.
3. $426\frac{1}{5} - 263\frac{3}{10}$.
4. $396\frac{2}{5} - 129\frac{2}{3}$.
5. $620\frac{5}{8} - 531\frac{1}{12}$.
6. $426\frac{2}{7} - 329\frac{5}{21}$.
7. $500\frac{4}{7} - 287\frac{13}{14}$.
8. $531\frac{3}{5} - 148\frac{14}{15}$.
9. $\frac{3}{7} + \frac{5}{14} + \frac{6}{35}$.
10. $\frac{4}{5} + \frac{3}{8} + \frac{1}{10}$.
11. $\frac{3}{8} + \frac{5}{12} + \frac{7}{16}$.
12. $\frac{4}{5} + \frac{3}{4} + \frac{7}{10}$.
13. $\frac{7}{12} + \frac{7}{20} + \frac{7}{10}$.
14. $\frac{6}{7} + \frac{3}{14} + \frac{2}{21}$.
15. $\frac{9}{20} + \frac{7}{30} + \frac{3}{10}$.
16. $\frac{3}{5} + \frac{7}{10} + \frac{4}{15}$.
17. $\frac{3}{14} + \frac{4}{21} + \frac{9}{35}$.
18. $\frac{7}{8} + \frac{3}{4} + \frac{1}{2} + \frac{5}{32}$.
19. $\frac{5}{12} + \frac{11}{72} + \frac{13}{144}$.
20. $4\frac{2}{3} + 9\frac{7}{8} + 23\frac{5}{12}$.
21. $\frac{2}{3} + \frac{3}{4} + \frac{5}{24} + \frac{7}{12}$.
22. $16\frac{4}{5} + 21\frac{1}{8} + 32\frac{1}{20}$.
23. $\frac{6}{7} + \frac{3}{5} + \frac{9}{35} + \frac{9}{14}$.
24. $\frac{3}{16}$ in. $+ \frac{7}{8}$ in. $- \frac{1}{4}$ in.

25. If Carrie has a strip of cloth $5\frac{1}{8}$ in. wide, how much must she cut off to make it $3\frac{1}{2}$ in. wide?

26. If Jack has a kite that is $14\frac{3}{4}$ in. on each of two of its sides, and $19\frac{7}{8}$ in. on each of the other two, how many inches around the kite?

27. If you are building a playhouse and need a stick $15\frac{7}{8}$ in. long for a window sill, and have a stick $17\frac{1}{4}$ in. long, how much must you saw off?

28. A farmer owns a triangular piece of land that is $175\frac{2}{3}$ rd. on one side, $167\frac{1}{2}$ rd. on another, and $37\frac{1}{8}$ rd. on the third. How many rods of fence will he need for it?

### WRITTEN EXERCISE

1. Frank is $4\frac{1}{8}$ ft. tall, and Rob lacks 4 in. of being as tall as Frank. How tall is Rob?

2. John is $11\frac{2}{3}$ yr. old, Mary is $\frac{1}{2}$ yr. older, and Tom is $1\frac{1}{4}$ yr. younger than Mary. How old is Tom?

3. If my finger nail is $\frac{7}{16}$ in. long, and the rest of my finger is $3\frac{3}{8}$ in. long, what is the total length?

4. If you weigh $56\frac{3}{4}$ lb., and your dog weighs $18\frac{2}{3}$ lb., and your books weigh $3\frac{1}{2}$ lb., what is the total weight?

5. Harry had a kite string $198\frac{2}{3}$ yd. long. His kite was up in a shower and the string shrank $1\frac{7}{8}$ yd. How long was it then?

6. A plate of glass $37\frac{1}{8}$ in. by $23\frac{1}{4}$ in. was set in a picture frame that covered $\frac{3}{8}$ in. from each edge. What are the dimensions of the glass not covered by the frame?

7. In making a dress ruffle $3\frac{7}{8}$ in. wide, enough cloth must be allowed to turn in $\frac{7}{8}$ in. on one side and $1\frac{1}{4}$ in. on the other. What is the total width of cloth needed?

8. In running a gas pipe into a schoolroom the gas fitter found he needed four pieces, 6 ft. $2\frac{1}{2}$ in., 5 ft. $3\frac{7}{8}$ in., 10 ft. 2 in., and a piece half as long as the third. What was the total length required?

9. A tenpenny nail is $2\frac{3}{4}$ in. long, and a fourpenny nail is $1\frac{3}{8}$ in. shorter; how long is a fourpenny nail? A sixpenny nail is $\frac{5}{8}$ in. longer than a fourpenny; how long is a sixpenny nail?

**107.** Work with fractions can be made much more interesting and real by supplying the class with rulers and with paper for folding. Let the pupils fold a piece 1 ft. square, fold a border $\frac{7}{8}$ in., fold again $1\frac{1}{2}$ in. farther from the edge, and so on, introducing reduction, addition, and subtraction.

### ORAL EXERCISE

*Add as indicated in Exs. 1–12:*

1. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
2. $\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$.
3. $\frac{1}{4} + \frac{3}{8} + \frac{1}{2}$.
4. $\frac{1}{3} + \frac{1}{6} + 1\frac{1}{2}$.
5. $\frac{2}{3} + \frac{5}{6} + 1\frac{7}{2}$.
6. $\frac{1}{6} + \frac{2}{3} + 1\frac{5}{2}$.
7. $\frac{1}{4} + \frac{1}{8} + \frac{1}{16}$.
8. $\frac{3}{4} + \frac{1}{8} + \frac{1}{16}$.
9. $\frac{3}{4} + \frac{7}{8} + \frac{1}{16}$.
10. $\frac{1}{6} + 1\frac{1}{2} + 2\frac{1}{3}$.
11. $\frac{5}{6} + 3\frac{1}{2} + 5\frac{2}{3}$.
12. $\frac{1}{8} + 5\frac{3}{4} + 7\frac{7}{8}$.

*Subtract as indicated in Exs. 13–28:*

13. $\frac{3}{4} - \frac{1}{2}$.
14. $\frac{3}{4} - \frac{3}{8}$.
15. $1\frac{5}{16} - \frac{3}{8}$.
16. $\frac{7}{20} - \frac{1}{10}$.
17. $\frac{1}{3} - \frac{1}{6}$.
18. $\frac{2}{3} - \frac{1}{6}$.
19. $1\frac{5}{12} - \frac{1}{6}$.
20. $\frac{3}{40} - \frac{1}{20}$.
21. $1\frac{1}{2} - \frac{1}{2}$.
22. $1\frac{7}{2} - \frac{1}{4}$.
23. $\frac{2}{3} - 1\frac{5}{2}$.
24. $\frac{5}{7} - 1\frac{3}{4}$.
25. $1\frac{5}{16} - \frac{1}{8}$.
26. $3\frac{5}{2} - \frac{1}{16}$.
27. $\frac{1}{8} - 6\frac{1}{4}$.
28. $1\frac{7}{11} - \frac{1}{22}$.

### WRITTEN EXERCISE

1. $5\frac{2}{9} + 3\frac{1}{12} + 1\frac{4}{27}$.
2. $\frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{7}{8}$.
3. $6\frac{3}{5} + 2\frac{1}{4} + \frac{3}{10} + 2\frac{1}{2}$.
4. $4\frac{3}{16} + \frac{2}{3} + 2\frac{5}{12} + \frac{1}{2}$.
5. $3\frac{5}{12} + \frac{3}{25} + 2\frac{1}{6} + 4\frac{7}{10}$.
6. $2\frac{1}{16} + 5\frac{4}{3} + \frac{2}{3} + 6\frac{3}{8}$.
7. $20\frac{1}{8} - 2\frac{1}{4}$.
8. $5\frac{3}{16} - 2\frac{7}{12}$.
9. $11\frac{1}{40} - 6\frac{5}{8}$.
10. $20\frac{3}{10} - 7\frac{5}{6}$.
11. $15\frac{3}{8} - 5\frac{2}{3}\frac{3}{2}$.
12. $42\frac{1}{16} - 38\frac{7}{12}$.
13. $37\frac{3}{5} - 19\frac{7}{8}$.
14. $41\frac{6}{7} - 19\frac{4}{5}$.
15. $28\frac{5}{8} - 19\frac{3}{3}\frac{1}{2}$.
16. $121\frac{3}{8} - 36\frac{7}{8}$.
17. $42\frac{5}{8} - 29\frac{3}{4}$.
18. $31\frac{1}{2} - 18\frac{7}{8}$.

19. What is the difference in length between a tack $\frac{1}{8}$ in. long and one $\frac{1}{4}$ in. long? Also between one $\frac{1}{4}$ in. long and one $\frac{3}{8}$ in. long? between one $\frac{3}{8}$ in. long and one $\frac{7}{16}$ in. long? Also between one $\frac{7}{16}$ in. long and one $\frac{9}{16}$ in. long? between one $\frac{9}{16}$ in. long and one $\frac{5}{8}$ in. long?

**108.** It should be remembered that the common fractions most frequently met are those with denominators 2, 4, 8, 16, 32, 64; 3, 6, 9, 12, 24, 36, 48; 5, 10.

### WRITTEN EXERCISE

1. A farmer has 57 hens. In 5 weeks in the spring they laid 855 eggs. How many eggs did they all average per week? How many did each average per week? for the 5 weeks?

2. He fed these hens 112 lb. of corn in the 5 weeks. At 56 lb. to the bushel, how many bushels did he feed? How much was it worth at 75 ct. a bushel?

3. He also fed them 96 lb. of wheat. At 60 lb. to the bushel, how many bushels did he feed? How much was it worth at 90 ct. a bushel?

4. He also fed them 40 lb. of oats. At 32 lb. to the bushel, how many bushels did he feed? How much was it worth at 48 ct. a bushel?

5. He also fed them 16 lb. of buckwheat. At 48 lb. to the bushel, what part of a bushel did he feed? How much was it worth at 60 ct. a bushel?

6. The corn, wheat, oats, and buckwheat fed to these hens amounted to how many pounds? bushels? How many pounds did each hen eat?

7. The rest of the feed cost $1.96. What was the total cost of feed for the 57 hens for 5 weeks? the average cost per hen? the average cost per egg?

8. The eggs sold at 20 ct. a dozen. How much did the farmer make per dozen above the cost of feed? How much on all the 855 eggs?

9. Allowing $4.25 for the use of the property, and $3 for the time spent in caring for the hens, how much did the farmer make on eggs during these 5 weeks? How much did he make per hen?

## Multiplication of Fractions

### ORAL EXERCISE

1. How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{4}{7}$?

2. How much is $\frac{1}{3}$ of 6 bu.? $\frac{1}{3}$ of 6 sevenths? $\frac{1}{3}$ of $\frac{6}{11}$?

3. How much is $\frac{1}{4}$ of $\frac{16}{17}$? $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{1}{4}$ of $\frac{24}{25}$? $\frac{1}{3}$ of $\frac{6}{5}$?

4. In each of these cases in which you found a fraction of a fraction, how did you proceed?

*State results in Exs. 5–12:*

5. $\frac{1}{7}$ of $\frac{4}{15}$. 6. $\frac{1}{5}$ of $\frac{5}{16}$. 7. $\frac{1}{6}$ of $\frac{3}{13}$. 8. $\frac{1}{2}$ of $\frac{8}{9}$.

9. $\frac{1}{8}$ of $\frac{6}{21}$. 10. $\frac{1}{3}$ of $\frac{8}{25}$. 11. $\frac{1}{10}$ of $\frac{20}{21}$ 12. $\frac{1}{12}$ of $\frac{24}{25}$.

13. Point to $\frac{1}{3}$ of $\frac{3}{4}$ of the sphere. What part of the sphere is it?

14. Why can you not find $\frac{1}{2}$ of $\frac{1}{4}$ in the same way?

15. Point to $\frac{1}{2}$ of $\frac{1}{4}$ of this sphere. What part of the sphere is it?

16. Draw on the blackboard a rectangle $2\frac{1}{2}$ ft. long and 1 ft. high; mark it off into fifths; mark off each fifth into halves. Then how much is $\frac{1}{2}$ of $\frac{1}{5}$? Also $\frac{1}{5}$ of $\frac{1}{2}$?

17. Draw another rectangle as in Ex. 16, marking off each fifth into thirds. How much is $\frac{1}{3}$ of $\frac{1}{5}$?

18. State the value of $\frac{1}{3}$ of $\frac{1}{5}$, $\frac{1}{5}$ of $\frac{1}{3}$; $\frac{1}{5}$ of $\frac{1}{7}$, $\frac{1}{7}$ of $\frac{1}{5}$; $\frac{1}{5}$ of $\frac{1}{9}$, $\frac{1}{9}$ of $\frac{1}{5}$; $\frac{1}{3}$ of $\frac{1}{10}$, $\frac{1}{10}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{2}$.

19. In each case in Ex. 18 what did you do to the two denominators?

20. Because $\frac{1}{4}$ of \$1 is 25 ct., $\frac{3}{4}$ of \$1 is how much?

21. Because $\frac{1}{4}$ of $\frac{1}{5}$ is $\frac{1}{20}$, $\frac{3}{4}$ of $\frac{1}{5}$ is how many times $\frac{1}{20}$?

22. Then how much is $\frac{3}{4}$ of $\frac{1}{5}$? $\frac{3}{4}$ of $\frac{1}{7}$? $\frac{3}{4}$ of $\frac{1}{10}$? $\frac{3}{4}$ of $\frac{1}{11}$?

### ORAL EXERCISE

1. What part of A is $\frac{1}{2}$ of $\frac{1}{2}$ of A?

These rectangles should be drawn on the blackboard.

2. What part of B is $\frac{1}{2}$ of $\frac{1}{3}$ of B?

3. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of B is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of B.

4. What part of C is $\frac{1}{3}$ of $\frac{1}{2}$ of C?

5. What part of D is $\frac{1}{3}$ of $\frac{1}{3}$ of D?

6. Each third of D equals how many ninths of D?

7. Point to $\frac{1}{2}$ of E; to $\frac{1}{4}$ of $\frac{1}{2}$ of E. Also point to $\frac{1}{2}$ of $\frac{1}{4}$ of E. What part of E is $\frac{1}{2}$ of $\frac{1}{4}$ of E, or $\frac{1}{4}$ of $\frac{1}{2}$ of E?

8. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of E; to $\frac{3}{4}$ of $\frac{1}{2}$ of E. What part of E is $\frac{3}{4}$ of $\frac{1}{2}$?

9. Point to $\frac{1}{3}$ of F; to $\frac{1}{4}$ of $\frac{1}{3}$ of F. Also point to $\frac{1}{4}$ of F, and to $\frac{1}{3}$ of $\frac{1}{4}$ of F. Each is what part of F?

10. Point to $\frac{3}{4}$ of F; to $\frac{1}{3}$ of $\frac{3}{4}$ of F. Also point to $\frac{1}{3}$ of F, and to $\frac{3}{4}$ of $\frac{1}{3}$ of F. Each is what part of F?

### WRITTEN EXERCISE

*Draw rectangles to show:*

1. $\frac{1}{3}$ of $\frac{1}{3}$, or $\frac{1}{3}$ of $\frac{1}{3}$.
2. $\frac{1}{2}$ of $\frac{1}{6}$, or $\frac{1}{6}$ of $\frac{1}{2}$.
3. $\frac{2}{3}$ of $\frac{1}{5}$, or $\frac{1}{5}$ of $\frac{2}{3}$.
4. $\frac{1}{2}$ of $\frac{5}{6}$, or $\frac{5}{6}$ of $\frac{1}{2}$.
5. $\frac{2}{5}$ of $\frac{2}{3}$, or $\frac{2}{3}$ of $\frac{2}{5}$.
6. $\frac{4}{5}$ of $\frac{1}{6}$, or $\frac{1}{6}$ of $\frac{4}{5}$.
7. $\frac{1}{8}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{8}$.
8. $\frac{3}{8}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{3}{8}$.
9. $\frac{5}{8}$ of $\frac{1}{5}$, or $\frac{1}{5}$ of $\frac{5}{8}$.
10. $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{2}{5}$.
11. $\frac{1}{10}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{10}$.
12. $\frac{3}{10}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{3}{10}$.

### ORAL EXERCISE

1. How much is 2 times 3 in.? 2 times $\frac{3}{7}$? 5 times $\frac{2}{11}$? How did you multiply these fractions by an integer?

2. How much is 4 times $\frac{1}{4}$? 4 times $\frac{3}{4}$? 2 times $\frac{3}{4}$? 5 times $\frac{7}{40}$? 7 times $\frac{4}{49}$? How did you multiply these fractions by an integer?

**109. How to multiply a fraction by an integer.** Therefore,

1. *To multiply a fraction by an integer, multiply the numerator by the multiplier.*

2. *Before actually multiplying, indicate the multiplication and cancel if possible.*

For example, to multiply $\frac{11}{63}$ by 18. Since we have 11 sixty-thirds, if we have 18 times as much we shall have

$$\frac{\overset{2}{\cancel{18}} \text{ times } 11}{\underset{7}{\cancel{63}}}, \text{ or } \frac{22}{7}, \text{ or } 3\tfrac{1}{7}.$$

### ORAL EXERCISE

*Multiply:*

| | | | |
|---|---|---|---|
| 1. $\frac{5}{9}$ by 9. | 2. $\frac{6}{7}$ by 7. | 3. $\frac{4}{5}$ by 5. | 4. $\frac{3}{8}$ by 8. |
| 5. $\frac{3}{4}$ by 2. | 6. $\frac{9}{10}$ by 5. | 7. $\frac{7}{8}$ by 4. | 8. $\frac{3}{10}$ by 5. |
| 9. $\frac{3}{8}$ by 16. | 10. $\frac{4}{5}$ by 10. | 11. $\frac{5}{7}$ by 21. | 12. $\frac{3}{11}$ by 44. |
| 13. $\frac{5}{8}$ by 40. | 14. $\frac{6}{7}$ by 28. | 15. $\frac{5}{9}$ by 36. | 16. $\frac{7}{12}$ by 24. |
| 17. $\frac{3}{17}$ by 34. | 18. $\frac{4}{13}$ by 26. | 19. $\frac{5}{11}$ by 99. | 20. $\frac{6}{25}$ by 100. |

### WRITTEN EXERCISE

*Multiply:*

| | | | |
|---|---|---|---|
| 1. $\frac{4}{21}$ by 35. | 2. $\frac{6}{35}$ by 28. | 3. $\frac{7}{36}$ by 27. | 4. $\frac{5}{81}$ by 63. |
| 5. $\frac{3}{41}$ by 82. | 6. $\frac{4}{11}$ by 121. | 7. $\frac{5}{49}$ by 63. | 8. $\frac{7}{64}$ by 96. |
| 9. $\frac{7}{72}$ by 16. | 10. $\frac{11}{54}$ by 36. | 11. $\frac{9}{56}$ by 49. | 12. $\frac{13}{88}$ by 121. |
| 13. $\frac{7}{108}$ by 24. | 14. $\frac{8}{121}$ by 99. | 15. $\frac{11}{128}$ by 192. | 16. $\frac{11}{144}$ by 132. |

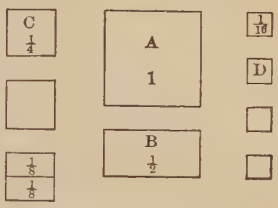
### ORAL EXERCISE

1. In the picture, call A *one*. Point to $\frac{1}{2}$ of $\frac{1}{2}$; to $\frac{1}{2}$ of $\frac{1}{4}$.

2. Point to $\frac{3}{4}$ (three squares); to $\frac{2}{3}$ of $\frac{3}{4}$.

3. Point to $\frac{1}{3}$ of $\frac{3}{4}$ and tell what fraction it equals.

4. Point to $\frac{1}{3}$ of $1\frac{1}{2}$ and tell what fraction it equals.




*Point to the following and tell what each equals:*

5. $\frac{1}{5}$ of $1\frac{1}{4}$. 6. $\frac{1}{7}$ of $1\frac{3}{4}$. 7. $\frac{3}{4}$ of $\frac{1}{4}$. 8. $\frac{1}{2}$ of $\frac{4}{16}$.

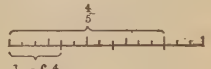
9. $\frac{1}{4}$ of $\frac{4}{16}$. 10. $\frac{1}{4}$ of $\frac{1}{4}$. 11. $\frac{1}{8}$ of $\frac{1}{2}$. 12. $\frac{1}{3}$ of $\frac{3}{16}$.

13. How much is $\frac{1}{2}$ of 4 ft.? $\frac{1}{2}$ of $\frac{4}{5}$? $\frac{1}{2}$ of $\frac{6}{7}$? $\frac{1}{4}$ of $\frac{4}{5}$? How did you take these fractional parts of fractions?

**110. Multiplication by a fraction.** To take a fractional part of a number is called *multiplying by a fraction.*

For example, to take $\frac{1}{3}$ of 4 is called multiplying 4 by $\frac{1}{3}$. It is indicated either by "$\frac{1}{3}$ of 4" or by "$\frac{1}{3} \times 4$," the $\times$ being read "of."

**111. How to multiply a fraction by a fraction.**

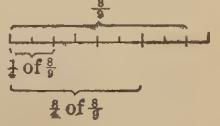



Since $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ is twice as much, or $\frac{8}{15}$. Therefore,

1. *To multiply a fraction by a fraction, multiply the numerators for a new numerator and the denominators for a new denominator.*

2. *Instead of actually multiplying, it is better to indicate the operation and cancel if possible.*

For example, $\frac{3}{4}$ of $\frac{8}{9} = \frac{\overset{2}{\cancel{3}} \times \overset{}{\cancel{8}}}{\underset{3}{\cancel{4}} \times \cancel{9}} = \frac{2}{3}$.



### ORAL EXERCISE

1. $\frac{1}{2}$ of $\frac{3}{5}$. 2. $\frac{1}{3}$ of $\frac{2}{7}$. 3. $\frac{1}{14}$ of $\frac{7}{8}$. 4. $\frac{2}{3}$ of $\frac{7}{5}$.

5. $\frac{1}{15}$ of $\frac{5}{7}$. 6. $\frac{1}{7}$ of $\frac{42}{50}$. 7. $\frac{1}{8}$ of $\frac{32}{35}$. 8. $\frac{1}{6}$ of $\frac{7}{8}$.

9. $\frac{1}{6}$ of $\frac{4}{11}$. 10. $\frac{1}{3}$ of $\frac{12}{13}$. 11. $\frac{1}{12}$ of $\frac{8}{15}$. 12. $\frac{1}{10}$ of $\frac{5}{6}$.

13. If a man owns a garden plot of $\frac{4}{5}$ acre, and sells $\frac{1}{3}$ of it, what part of an acre does he sell?

14. If one of you lives $1\frac{2}{3}$ miles from here, and I live $\frac{1}{5}$ as far away, how far from here do I live?

15. If you live $\frac{4}{5}$ of a mile from school, and Jack lives a quarter as far, how far does he live from school?

16. If Kate lives $1\frac{1}{2}$ miles away, and Mary lives $\frac{1}{3}$ as far, how far away does Mary live? If Jennie lives $\frac{1}{2}$ as far as Mary, how far away does Jennie live?

### WRITTEN EXERCISE

1. $\frac{2}{3}$ of $\frac{4}{5}$. 2. $\frac{3}{7}$ of $\frac{5}{8}$. 3. $\frac{3}{4}$ of $\frac{6}{7}$. 4. $\frac{5}{6}$ of $1\frac{2}{3}$.

5. $\frac{7}{8}$ of $1\frac{2}{21}$. 6. $\frac{5}{7}$ of $1\frac{4}{15}$. 7. $\frac{3}{5}$ of $\frac{6}{7}$. 8. $\frac{4}{7}$ of $1\frac{1}{2}$.

9. $2\frac{1}{22}$ of $1\frac{1}{7}$. 10. $\frac{6}{5}$ of $1\frac{5}{16}$. 11. $\frac{2}{3}$ of $\frac{9}{4}$. 12. $\frac{6}{5}$ of $1\frac{5}{2}$.

13. $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{10}{11}$. 14. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. 15. $\frac{3}{7}$ of $\frac{1}{6}$ of 14.

16. If a yard of cloth costs $\$\frac{3}{4}$, what will $\frac{5}{8}$ yd. cost?

17. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long will it take it to run $\frac{7}{8}$ mi. at the same rate?

18. If a train runs $\frac{1}{3}$ of a certain distance in 1 hr., what part of the distance can it run in 45 min.?

19. Draw a rectangle 2 in. wide and 4 in. long. Divide it into square inches. How many are there? Show that you have either 2 times 4 sq. in. or 4 times 2 sq. in.

20. Find the area of a rectangle 17 ft. by 19 ft.; of one $\frac{3}{4}$ in. by 16 in.; of one 35 in. by $\frac{1}{5}$ in.

### ORAL EXERCISE

1. How much is 4 times 5? 4 times $\frac{2}{3}$? 4 times $5\frac{2}{3}$?
2. How much is $2 \times 8$? $\frac{3}{4}$ of 8? $2\frac{3}{4}$ times 8?
3. How much is $\frac{1}{3}\frac{0}{}$ of $\frac{6}{5}$? Then how much is $3\frac{1}{3} \times 1\frac{1}{5}$?

**112. Mixed numbers.**

1. *To multiply a mixed number by an integer, multiply the whole and the fractional parts separately and add the products.*

For example, 6 times $3\frac{2}{5}$ equals $18\frac{12}{5}$, or $20\frac{2}{5}$.

2. *To multiply an integer by a mixed number, multiply by the whole and the fractional parts separately and add the products.*

For example, $3\frac{2}{5}$ times 6 equals $3 \times 6$, and $\frac{2}{5}$ of 6, or $18\frac{12}{5}$, or $20\frac{2}{5}$.

3. *To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the results.*

For example, $5\frac{2}{5} \times 3\frac{3}{4} = \frac{27}{\cancel{5}} \text{ of } \frac{\overset{3}{\cancel{15}}}{4} = \frac{81}{4} = 20\frac{1}{4}$.

### WRITTEN EXERCISE

| | | |
|---|---|---|
| 1. $17 \times 2\frac{1}{3}\frac{9}{4}$. | 2. $27 \times 41\frac{5}{9}$. | 3. $49 \times 81\frac{6}{7}$. |
| 4. $81 \times 62\frac{5}{9}$. | 5. $48 \times 37\frac{3}{1}\frac{}{0}$. | 6. $26\frac{3}{4} \times 32$. |
| 7. $37\frac{4}{5} \times 45$. | 8. $29\frac{4}{5} \times 35$. | 9. $49\frac{7}{8} \times 68$. |
| 10. $25\frac{2}{3} \times 19$. | 11. $47\frac{1}{1}\frac{}{0} \times 65$. | 12. $26\frac{1}{2} \times 63$. |
| 13. $35\frac{7}{8} \times 36$. | 14. $6\frac{7}{8} \times 7\frac{7}{8}$. | 15. $9\frac{2}{3} \times 15\frac{7}{8}$. |
| 16. $16\frac{3}{4} \times 14\frac{1}{3}$. | 17. $15\frac{7}{9} \times 14\frac{3}{7}$. | 18. $14\frac{3}{4} \times 16\frac{4}{5}$. |
| 19. $18\frac{4}{7} \times 23\frac{7}{8}$. | 20. $19\frac{2}{3} \times 27\frac{4}{5}$. | 21. $16\frac{3}{4} \times 28\frac{4}{5}$. |

22. At $\$2\frac{3}{4}$ a yard, how much will $1\frac{1}{2}$ yd. of cloth cost?

## Cancellation

### ORAL EXERCISE

1. What is the value of $\frac{2 \times 3}{5 \times 3}$? What factor may you cancel?

2. State the value of: $\frac{2 \times 7}{2 \times 8}$, $\frac{3 \times 11}{4 \times 11}$, $\frac{5 \times 6}{6 \times 7}$.

3. If there is the same factor in both numerator and denominator, what do you do with it?

**113.** We found, in § 86, that we may cancel equal factors from both terms of a fraction.

**114. Cancellation.** Since a fraction is an indicated division, *equal factors may be canceled from both dividend and divisor without changing the quotient.*

Divide $2 \times 7 \times 15$ by $21 \times 5 \times 6$.

Writing this as a fraction (§ 95), 2 is canceled from 2 and 6 (§ 86); 7 is canceled from 7 and 21; 3 is canceled from 3 (left from the 21) and 15; 5 is canceled from 5 (left from the 15) and 5. Therefore the quotient is $\frac{1}{3}$, for 1 is a factor of the numerator.

$$\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{7}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{15}}}{\underset{3}{\cancel{21}} \times \underset{1}{\cancel{5}} \times \underset{3}{\cancel{6}}} = \frac{1}{3}$$

### WRITTEN EXERCISE

*Divide, using cancellation:*

1. $2 \times 3 \times 6$ by $5 \times 9 \times 4$.
2. $3 \times 14 \times 20$ by $15 \times 21 \times 7$.
3. $4 \times 15 \times 27$ by $9 \times 5 \times 24$.
4. $7 \times 21 \times 77 \times 6$ by $49 \times 33 \times 42$.
5. $20 \times 25 \times 32 \times 48$ by $27 \times 40 \times 15 \times 12$.
6. $32 \times 27 \times 44 \times 39$ by $55 \times 72 \times 26 \times 8 \times 3$.
7. $125 \times 108 \times 51 \times 8$ by $40 \times 45 \times 84 \times 12 \times 3$.

**115. Practical uses of cancellation.** We found in §§ 109–112 that cancellation is useful in the multiplication of fractions and mixed numbers. In this it is of much practical importance, as will be seen on page 60.

What is the product of $\frac{1}{15}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

Reducing to improper fractions, $45\frac{1}{2} = \frac{91}{2}$, $33\frac{1}{3} = \frac{100}{3}$, $16\frac{7}{8} = \frac{135}{8}$. Proceeding, as in § 111, and canceling, the product is $1706\frac{1}{4}$.

$$\frac{\overset{25}{\cancel{1}} \times 91 \times \overset{\overset{27}{\cancel{100}}}{\cancel{100}} \times \overset{9}{\cancel{135}}^{3}}{\underset{3}{\cancel{15}} \times 2 \times \underset{1}{\cancel{3}} \times \underset{2}{\cancel{8}}} = \frac{6825}{4} = 1706\tfrac{1}{4}$$

WRITTEN EXERCISE

1. $20\frac{1}{2} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
2. $15\frac{2}{3} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
3. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
4. $42\frac{2}{3} \times 66\frac{2}{3} \times 75\frac{1}{16}$.
5. $35\frac{1}{16} \times 22\frac{1}{32} \times 12\frac{4}{5}$.
6. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
7. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6}$.
8. $3\frac{1}{2} \times 4\frac{2}{3} \times 5\frac{3}{4} \times 6\frac{4}{5} \times 7\frac{5}{6}$.
9. $8\frac{2}{3} \times 7\frac{1}{2} \times 5\frac{7}{16} \times 1\frac{3}{13}$.
10. $2\frac{2}{3} \times 3\frac{4}{5} \times 5\frac{6}{7} \times 6\frac{7}{8} \times 7\frac{1}{7}$.

11. Divide the product of 4, 7, 19, 21, and 64 by the product of 14, 38, 28, and 3.

12. Divide the product of 3, 9, 15, 38, 42, and 75 by the product of 19, 7, 3, 5, 27, 60, and 8.

13. Divide the product of 2, 3, 4, 5, 6, 7, 8, and 9 by the product 10, 12, 14, 15, and 16.

14. How many times as much will it cost to transport 12 boxes weighing 300 lb. each 200 mi., as to transport 6 boxes weighing 400 lb. each 75 mi., at the same rate per pound for each mile?

15. How many times as much will it cost to transport 3 boxes of merchandise weighing 400 lb. each a given distance, as to transport 2 boxes weighing 300 lb. each the same distance, at the same rate per pound?

**116. Area of rectangle.** If each of these orange boxes is 20 in. long and 10 in. wide, how many square inches in the area of the base?

If the base were 1 in. long and 1 in. wide, the area would be 1 sq. in. But it is 20 times as long as this, and 10 times as wide. Therefore the area is $20 \times 10$ times 1 sq. in. $= 200$ sq. in.

In all such examples the *inside measure* is meant.

**117. Volume of a box.** How many cubic inches in a box $3\frac{1}{2}$ in. long, $2\frac{1}{2}$ in. wide, and $1\frac{1}{3}$ in. deep?

If it were 1 in. long and 1 in. wide and 1 in. thick, it would contain 1 cu. in. But it is $3\frac{1}{2}$ times as long as this, $2\frac{1}{2}$ times as wide, and $1\frac{1}{3}$ times as deep. Therefore it contains $3\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{3}$ times 1 cu. in., or

$$\frac{7}{2} \times \frac{5}{2} \text{ times } \frac{4}{3} \text{ cu. in.} = \frac{7 \times 5 \times \cancel{4}}{\cancel{2} \times \cancel{2} \times 3} \text{ cu. in.} = \frac{35}{3} \text{ cu. in.} = 11\tfrac{2}{3} \text{ cu. in.}$$

#### WRITTEN EXERCISE

1. Figs are shipped in a box 20 in. by 2 in. by $3\frac{1}{2}$ in. How many cubic inches in a box?

2. California cherries are shipped in a box $15\frac{1}{2}$ in. by $8\frac{1}{2}$ in. by $3\frac{1}{2}$ in. How many cubic inches in a box?

3. The standard Florida orange box for shipping is 12 in. by 12 in. by $26\frac{5}{8}$ in. How many cubic inches does it contain?

4. A bushel contains $2150\frac{1}{2}$ cu. in. The capacity of one of the Florida orange boxes is how many cubic inches more than $1\frac{3}{4}$ bu.?

5. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This contains how many cubic inches more than 1 bu.?

6. In New Jersey a bushel cranberry box is 12 in. by $8\frac{3}{4}$ in. by 22 in. How does this differ in capacity from the standard bushel?

7. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the standard bushel?

8. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. How many cubic inches does it contain?

9. The California orange box is sometimes $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. by $11\frac{1}{4}$ in., and sometimes 22 in. by $7\frac{3}{4}$ in. by $17\frac{1}{2}$ in. Which holds the more, and how many cubic inches more?

10. Dried fruits are shipped in 50-lb. or 25-lb. boxes. The 50-lb. box is $15\frac{1}{4}$ in. by 9 in. by 9 in. How many cubic inches in the box?

11. The 25-lb. box mentioned in Ex. 10 is sometimes $13\frac{3}{4}$ in. by $9\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

12. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by 4 in.
Cherries, $16\frac{1}{2}$ in. by $10\frac{3}{8}$ in. by $2\frac{1}{2}$ in.
Pears, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $8\frac{1}{2}$ in.
Peaches, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $4\frac{7}{8}$ in.

Find the number of cubic inches in each box.

## Division of Fractions

### ORAL EXERCISE

1. How much is 10 ft. $\div 5$? $\frac{10}{11} \div 5$? $\frac{10}{13} \div 5$? $\frac{14}{15} \div 7$? What is the effect of dividing the numerator?

2. How much is $\frac{1}{2}$ of $\frac{1}{5}$? Then how much is $\frac{1}{5} \div 2$? $\frac{1}{5} \div 3$? What is the effect of multiplying the denominator?

A B C

3. In this picture, if A is called 1, point to $\frac{5}{8}$. How much is $\frac{5}{8} \div 5$? Point to $\frac{1}{2}$. How much is $\frac{1}{2} \div 4$?

4. If B is called 1, point to $1\frac{3}{5}$. Point to that part which shows $1\frac{3}{5} \div 8$. Point to $\frac{4}{5}$; to $\frac{4}{5} \div 4$.

**118. Dividing a fraction by an integer.** We see that,

1. *Dividing the numerator by an integer divides the fraction by the integer.*

2. *Multiplying the denominator by an integer divides the fraction by the integer.*

**119.** It is sometimes easier to divide mixed numbers than improper fractions.

While $10\frac{1}{7} \div 3 = \frac{71}{7} \div 3 = \frac{71}{21} = 3\frac{8}{21}$, with a number like $244\frac{1}{2}$ to be divided by 3 we would not do this, for it would be too long. Here we should say, "$244\frac{1}{2} \div 3 = 81 + \frac{1}{3}$ of $1\frac{1}{2} = 81\frac{1}{2}$."

### WRITTEN EXERCISE

| | | | |
|---|---|---|---|
| 1. $2\frac{5}{36} \div 5$. | 2. $\frac{3}{17} \div 5$. | 3. $\frac{5}{21} \div 15$. | 4. $1\frac{13}{36} \div 7$. |
| 5. $2\frac{3}{16} \div 5$. | 6. $3\frac{5}{12} \div 41$. | 7. $2\frac{7}{24} \div 11$. | 8. $306\frac{1}{2} \div 9$. |
| 9. $225\frac{1}{3} \div 5$. | 10. $175\frac{1}{3} \div 6$. | 11. $273\frac{1}{8} \div 2$. | 12. $826\frac{1}{5} \div 5$. |

### ORAL EXERCISE

1. How many thirds of an inch in 1 in.? Then $1 \div \frac{1}{3}$ = how many?

2. How many fifths of an inch in 1 in.? Then $1 \div \frac{1}{5}$ = how many?

3. Tell the values of

$1 \div \frac{1}{8}$ $\quad 1 \div \frac{1}{9}$ $\quad 1 \div \frac{1}{7}$ $\quad 1 \div \frac{1}{10}$ $\quad 1 \div \frac{1}{20}$

4. If there are five fifths in 1, how many fifths are there in 2? in 3? Then $2 \div \frac{1}{5}$ = how many? $3 \div \frac{1}{5}$ = how many? $4 \div \frac{1}{5}$ = how many? $5 \div \frac{1}{7}$ = how many?

5. Tell the values of

$2 \div \frac{1}{4}$ $\quad 3 \div \frac{1}{7}$ $\quad 2 \div \frac{1}{10}$ $\quad 4 \div \frac{1}{6}$ $\quad 9 \div \frac{1}{8}$

6. Instead of dividing 2 by $\frac{1}{4}$, you will get the same result by multiplying 2 by what number?

7. Then $5 \div \frac{1}{7} = 5$ multiplied by what number?
$7 \div \frac{1}{8} = 7$ multiplied by what number?
$9 \div \frac{1}{12} = 9$ multiplied by what number?

**120. Dividing by a unit fraction.** There is, therefore, this easy way to divide by a unit fraction:

*To divide by a unit fraction, multiply by the denominator.*

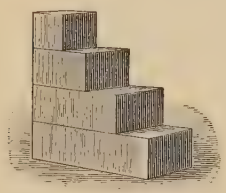
For example, $13 \div \frac{1}{3} = 13 \times 3 = 39$.

### WRITTEN EXERCISE

| | | |
|---|---|---|
| 1. $81 \div \frac{1}{8}$. | 2. $16 \div \frac{1}{9}$. | 3. $23 \div \frac{1}{7}$. |
| 4. $32 \div \frac{1}{6}$. | 5. $54 \div \frac{1}{4}$. | 6. $29 \div \frac{1}{3}$. |
| 7. $48 \div \frac{1}{5}$. | 8. $63 \div \frac{1}{7}$. | 9. $27 \div \frac{1}{9}$. |
| 10. $17 \div \frac{1}{12}$. | 11. $97 \div \frac{1}{12}$. | 12. $39 \div \frac{1}{20}$. |
| 13. $125 \div \frac{1}{16}$. | 14. $144 \div \frac{1}{32}$. | 15. $240 \div \frac{1}{64}$. |

### ORAL EXERCISE

1. If B is 1, which block is $\frac{1}{2}$? $1\frac{1}{2}$? 2? How many $\frac{1}{2}$'s in 1? in $1\frac{1}{2}$, or $\frac{3}{2}$? in 2?




2. If C is 1, which block is $\frac{1}{3}$? $1\frac{1}{3}$? $\frac{2}{3}$? How many $\frac{1}{3}$'s in 1? in $\frac{2}{3}$? in $1\frac{1}{3}$?

3. If C is 1, what is B? D? What is $1\frac{1}{3} \div \frac{2}{3}$?

4. If C is called 1, how much is C ÷ B, or $1 \div \frac{2}{3}$? If D is called 1, how much is D ÷ C, or $1 \div \frac{3}{4}$?

5. If $1 \div \frac{2}{3} = \frac{3}{2}$, how much is $2 \div \frac{2}{3}$? $3 \div \frac{2}{3}$? $\frac{3}{5} \div \frac{2}{3}$?

6. If $1 \div \frac{2}{3} = \frac{3}{2}$, then $\frac{7}{8} \div \frac{2}{3} = \frac{7}{8}$ of $\frac{3}{2}$ = how much?

**121. Reciprocal.** The fraction $\frac{3}{2}$ is called the *reciprocal* of $\frac{2}{3}$.

**122. How to divide by a fraction.** We see that *to divide by a fraction we may multiply by its reciprocal.*

Thus, $$\frac{5}{8} \div \frac{3}{4} = \frac{5}{\cancel{8}_{2}} \times \frac{\cancel{4}}{3} = \frac{5}{6}.$$

Also, $$4\tfrac{2}{3} \div 2\tfrac{4}{5} = \frac{14}{3} \div \frac{14}{5} = \frac{\cancel{14}}{3} \times \frac{5}{\cancel{14}} = \frac{5}{3}.$$

### WRITTEN EXERCISE

1. $\frac{7}{8} \div \frac{2}{3}$.
2. $\frac{5}{6} \div \frac{3}{4}$.
3. $\frac{9}{10} \div \frac{4}{5}$.
4. $\frac{5}{12} \div \frac{7}{8}$.
5. $\frac{3}{8} \div \frac{5}{12}$.
6. $\frac{7}{16} \div \frac{3}{32}$.
7. $\frac{5}{64} \div \frac{7}{32}$.
8. $\frac{9}{24} \div \frac{5}{16}$.
9. $\frac{3}{40} \div \frac{6}{25}$.
10. $\frac{7}{48} \div \frac{7}{12}$.
11. $\frac{3}{4} \div 1\frac{1}{16}$.
12. $2\frac{7}{18} \div 3\frac{1}{3}$.
13. $10\frac{7}{36} \div 9\frac{1}{4}$.
14. $5\frac{1}{4} \div 19\frac{3}{8}$.
15. $3\frac{9}{20} \div 2\frac{3}{10}$.
16. $4\frac{5}{9} \div 12\frac{1}{3}$.
17. $7\frac{3}{10} \div 1\frac{4}{5}$.
18. $4\frac{9}{10} \div 12\frac{5}{6}$.

## Kitchen Problems

### WRITTEN EXERCISE

1. Butter is worth 24 ct. a pound, and I bought \$1.86 worth to-day. How many pounds did I buy?

2. A recipe calls for dividing 6 qt. 1 pt. of milk into 4 equal parts. How many quarts and pints in each part?

3. I have 11 qt. $\frac{3}{4}$ pt. of cream, and a recipe calls for $1\frac{5}{8}$ pt. to each person. How many portions can I make?

4. How much does a purchaser gain by buying a barrel of flour (196 lb.) for \$5.75, over buying the same amount @ $3\frac{1}{2}$ ct. a pound?

5. We ordered from the grocer this morning, 10 lb. of sugar @ $5\frac{1}{2}$ ct., $2\frac{1}{2}$ lb. of tea @ 56 ct., and 8 lb. of coffee @ $32\frac{1}{2}$ ct. How much was the bill?

6. The weight of materials for a recipe for brown bread is as follows: corn meal 6 oz., rye meal 6 oz., flour 8 oz., sour milk $1\frac{1}{2}$ lb., molasses 12 oz., and $\frac{1}{2}$ oz. each of soda and salt. Required the total weight.

7. The cost is as follows: corn meal $1\frac{1}{4}$ ct., rye meal 1 ct., flour $1\frac{1}{2}$ ct., sour milk 4 ct., molasses $3\frac{1}{4}$ ct., and the soda and salt together $\frac{1}{4}$ ct. Required the total cost.

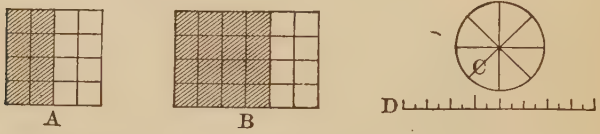
8. The weight of materials for a recipe for blueberry pudding for a family of 6 is as follows: bread crumbs 12 oz., blueberries $1\frac{1}{2}$ lb., sugar 8 oz., butter 2 oz. Required the total weight of these materials. What would be the weight of each of the materials for a family of 12?

9. The cost of the materials in Ex. 8 is as follows: bread crumbs 5 ct., blueberries 12 ct., sugar $2\frac{1}{2}$ ct., butter $2\frac{1}{2}$ ct., and the sauce for the pudding 3 ct. This being served to 6 persons, how much was the cost apiece?

## Summary of Principles of Fractions

### ORAL EXERCISE

1. In line D, point to $\frac{3}{16}$ of the line; also to $\frac{6}{16}$. We see that $\frac{6}{16}$ is how many times $\frac{3}{16}$? How does multiplying the numerator by 2 change the fraction? Then how does dividing the numerator by 2 change the fraction?




These figures can be used more easily if drawn on the board.

2. Point to $\frac{1}{2}$ of square A; also to $\frac{1}{4}$. We see that $\frac{1}{2}$ is how many times $\frac{1}{4}$? How does multiplying the denominator by 2 change the fraction? Then how does dividing the denominator by 2 change the fraction?

3. In rectangle B, show that $\frac{2}{3} = \frac{4}{6}$. Then does it change the value of a fraction to multiply or divide both terms by the same number? Show this with the circle C.

**123. Multiplying and dividing the terms of a fraction.**

*Multiplying the numerator by any number multiplies the fraction by that number.*

*Dividing the numerator by any number divides the fraction by that number.*

*Multiplying the denominator by any number divides the fraction by that number.*

*Dividing the denominator by any number multiplies the fraction by that number.*

*Dividing both terms of a fraction by the same number does not change the value of the fraction.*

ORAL EXERCISE

1. How many cents in $\$\frac{1}{8}$? Then 16 times $\$\frac{1}{8}$ is the same as 16 times how many cents?

2. How many cents in $\$\frac{1}{3}$? Then 9 times $\$\frac{1}{3}$ is the same as 9 times how many cents?

3. How many cents in $\$\frac{1}{6}$? Then 24 times $\$\frac{1}{6}$ is the same as 24 times how many cents?

**124. Multiplying by aliquot parts.** It is easier to multiply $\$\frac{1}{8}$, $\$\frac{1}{6}$, and $\$\frac{1}{3}$, than it is to multiply $12\frac{1}{2}$ ct., $16\frac{2}{3}$ ct., and $33\frac{1}{3}$ ct.

At $33\frac{1}{3}$ ct. each, 9 neckties cost 9 times $\$\frac{1}{3}$, or \$3.

At $16\frac{2}{3}$ ct. each, 12 bottles of ink cost 12 times $\$\frac{1}{6}$, or \$2.

At $12\frac{1}{2}$ ct. each, 9 balls cost 9 times $\$\frac{1}{8}$, or $\$1\frac{1}{8}$, or $\$1.12\frac{1}{2}$.

While goods are not marked $16\frac{2}{3}$ ct., they are often marked 6 for \$1, which is the same thing.

4. At $12\frac{1}{2}$ ct. a yard, what will 16 yd. of cloth cost?

5. At $16\frac{2}{3}$ ct. a yard, what will 18 yd. of cloth cost?

6. At $33\frac{1}{3}$ ct. a yard, what will 21 yd. of cloth cost?

7. At 8 doz. marbles for \$1, what will 6 doz. cost?

8. At \$1 per half dozen, what will 3 doz. bottles of mucilage cost?

9. At the rate of 3 for \$1, what must the school pay for 16 arithmetics?

10. At $16\frac{2}{3}$ ct. each, find the cost of 9 water pitchers; of a dozen plants for the school; of 6 yards of ribbon.

11. At $12\frac{1}{2}$ ct. each, find the cost of 10 glasses; of 4 lamp chimneys; of 20 cans of soup; of 32 yards of calico.

12. How many jackets can be cut from $17\frac{1}{2}$ yd. of cloth, allowing $2\frac{1}{2}$ yd. to a jacket?

### ORAL EXERCISE

1. How much is 8 times $\$\frac{1}{8}$? $\$1\frac{1}{8}$? $\$1.12\frac{1}{2}$?

2. How much is 6 times $\$\frac{1}{6}$? $\$1\frac{1}{6}$? $\$1.16\frac{2}{3}$?

3. How much is 3 times $\$\frac{1}{3}$? $\$1\frac{1}{3}$? $\$1.33\frac{1}{3}$?

4. At $\$1.33\frac{1}{3}$ a yard, how much will 15 yd. of lace cost?

5. At $\$1.16\frac{2}{3}$ a yard, how much will 9 yd. of silk cost?

6. At $\$1.12\frac{1}{2}$ a yard, how much will 6 yd. of velvet cost?

7. Find the cost of 12 books at $\$1.12\frac{1}{2}$ each; at $\$1.16\frac{2}{3}$.

8. At $\$1.33\frac{1}{3}$ each, find the cost of 3 chairs; of 18 chairs; of 33 chairs; of 36 chairs.

9. At $\$1.16\frac{2}{3}$ each, find the cost of 6 maps; of 12 maps; of 30 maps; of 54 maps.

10. At $\$1.12\frac{1}{2}$ each, find the cost of 8 books; of 40 books; of 32 books; of 20 books.

**125.** Therefore, at $\$1.12\frac{1}{2}$ each, 7 books cost $\$7 + \$\frac{7}{8}$, or $\$7.87\frac{1}{2}$ ($7.88).

At $\$1.16\frac{2}{3}$ each, 5 books cost $\$5 + \$\frac{5}{6}$, or $\$5.83\frac{1}{3}$ ($5.83).

At $\$1.33\frac{1}{3}$ each, 8 articles cost $\$8 + \$\frac{8}{3}$, or $\$10\frac{2}{3}$ ($10.67).

### WRITTEN EXERCISE

1. At $12\frac{1}{2}$ ct. each, find the cost of 147 balls.

2. At $16\frac{2}{3}$ ct. each, find the cost of 113 notebooks.

3. At $33\frac{1}{3}$ ct. each, find the cost of 101 handkerchiefs.

4. At $\$1.33\frac{1}{3}$ a volume, find the cost of 165 books.

5. At $\$1.16\frac{2}{3}$ a yard, find the cost of 142 yd. of lace.

6. At $\$1.12\frac{1}{2}$ each, find the cost of 112 yd. of velvet.

7. At $\$2.16\frac{2}{3}$ each, find the cost of 72 umbrellas.

8. What is the difference in cost of 144 books at $\$1.16\frac{2}{3}$ a volume, and the same number at $\$1.12\frac{1}{2}$ a volume?

## Review Drill

1. $47\frac{1}{2} + 93\frac{1}{3}$.
2. $72\frac{1}{4} + 83\frac{1}{6}$.
3. $29\frac{3}{4} + 47\frac{1}{5}$.
4. $62\frac{1}{5} + 35\frac{1}{3}$.
5. $85\frac{4}{5} + 16\frac{2}{3}$.
6. $87\frac{5}{6} + 19\frac{2}{3}$.
7. $67\frac{4}{5} + 29\frac{1}{6}$.
8. $89\frac{3}{7} + 23\frac{1}{2}$.
9. $67\frac{4}{5} + 49\frac{1}{2}$.
10. $39\frac{9}{10} + 28\frac{3}{4}$.
11. $87\frac{3}{5} + 62\frac{7}{10}$.
12. $37\frac{4}{7} + 18\frac{1}{14}$.
13. $79\frac{4}{5} - 22\frac{1}{2}$.
14. $137\frac{1}{2} - 86\frac{2}{3}$.
15. $48\frac{3}{4} - 23\frac{7}{8}$.
16. $92\frac{1}{4} - 37\frac{2}{5}$.
17. $30\frac{2}{3} - 19\frac{7}{8}$.
18. $82\frac{1}{2} - 68\frac{7}{8}$.
19. $72\frac{3}{5} - 56\frac{1}{8}$.
20. $41\frac{3}{8} - 16\frac{5}{6}$.
21. $72 - 46\frac{2}{3}$.
22. $27 \times 14\frac{2}{3}$.
23. $32 \times 92\frac{1}{2}$.
24. $67 \times 74\frac{2}{5}$.
25. $18 \times 96\frac{3}{4}$.
26. $30\frac{1}{2} \times 68$.
27. $40\frac{1}{3} \times 96$.
28. $32\frac{2}{5} \times 85$.
29. $41\frac{3}{4} \times 96$.
30. $\frac{2}{3}$ of $\frac{7}{8}$ of $1\frac{5}{7}$.
31. $10\frac{1}{3} \times 12\frac{1}{2}$.
32. $6\frac{1}{3} \times 7\frac{1}{5}$.
33. $4\frac{1}{4} \times 8\frac{1}{5}$.
34. $5\frac{3}{4} \times 9\frac{4}{5}$.
35. $29 \times 635\frac{1}{4}$.
36. $42 \times 408\frac{3}{4}$.
37. $52 \times 607\frac{3}{4}$.
38. $73 \times 429\frac{3}{7}$.
39. $87 \times 602\frac{4}{5}$.
40. $\frac{4}{5} \div \frac{2}{3}$.
41. $\frac{2}{3} \div \frac{4}{5}$.
42. $\frac{3}{7} \div \frac{2}{3}$.
43. $\frac{2}{3} \div \frac{3}{7}$.
44. $\frac{7}{8} \div \frac{4}{5}$.
45. $\frac{4}{5} \div \frac{7}{8}$.
46. $\frac{6}{7} \div \frac{3}{4}$.
47. $\frac{3}{4} \div \frac{6}{7}$.
48. $\frac{3}{4} \div 5\frac{1}{3}$.
49. $\frac{5}{7} \div 2\frac{1}{5}$.
50. $\frac{2}{3} \div 4\frac{2}{3}$.
51. $7\frac{6}{7} \div 1\frac{3}{4}$.
52. $9\frac{2}{3} \div 3\frac{1}{3}$.
53. $10\frac{4}{5} \div 3\frac{1}{2}$.
54. $5\frac{5}{7} \div 1\frac{1}{7}$.
55. $29\frac{1}{3} \div \frac{1}{3}$.
56. $3\frac{9}{10} \div 2\frac{1}{3}$.
57. $2\frac{4}{5} \div 3\frac{1}{2}$.
58. $4\frac{7}{8} \div 2\frac{2}{3}$.
59. $4\frac{6}{7} \div 1\frac{3}{14}$.
60. $16\frac{2}{3} \div 2\frac{1}{5}$.
61. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
62. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of 1 cu. ft.
63. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft.
64. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in.
65. $\frac{1}{2}$ of 5 times 8 sq. ft.
66. $\frac{3}{5}$ of 15 times 48 sq. ft.
67. $\frac{7}{8}$ of 4 times 16 sq. in.
68. $\frac{2}{7}$ of 13 times 14 sq. in.

Examples like the above should be given, a few at a time, with concrete problems like those on page 65, or should be assigned in groups for seat work, a certain number for ten minutes.

## Compound and Complex Fractions

**126. Compound fraction.** An indicated multiplication of fractions is sometimes called a *compound fraction.*

**127. Complex fraction.** An indicated division of fractions is sometimes called a *complex fraction.*

For example, $\frac{2}{3}$ of $\frac{4}{5}$ is a compound fraction, and $\frac{\frac{2}{3}}{\frac{4}{5}}$ and $\frac{\frac{3}{7}}{5}$ are complex fractions.

These terms are not of great value, but they are used in certain courses of study.

**128.** Such forms are treated the same as ordinary multiplications and division of fractions.

For example, $\frac{\frac{2}{3}}{5} = \frac{2}{3} \div 5 = \frac{2}{15}$.

### WRITTEN EXERCISE

*Reduce to the simpler forms in Exs. 1–24:*

1. $\frac{\frac{2}{3}}{4}$.
2. $\frac{\frac{3}{5}}{3}$.
3. $\frac{\frac{4}{7}}{14}$.
4. $\frac{\frac{6}{8}}{15}$.
5. $\frac{\frac{5}{8}}{\frac{7}{8}}$.
6. $\frac{6\frac{6}{11}}{2\frac{3}{22}}$.
7. $\frac{\frac{5}{9}}{1\frac{3}{18}}$.
8. $\frac{2\frac{1}{2}}{3\frac{1}{3}}$.
9. $\frac{\frac{3}{5}}{\frac{2}{3}}$.
10. $\frac{3\frac{4}{5}}{6\frac{7}{8}}$.
11. $\frac{8\frac{2}{3}}{16\frac{1}{4}}$.
12. $\frac{2}{3}$ of $\frac{3\frac{1}{3}}{4}$.
13. $\frac{3}{5}$ of $\frac{4\frac{1}{2}}{6}$.
14. $\frac{7}{8}$ of $\frac{2}{3\frac{1}{2}}$.
15. $\frac{5}{56\frac{1}{4}}$.
16. $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{5}+\frac{1}{3}}$.
17. $\frac{2\frac{1}{2}+\frac{2}{3}}{3\frac{1}{2}-\frac{1}{3}}$.
18. $\frac{4\frac{1}{2}+\frac{2}{3}}{4\frac{1}{2}-\frac{2}{3}}$.
19. $\frac{3}{5}$ of $\frac{1}{16}$ of $\frac{8}{9}$.
20. $\frac{2}{7}$ of $\frac{3}{4}$ of $2\frac{1}{2}$.
21. $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{5}{10}$.
22. $\frac{6}{11}$ of $\frac{22}{7}$ of $\frac{21}{2}$.
23. $\frac{5}{21}$ of $\frac{7}{15}$ of $\frac{3}{8}$.
24. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $2\frac{1}{2}$.

25. If Mr. A buys $\frac{3}{4}$ of the share of Mr. B who owns $\frac{5}{16}$ of a farm, and then sells $\frac{2}{3}$ of what he buys to Mr. C, what part of the farm does Mr. C own?

## Fractional Relations

### ORAL EXERCISE

1. What part of 4 is 2? What part of 10 is 8? How do you find what part one number is of another?

2. What part of 7 is 5? of 9 is 4? of 7 is 6? of 16 is 12? of 25 is $12\frac{1}{2}$? of 100 is $33\frac{1}{3}$? of 100 is $16\frac{2}{3}$?

**129. Fractional parts.** It therefore appears that

*To find what part the second of two numbers is of the first, divide the second by the first.*

For example, what part of 51 is 17? 17 is $\frac{17}{51}$, or $\frac{1}{3}$ of 51.

### WRITTEN EXERCISE

*In Exs. 1–24 the second number is what part of the first?*

| | | |
|---|---|---|
| 1. 14, 8. | 2. 18, 14. | 3. $\frac{7}{8}$, $\frac{2}{3}$. |
| 4. $\frac{2}{7}$, $\frac{6}{11}$. | 5. $3\frac{1}{3}$, $1\frac{2}{3}$. | 6. 17, $8\frac{1}{2}$. |
| 7. 82, $9\frac{1}{9}$. | 8. $41\frac{3}{8}$, $5\frac{1}{8}$. | 9. $66\frac{2}{3}$, $16\frac{2}{3}$. |
| 10. 243, 27. | 11. 100, $37\frac{1}{2}$. | 12. $45\frac{3}{5}$, $7\frac{3}{5}$. |
| 13. $16\frac{5}{8}$, $2\frac{3}{8}$. | 14. $42\frac{1}{2}$, $14\frac{1}{6}$. | 15. $62\frac{1}{8}$, $7\frac{7}{8}$. |
| 16. $66\frac{2}{3}$, $33\frac{1}{3}$. | 17. 225, $37\frac{1}{2}$. | 18. $22\frac{1}{2}$, $16\frac{2}{3}$. |
| 19. $41\frac{7}{8}$, $21\frac{3}{8}$. | 20. $37\frac{1}{2}$, $33\frac{1}{3}$. | 21. 421, $60\frac{1}{7}$. |
| 22. $33\frac{1}{3}$, $16\frac{2}{3}$. | 23. 427, $47\frac{4}{9}$. | 24. 325, $29\frac{6}{11}$. |

25. Frank is 12 yr. old. His age is what part of his brother's, who is 18?

26. The age of his brother, who is 18, is what part of his uncle's, who is 54?

27. If Frank is 12 and his brother is 18, Frank's age was what part of his brother's 6 yr. ago? It will be what part of his brother's 6 yr. from now?

### ORAL EXERCISE

1. 10 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number?

2. 12 is $\frac{1}{3}$ of what number? If 12 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of the number? What is $\frac{3}{3}$ of it?

3. 15 is $\frac{3}{7}$ of some number. What is $\frac{1}{7}$ of it? all of it?

4. 21 is $\frac{7}{8}$ of what number? $\frac{3}{5}$ of what number?

**130. Illustrative problem.** If $33\frac{1}{3}$ is $\frac{2}{3}$ of some number, what is the number?

Because $\frac{2}{3}$ of some number $= 33\frac{1}{3}$, or $\frac{100}{3}$,
therefore $\frac{1}{3}$ of the number $= \frac{1}{2}$ of $\frac{100}{3} = \frac{50}{3}$,
and $\frac{3}{3}$ of the number $= 3$ times $\frac{50}{3} = 50$.

Therefore $33\frac{1}{3}$ is $\frac{2}{3}$ of 50.

### WRITTEN EXERCISE

*Find the number of which*

| | | |
|---|---|---|
| 1. 23 is $\frac{1}{3}$. | 2. 38 is $\frac{2}{3}$. | 3. 63 is $\frac{7}{8}$. |
| 4. 75 is $\frac{3}{8}$. | 5. 45 is $\frac{9}{10}$. | 6. 150 is $\frac{3}{8}$. |
| 7. 222 is $\frac{6}{7}$. | 8. 125 is $\frac{3}{3}\frac{5}{2}$. | 9. 144 is $\frac{9}{16}$. |
| 10. \$150 is $\frac{3}{5}$. | 11. \$2.25 is $\frac{5}{8}$. | 12. \$10.25 is $\frac{5}{3}$. |

13. If there are 85 girls in a school, and they are $\frac{5}{9}$ of the total number of pupils, how many pupils are there?

14. In my library are 164 books on history, and these are $\frac{27}{107}$ of all my books. How many books have I?

15. If a man spends \$1687.50 this year, and this is $\frac{3}{4}$ of his income, how much is his income?

16. A man left to his son $\frac{3}{8}$ of his estate, this share amounting to \$3210. To his daughter he left $\frac{1}{10}$ of his estate, and the rest to his wife. How much did the wife and daughter each receive?

### ORAL EXERCISE

1. What will $2\frac{1}{2}$ lb. of coffee cost at 24¢ a pound?

2. What will $\frac{3}{4}$ lb. of pepper cost at 24¢ a pound?

3. What will $2\frac{1}{2}$ lb. of butter cost at 26¢ a pound?

4. What will 5 oz. of cinnamon cost at 32¢ a pound?

5. What will 20 lb. of loaf sugar cost at $6\frac{1}{4}$¢ a pound?

6. What will $\frac{3}{4}$ doz. hats cost at $36 a dozen?

7. At 32¢ a gallon, what will $12\frac{1}{2}$ gal. of vinegar cost?

8. What will $1\frac{1}{2}$ lb. of cocoa cost at 22¢ per half-pound can?

9. At 32¢ a pound, what will $1\frac{1}{4}$ lb. of ground ginger cost?

10. At the rate of 80¢ for 10 lb. of rice, how much must I pay for 15 lb.?

11. At the rate of $1.64 for $\frac{1}{2}$ gal. of olive oil, how much will $2\frac{1}{2}$ gal. cost?

12. I can buy a 14-lb. box of oatmeal for $1.54. At this rate, what do I pay for 5 lb.?

13. At 30¢ a pound, how much will $\frac{3}{4}$ lb. of mustard cost? (Always count the half cent as a cent.)

14. I can buy a box of 60 cakes of soap for $2.94, or the same soap at $5\frac{1}{2}$¢ a cake. What shall I gain on 60 cakes by buying the whole box at once?

15. I can buy a 5-lb. can of baking powder for $1.95. How much cheaper is this than buying the same amount in half-pound cans at 23¢ each?

16. A hotel needs a pint and a half of vanilla extract this week. How much cheaper is it to buy a pint bottle for 98¢ and a half-pint bottle for 54¢, than to buy it in half-pint bottles?

### WRITTEN EXERCISE

1. At \$1.38 a yard, what will 37 yd. of carpet cost?

2. In buying 2 doz. cans of peaches, how much is gained by buying @ \$3.73 a dozen over buying at 3 cans for a dollar?

3. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ 16¢, $3\frac{1}{2}$ lb. of codfish @ 12¢, and 4 cans of salmon at the rate of 2 for a quarter?

4. I bought $\frac{1}{2}$ gal. of olive oil @ \$3.30 a gallon, but $\frac{1}{4}$ of what I bought leaked out. What rate per gallon do I pay for what is left?

5. In buying 48 cans of tomatoes, how much is gained by buying two cases of 2 doz. each, @ \$2.89 a case, over buying at the rate of 3 cans for 40¢?

6. I bought 3 gal. of best maple sirup @ $\$1.12\frac{1}{2}$ a gallon, $2\frac{1}{2}$ gal. New Orleans molasses @ 62¢ a gallon, and gave the grocer a \$5 bill. How much change did I get?

7. I bought 6 lb. of prunes @ $12\frac{1}{2}$¢, 8 lb. of dried apricots @ $12\frac{1}{2}$¢, $2\frac{1}{2}$ lb. of raisins @ 16¢, $1\frac{1}{2}$ lb. of figs @ 22¢, and $\frac{1}{2}$ lb. English walnuts @ 18¢, and gave the grocer a \$5 bill. How much change did I get?

8. I bought 8 cans of peaches @ \$3.75 a dozen, 15 cans of tomatoes @ $\$1.37\frac{1}{2}$ a dozen, 9 cans of soup @ \$1.50 a dozen, $3\frac{1}{2}$ lb. of raisins @ 18¢, $2\frac{1}{2}$ lb. of nuts @ 17¢, $1\frac{1}{2}$ gal. of sirup @ 60¢, $1\frac{1}{2}$ doz. cakes of soap @ 42¢ a dozen. How much was my bill?

9. I bought 10 lb. of white beans @ $6\frac{1}{4}$¢, $3\frac{1}{2}$ lb. split peas @ 7¢, 3 cans of soup @ $12\frac{1}{2}$¢, and $2\frac{1}{2}$ lb. of paraffin candles @ $12\frac{1}{2}$¢. How much was my bill? (Neglect fractions less than $\frac{1}{2}$¢, although in a bill, as on page 75, a fraction of a cent is usually called a cent.)

## Cash Checks in Large Stores

**131. Cash checks.** In large stores the clerk fills out a small cash check like the following:

| Sold by No. 127 | | Am't rec'd $15.00 | | |
|---|---|---|---|---|
| ½ yd. | Silk | .57 | | 29 |
| 4⅜ " | Summer Silk | .38 | 1 | 67 |
| 11½ " | Silk | .99 | 11 | 39 |
| | Total | | 13 | 35 |
| | Change due | | 1 | 65 |
| | | | $15 | 00 |

### WRITTEN EXERCISE

*Make out cash checks for the following sales:*

1. 14½ yd. gingham @ 25¢, 9 yd. cotton @ 12½¢, 10½ yd. madras @ 35¢. Amount received, $10.

2. 6½ yd. cheviot @ $1.10, 5⅜ yd. silk @ $1.25, 9½ yd. velveteen @ 98¢. Amount received, $25.

3. 11½ yd. Persian lawn @ $1.95, 6⅝ yd. dimity @ 25¢, 12⅛ yd. linen suiting @ 75¢. Amount received, $40.

4. 9½ yd. Persian lawn @ $1.37½, 5⅛ yd. cheviot @ $1.25, 15 yd. cotton @ 12½¢. Amount received, $30.

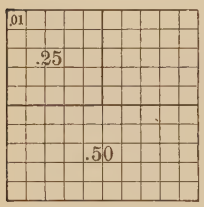
5. 7 yd. muslin @ 25¢, 12½ yd. lining @ 11¢, 6⅝ yd. lawn @ $1.50, 7 yd. suiting @ 75¢. Amount received, $20.

6. 16 yd. velvet @ $2.25, 14½ yd. suiting @ 48¢, 23 yd. cotton @ 15¢, 6⅜ yd. dimity @ 24¢, 7¼ yd. ribbon @ 25¢. Amount received, $50.

## DECIMAL FRACTIONS

### ORAL EXERCISE

1. In the picture each column of squares is what part of the large square? Point to $\frac{5}{10}$ of the large square.




2. Each small square is what part of a column? of the large square? Point to $\frac{50}{100}$ of the large square; to $\frac{25}{100}$.

3. If the large square represents \$1, what part represents 1 dime? 1 cent?

**132. Meaning of decimals.** Just as \$3.25 means $\$3 + \$\frac{25}{100}$, so 3.25 ft. means 3 ft. + $\frac{25}{100}$ ft., or $3\frac{1}{4}$ ft.; 4.75 in. means 4 in. + $\frac{75}{100}$ in., or $4\frac{3}{4}$ in.; 2.5 means $2 + \frac{5}{10}$, or $2\frac{1}{2}$; 0.5, or .5, means $\frac{5}{10}$, or $\frac{50}{100}$, or .50; 0.125, or .125, means $\frac{125}{1000}$, or $\frac{1}{8}$.

**133. Decimal fractions.** A fraction whose denominator is not written, but is some power of 10, is called a *decimal fraction.*

**134. Decimals.** Decimal fractions are usually called *decimals.*

**135. Decimal point.** The period placed at the left of tenths is called the *decimal point.*

Thus, $0.2 = \frac{2}{10}$, or $\frac{1}{5}$; $0.7 = \frac{7}{10}$; $0.8 = \frac{8}{10}$, or $\frac{4}{5}$; $.06 = \frac{0}{10} + \frac{6}{100}$, or $\frac{3}{50}$; $.08 = \frac{8}{100} = \frac{4}{50} = \frac{2}{25}$; $.007 = \frac{7}{1000}$; $.125 = \frac{125}{1000} = \frac{1}{8}$; $.1231 = \frac{1231}{10000}$.

**136.** The expression 0.5 means the same as .5. The 0 may be written or not as we please.

**137. Reading decimals.** 125.375 is read, "one hundred twenty-five *and* three hundred seventy-five thousandths."

Pupils should be given much exercise in reading such numbers from the blackboard, avoiding the use of "and" except between the units and tenths, and the vulgarism of "aught" for "naught."

### ORAL EXERCISE

*Read the numbers in Exs. 1–12:*

| | | | |
|---|---|---|---|
| 1. 1.25. | 2. 3.78. | 3. 2.95. | 4. 6.33. |
| 5. 0.125. | 6. 48.675. | 7. 0.375. | 8. 67.005. |
| 9. 32.375. | 10. 100.025. | 11. 91.075. | 12. 300.075. |

13. What is the name of the place to the right of tenths? to the right of that? What is the name of the fourth decimal place? the fifth? the sixth?

**138.** Since we rarely read aloud the names of numbers beyond three decimal places, even when we write them, the chief drill should be on reading decimal fractions like those in Exs. 1–12.

*Read the following:*

14. 16.004. 15. 82.3745. 16. 91.8273. 17. 68.9982.

18. 75.1375. 19. 12.31478. 20. 16.42398. 21. 14.763458.

### WRITTEN EXERCISE

*Write decimally the fractions in Exs. 1–5:*

1. Three tenths; five tenths; nine tenths.

2. Four and four tenths; seven and eight tenths.

3. Seventy-five hundredths; thirty-three and a third hundredths; three hundred and three hundredths.

4. One hundred forty-four thousandths; six hundred seventy-five thousandths; four ten-thousandths.

5. Five and seven thousandths; eight and forty-two thousandths; twenty-five thousandths; one millionth.

6. Reduce to lowest terms the fractions $\frac{30}{100}$ and $\frac{300}{10000}$. What does this tell you of the values of 0.30 and 0.300?

7. As in Ex. 6, show that $0.250 = 0.25 = \frac{1}{4}$; that $0.500 = 0.50 = 0.5 = \frac{1}{2}$; that $0.7500 = 0.750 = 0.75 = \frac{3}{4}$.

## Reduction of Decimal Fractions

### ORAL EXERCISE

1. Express $\frac{5}{10}$ as hundredths; as thousandths. Write the results as decimal fractions on the blackboard.

2. Express as tenths: $\frac{1}{2}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{2}$, $\frac{4}{2}$ or 2, $\frac{5}{2}$ or $2\frac{1}{2}$.

3. Express as hundredths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{50}$, $\frac{3}{25}$, $\frac{11}{50}$.

**139. Annexing zeros to decimals.** Because $\frac{5}{10} = \frac{50}{100}$, or $.5 = .50$, we see that

*Annexing a zero to a decimal fraction does not change its value.*

**140. Reducing decimals to common fractions.** Because $.25 = \frac{25}{100}$, which equals $\frac{1}{4}$,

*To reduce a decimal to a common fraction, write the denominator of the decimal and then reduce the fraction to lowest terms.*

### WRITTEN EXERCISE

*Reduce to common fractions in lowest terms, or to mixed numbers, in Exs. 1–16:*

| | | | |
|---|---|---|---|
| 1. .4. | 2. .6. | 3. .2. | 4. .8. |
| 5. .25. | 6. .75. | 7. .45. | 8. .55. |
| 9. .125. | 10. .375. | 11. .625. | 12. .875. |
| 13. 1.25. | 14. 2.75. | 15. 5.125. | 16. 10.375. |

17. Express as hundredths: 0.7, 0.6, 0.1, 0.8, 4, 63.

18. Express as thousandths: 0.1, 0.25, 0.37, 0.09, 31.

19. Express as hundredths: .470, .680, .100, .040, 7.8.

20. Express 0.125 as eighths. Find 0.125 of $1600.

### ORAL EXERCISE

1. Express 0.25 as a common fraction; $\frac{3}{50}$ as a decimal.
2. Express 0.40 as a common fraction; $\frac{3}{25}$ as a decimal.

*Express the numbers in Exs. 3–12 as common fractions:*

3. 0.2. 4. 0.4. 5. 1.5. 6. 0.6. 7. 0.8.
8. 0.20. 9. 0.40. 10. 0.75. 11. 1.125. 12. 0.250.

*Express as decimals:*

13. $\frac{1}{5}$. 14. $\frac{2}{5}$. 15. $\frac{1}{2}$. 16. $\frac{3}{8}$. 17. $\frac{4}{5}$.
18. $\frac{1}{4}$. 19. $\frac{3}{4}$. 20. $\frac{1}{8}$. 21. $\frac{3}{50}$. 22. $\frac{7}{20}$.

**141. Reducing common fractions to decimals.** Because

$$\frac{1}{3} = \frac{1.0}{3} = 0.3\frac{1}{3}, \quad \text{and} \quad \frac{1}{3} = \frac{1.00}{3} = 0.33\frac{1}{3},$$

*To reduce a common fraction to a decimal fraction, divide the numerator by the denominator.*

Write the decimal point, and annex zeros as necessary.

**142. Exact decimals not always possible.** A common fraction cannot always be reduced to an exact decimal.

Thus, $\frac{1}{3} = 0.3\frac{1}{3} = 0.33\frac{1}{3} = 0.333\frac{1}{3}$, and so on, or 0.333+.

No common fraction equals an exact decimal if the denominator contains other prime factors than 2 and 5, when in lowest terms.

### WRITTEN EXERCISE

*Express as decimals, carrying the reduction only to thousandths in case of inexact decimals:*

1. $\frac{7}{8}$. 2. $\frac{3}{20}$. 3. $\frac{4}{15}$. 4. $\frac{5}{6}$. 5. $\frac{6}{7}$.
6. $\frac{5}{9}$. 7. $\frac{7}{11}$. 8. $\frac{3}{28}$. 9. $\frac{5}{19}$. 10. $\frac{12}{35}$.
11. $\frac{125}{300}$. 12. $\frac{121}{150}$. 13. $\frac{32}{111}$. 14. $\frac{412}{675}$. 15. $3\frac{47}{61}$.
16. $3.07\frac{1}{2}$. 17. $4.05\frac{1}{5}$. 18. $6.75\frac{2}{5}$. 19. $4.00\frac{1}{2}$. 20. $7.25\frac{3}{5}$.

## Addition and Subtraction

### ORAL EXERCISE

*Write on the blackboard and add aloud the numbers in Exs. 1–10:*

| | | | | |
|---|---|---|---|---|
| 1. 2 ft. | 2. $2 | 3. $0.02 | 4. $2.22 | 5. .02 ft. |
| 6 | 6 | .06 | 6.66 | .06 |
| 6. .02 | 7. .04 | 8. .3 | 9. .6 | 10. .60 |
| .06 | .05 | .4 | .7 | .70 |

An abbreviation like ft., at the top of a column, refers to all numbers in the column.

*Write on the blackboard and subtract aloud the following:*

| | | | | |
|---|---|---|---|---|
| 11. 7 ft. | 12. $0.07 | 13. .07 | 14. .7 | 15. 7.77 |
| 2 | .02 | .02 | .2 | 2.22 |
| 16. $0.11 | 17. .115 | 18. 1.1 | 19. 1.512 | 20. 1.251 |
| .04 | .041 | .4 | .602 | .851 |

**143. Adding and subtracting decimals.** We see that to add or subtract numbers containing decimal fractions we

*Write like orders under one another and add or subtract as with integers.*

**144.** To add 2.5, 3.06, 9.475, and .1.

If the decimal points are in a column, units will come under units, tenths under tenths, and so on. Read the columns like a word, saying, "5, 13, 11, 15."

```
 2.5
 3.06
 9.475
  .1
------
15.135
```

**145.** To subtract .26 from 4.1.

Arranging the decimal points in a column and writing 4.1 as 4.10 (§ 139), the subtraction is the same as with United States money.

```
 4.10
  .26
------
 3.84
```

### ORAL EXERCISE

1. Add: $\frac{5}{10} + \frac{6}{10}$, $1\frac{5}{100} + 1\frac{6}{100}$, $0.5 + 0.6$, $0.05 + 0.06$.
2. Add: $1\frac{6}{10} + 3\frac{9}{10}$, $1\frac{6}{100} + 3\frac{9}{100}$, $1.6 + 3.9$, $1.06 + 3.09$.
3. Subtract: $\frac{6}{25} - \frac{9}{25}$, $\frac{6}{10} - \frac{9}{10}$, $1\frac{6}{10} - \frac{9}{10}$, $1.6 - 0.9$.
4. Subtract: $125 - 50$, $\$1.25 - \$0.50$, $1.25 - 0.50$.

### WRITTEN EXERCISE

See how many of these examples you can solve in ten minutes.

*Add the numbers in Exs. 1–8:*

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| \$2.47 | \$15.25½ | 28.92 ft. | 68. | 432. |
| 3.62 | 7.42½ | 6.87 | 0.09¼ | 41.025 |
| 4.81 | 19.00 | 9.09 | 7.4 | 7.875 |
| 9. | 6.04 | 125. | 321. | .905 |
| 28.06 | 7.87 | 7.75 | 40.08¾ | .095 |

6. $132 + 0.96 + 8.04 + 19.6 + 0.05 + 98.03$.
7. $42.06 + 98.03 + 925 + 0.6 + 0.85 + 2.5 + 3.4$.
8. $243 + 489 + 0.09 + 0.16 + 0.49 + 0.98 + 27.62$.

*Subtract as indicated in Exs. 9–29:*

| 9. | 10. | 11. | 12. |
|---|---|---|---|
| \$0.25½ | \$7.25 | \$9.63 | \$5.70 |
| 0.07½ | 1.50 | 2.78 | 2.95 |
| **13.** | **14.** | **15.** | **16.** |
| 67.025 | 42.57 | 48.054 | 540. |
| 9.8 | 6.64 | 6.97 | 0.675 |
| **17.** | **18.** | **19.** | **20.** |
| \$128.42 | \$142. | 37.204 | 203. |
| 7.06 | 4.92 | 9.32 | 94.602 |

21. $78.2 - 9.3$.
22. $80 - 0.07$.
23. $68.04 - 7.6$.
24. $40.04 - 0.4$.
25. $628 - 519.1$.
26. $78.1 - 8.01$.
27. $600.06 - 66.6$.
28. $625 - 500.25$.
29. $37.27 - 28.19$.

### ORAL EXERCISE

1. A small geography cost \$0.60 and a music book \$0.35. How much did the two cost?

2. An arithmetic cost \$0.65 and a paint box \$0.35. How much did the two cost?

3. A history cost \$0.60. I gave the dealer a 50-ct. piece and a quarter of a dollar. How much change did I receive?

4. I bought a large geography for \$1.25, and gave the dealer a dollar and a half. How much change did I receive?

5. How much more did the large geography cost than the small one? How much more than the arithmetic?

### WRITTEN EXERCISE

1. What was the total cost of all the purchases mentioned in the oral examples?

2. There are 15 girls in the class, and each bought an arithmetic. How much did these books cost them?

3. There are 14 boys in the class, and each bought a large geography. How much did these books cost them?

4. If each of the boys and each of the girls bought a paint box, how much was paid for paint boxes?

5. John paid during the term \$0.08 for pencils, \$0.24 for paper, and \$0.15 for a drawing book. Required the cost of these materials.

6. Make a list of the materials used by you this year in school, with the cost of each book, pencil, and other article, whether paid by you or the school. Add and find the total cost.

## The Farmer's Potato Crop

### WRITTEN EXERCISE

1. The greatest recent average state yield of potatoes per acre was in Nevada, where 1753 acres produced 190 bu. to the acre. What was the total yield?

2. When the average value of potatoes per acre in Michigan was \$25.11, and 171,453 acres were devoted to this product, what was the value of the total crop?

3. When New York had 339,000 acres in potatoes, and produced 27,628,500 bu. a year, what was the average number of bushels per acre?

4. If these New York potatoes sold at an average price of \$0.48 a bushel, what was the value of the total crop?

5. The state having the highest recent average yield of potatoes per acre in a year is Nevada, 190 bu. Montana averaged 170 bu., Wyoming 167 bu., and Maine 165 bu. What is the average of these amounts?

6. There are a few records of 500 bu. to the acre. If a man had 10 acres of such land, and the potatoes sold for 47 ct. a bushel, and it cost the farmer \$46.25 an acre to plant and cultivate the crop, what would he make a year?

7. A bushel of potatoes weighs 60 lb. How much would the potatoes raised on a 16-acre lot weigh, the average yield being 213.5 bu. per acre?

## Multiplication

### ORAL EXERCISE

1. Multiply by 7: 2 ft., .2, \$30, .030, .30, .003.
2. Multiply by 2: 7 in., $\frac{7}{10}$, .8, .9, .07, .007, .006.
3. Multiply by 5: \$11, .11, $\frac{1}{10}$, 1.1, .011, .01, .002.
4. Multiply by 10: $\frac{1}{10}$, $\frac{1}{10}$, 1.1, 1.25, .25, .2, .03, .07.

**146. Multiplying by integers.** Multiplying decimal fractions by integers is already familiar, especially in working with United States money. Thus, to multiply 24.125 by 35,

$$\begin{array}{r} 24.125 \\ 35 \\ \hline 120\,625 \\ 723\,75 \\ \hline 844.375 \end{array}$$

We know where to put the decimal point, because thousandths multiplied by a whole number must be thousandths.

This subject is considered more fully on page 106.

### WRITTEN EXERCISE

*Multiply in Exs. 1–12:*

1. 27.25 by 25.
2. 65.23 by 32.
3. 13.45 by 72.
4. 61.82 by 37.
5. 4.375 by 81.
6. 6.925 by 93.
7. 42.125 by 45.
8. 68.325 by 50.
9. 20.005 by 75.
10. 102.002 by 35.
11. 42.175 by 62.
12. 29.407 by 87.
13. If a 12-story city building is 10.62 ft. to a story, how tall is it?
14. If one side of a square is 21.33 in., how long is the perimeter (sum of the sides)?
15. If a triangle has three equal sides, each 14.72 in., how long is the perimeter?
16. The measure called the meter, studied later, is 39.37 in. long. How many inches in 62 meters?

## Multiplying by Powers of 10

### ORAL EXERCISE

1. How much is $2 \times \frac{1}{2}$? $3 \times \frac{1}{3}$? $5 \times \frac{1}{5}$? $10 \times \frac{1}{10}$? $10 \times 0.1$?

2. How much is $10 \times 0.2$? $10 \times 0.6$? $10 \times \frac{1}{100}$? $10 \times 0.01$?

3. How much is $100 \times \frac{1}{100}$? $100 \times 0.01$? $100 \times 0.02$?

4. How much is $10 \times \frac{25}{100}$? $100 \times \frac{25}{100}$? $10 \times 0.25$? $100 \times 0.38$? $10 \times 0.50$? $100 \times 0.75$? $1000 \times \frac{25}{1000}$? $1000 \times 0.025$?

5. We have found that to multiply a decimal fraction by 10, we need only move the decimal point one place in which direction? How is it when we multiply by 100? by 1000?

6. Multiply by 10: 9.6, 98.4, 0.96; also by 100.

**147. Multiplying by powers of 10.** We found on page 17 a rule for multiplying by powers of 10. What was it?

**148. Multiplying by multiples of 10.** Therefore, to multiply 487.7 by 600 is the same as to multiply by 6, moving the decimal point 2 places to the right (see page 17).

In the same way, to multiply 89.07 by 740 is the same as to multiply by 74, moving the decimal point 1 place to the right.

### WRITTEN EXERCISE

*Multiply:*

| | | |
|---|---|---|
| 1. $40 \times 6.73$. | 2. $30 \times 4.87$. | 3. $60 \times 9.81$. |
| 4. $90 \times 86.4$. | 5. $80 \times 36.5$. | 6. $70 \times 625.3$. |
| 7. $800 \times 65.42$. | 8. $900 \times 37.08$. | 9. $400 \times 61.8$. |
| 10. $1000 \times 1.25$. | 11. $2000 \times 6.725$. | 12. $6970 \times 432$. |
| 13. $9250 \times 67.25$. | 14. $8220 \times 350.4$. | 15. $6700 \times 495.4$. |
| 16. $2500 \times 374.65$. | 17. $3500 \times 408.08$. | 18. $35{,}000 \times 1.25$. |

## Division

### ORAL EXERCISE

1. Divide by 4: 16 ft., $\frac{1}{25}$, .16, .016, 1.6, .160.
2. Divide by 5: 55 in., $\frac{55}{64}$, .55, .055, 5.5, .550.
3. Divide by 7: \$63, $\frac{63}{64}$, .63, .063, $\frac{63}{10}$, 6.3, .630.
4. Divide by 10: \$450, $\frac{450}{1000}$, 4.50, 4.5, .450, .45, .75.

**149. Dividing by integers.** We have seen that *we write United States money in decimal fractions*, \$2.50 meaning merely $\$2+\$\frac{50}{100}$, or \$2+\$0.50. And since we have learned how to divide a number representing United States money (page 22), we know how to divide a decimal fraction. For example, to divide 844.375 by 35.

This subject is taken up again on page 110, this being only an introduction.

```
  24.125
35)844.375
   70
   144
   140
    43
    35
    87
    70
    175
    175
```

### WRITTEN EXERCISE

1. $1795.5 \div 42$.
2. $48.255 \div 15$.
3. $1583.25 \div 75$.
4. $691.53 \div 89$.
5. $1961.96 \div 98$.
6. $1693.44 \div 36$.
7. $4416.93 \div 63$.
8. $23{,}340.75 \div 75$.

9. The perimeter of a square is 851 in. What is the length of each side?

10. The perimeter of a triangle whose sides are all equal is 1237.9 in. What is the length of each side?

11. The perimeter of a six-sided figure, with equal sides, is 7.62 in. What is the length of each side?

## Dividing by Powers of 10

### ORAL EXERCISE

*State the results in Exs. 1–8:*

| | | | |
|---|---|---|---|
| 1. $10 \times 0.4$, | $4 \div 10$. | 2. $10 \times 1.4$, | $14 \div 10$. |
| 3. $10 \times \$0.04$, | $\$0.40 \div 10$. | 4. $10 \times 0.04$, | $0.4 \div 10$. |
| 5. $10 \times \$0.25$, | $\$2.50 \div 10$. | 6. $10 \times 0.25$, | $2.5 \div 10$. |
| 7. $10 \times 12.5$, | $125 \div 10$. | 8. $100 \times 0.04$, | $4 \div 100$. |

**150. Dividing by powers of 10.** From the above exercise and from page 23, the following is found:

1. *To divide a number by 10, move the decimal point one place to the left.*

Any integer may be thought of as having a decimal point after units. That is, 6 is the same as 6., or 6.0, or 6.00.

2. *To divide a number by 100, move the decimal point two places to the left.*

Thus, $7 \div 100 = .07$.

3. *To divide a number by 1000, move the decimal point three places to the left.*

Therefore, to divide 216 by 600 is the same as to divide 2.16 by 6.

$$6\text{ø}\text{ø}\overline{)2|16}\quad .36$$

### WRITTEN EXERCISE

| | | |
|---|---|---|
| 1. $133 \div 70$. | 2. $10.8 \div 60$. | 3. $32.9 \div 70$. |
| 4. $3745 \div 500$. | 5. $2291 \div 290$. | 6. $296.1 \div 470$. |
| 7. $273.6 \div 120$. | 8. $2984 \div 4000$. | 9. $7344 \div 9000$. |
| 10. $625 \div 25{,}000$. | 11. $961 \div 31{,}000$. | 12. $882 \div 21{,}000$. |
| 13. $1728 \div 1200$. | 14. $144 \div 60{,}000$. | 15. $872 \div 22{,}000$. |
| 16. $7260 \div 2500$. | 17. $154 \div 12{,}000$. | 18. $145.7 \div 4700$. |

### WRITTEN EXERCISE

In this exercise pupils should state the approximate answers orally, and these should be written down. The problems should then be solved on paper and the real result compared with the approximate one. This is one of the best checks against absurd results, and should frequently be made a subject of class drill with other written exercises. Teachers will not expect the same approximations from all pupils. $3\frac{1}{2}$ times 22¢ is a natural approximation for Ex. 1.

1. At 22¢ a pound, what will $3\frac{5}{8}$ lb. of steak cost?

2. At 18¢ a gallon, what will 6.25 gal. of oil cost?

3. At 26¢ a pound, what will $6\frac{7}{8}$ lb. of butter cost?

4. At $12\frac{1}{2}$¢ a yard, what will $8\frac{3}{4}$ yd. of ribbon cost?

5. At $2.50 a yard, what will 2.8 yd. of velvet cost?

6. If 6 yd. of silk cost $6.75, what is the price per yard?

7. If 8 doz. men's hose cost $55, what is the price per dozen?

8. A man drives 39.6 mi. in $5\frac{1}{2}$ hr. What is the rate per hour?

9. If 48 doz. boys' hose cost $57.60, what is the price per dozen?

10. At 14¢ a gross, how many gross of buttons can be bought for $3.22?

11. A dealer buys 200 lb. of raisins for $19. How much do they cost a pound?

12. A dealer pays $18.83 for 7 rocking-chairs. How much does he pay for each?

13. A dealer sells 17 pails of mackerel for $40.80. What is the price per pail?

14. A man pays $1110 a year for the rent of a city house. How much does he pay a month?

15. Traveling at the rate of 47 mi. an hour, how far will a train go between 8.45 A.M. and 11.30 A.M.?

16. A grocer buys a 50-gal. barrel of Porto Rico molasses for $11.50. How much does it cost per gallon?

17. A grocer sells a half chest of tea, 140 lb. to the chest, for $19.60. What is the price per pound?

18. A merchant sold a 40-yd. piece of cloth for $45, gaining $12\frac{1}{2}¢$ a yard. How much did the piece cost him?

19. A grocer buys 12 doz. 1-lb. cans of cocoa at $43¢$ each, and sells them at $58¢$ each. How much does he gain?

20. A dry goods dealer sells 75 yd. of cloth for $112.50, thereby gaining $28.50. How much did it cost him per yard?

21. A grocer buys six 50-lb boxes of dried apples at $4\frac{1}{2}¢$ a pound, and sells them at $6¢$ a pound. How much does he gain?

22. A salesman receives $15 a week. He works 49 weeks a year. His expenses are $396 a year. How much does he save a year?

23. If 15 men can do a piece of work in 4 da. 7 hr., a working day being 8 hr., how long will it take 3 men, working at the same rate?

24. Four places, A, B, C, D, are in a straight line. It is 17.42 mi. from A to B, 13.74 mi. from B to C, and 52.11 mi. from A to D. How far is it from C to D?

25. A merchant pays $38.40 for a piece of cloth. If there had been 3 yd. more in the piece, it would have cost him $42. How many yards were there in the piece?

26. Mr. A and Mr. B are in partnership, Mr. A contributing $\frac{3}{5}$ of the money and labor to run the business. If they make $6725 this year, what is the share of each?

## DENOMINATE NUMBERS

### Review of the Common Tables

#### ORAL EXERCISE

1. How many ounces in a pound? An ounce is what part of a pound?

2. Tell the number of gills in a pint, pints in a quart, and quarts in a gallon.

3. Tell the number of inches in a foot and feet in a yard. An inch is what part of a foot?

4. Tell the number of quarts in a peck and pecks in a bushel. A peck is what part of a bushel?

5. How many seconds in a minute? minutes in an hour? hours in a day? days in a week? days in a common year? days in a leap year? What months have 30 days?

*Express the following as indicated:*

6. 6 pt. as quarts. 7. 8 qt. as gallons. 8. 3 yd. as feet.

9. 14 da. as weeks. 10. 8 pk. as bushels. 11. 12 ft. as yards.

12. $1\frac{1}{2}$ lb. as ounces. 13. 15 pt. as quarts. 14. 4 wk. as days.

15. 48 hr. as days. 16. 32 oz. as pounds. 17. $2\frac{1}{2}$ qt. as pints.

#### WRITTEN EXERCISE

1. Write the table of weight, — ounces, pounds, tons.

2. Write the table of length, — inches, feet, yards, rods, miles.

3. Write the table of dry measure, — quarts, pecks, bushels.

4. Write the table of liquid measure, — gills, pints, quarts, gallons.

TE 154
.6827

**151. Tables for reference.** While these tables have been learned and used in primary arithmetic, they are here inserted for reference and for such review as may be necessary.

**152. Table of length:**

**12 inches (in.) = 1 foot (ft.).**
**3 feet = 1 yard (yd.).**
**5½ yards, or 16½ feet = 1 rod (rd.).**
**320 rods, or 5280 feet = 1 mile (mi.).**

The *hand* (4 in.) is used in measuring the height of horses at the shoulder. Sailors use the fathom (6 ft.) and cable length (120 fathoms) for measuring depths, and the knot (nautical mile, 1.15 common or statute miles, or 6080.27 ft.) for distances at sea.

**153. Table of square measure:**

**144 square inches (sq. in.) = 1 square foot (sq. ft.).**
**9 square feet = 1 square yard (sq. yd.).**
**30¼ square yards = 1 square rod (sq. rd.).**
**160 square rods = 1 acre (A.).**
**640 acres = 1 square mile (sq. mi.).**

Carpenters, architects, and mechanics often write 8″ for 8 in., and 5′ for 5 ft. They also use sq.″ and sq.′ for square inches and square feet. In this book both of these forms are used.

**154. Table of cubic measure:**

**1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).**
**27 cubic feet = 1 cubic yard (cu. yd.).**
**128 cubic feet = 1 cord (cd.).**

A perch of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and 1½ ft. thick, and it contains 24¾ cu. ft. It varies, however, in different parts of the country. A cubic yard of earth is considered a load.

**155. Table of weight:**

**16 ounces (oz.) = 1 pound (lb.).**
**2000 pounds = 1 ton (T.).**

The ton of 2000 lb. is sometimes called the *short ton*, there being a *long ton* of 2240 lb. which is used in some wholesale transactions in mining products.

Goldsmiths still use an old table of *Troy weight*, but it is not of enough importance to justify teaching. In this table

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The avoirdupois pound contains 7000 gr., the Troy pound 5760 gr. Therefore 1 lb. of iron is heavier than 1 lb. of gold.

A *carat* weight, used in weighing diamonds, varies, but is commonly taken in the United States as 3.2 Troy grains. The word *carat* is also used in speaking of the purity of gold, meaning $\frac{1}{24}$, "16 carats fine" meaning $\frac{16}{24}$ pure gold.

There is also a table of *Apothecaries' weight*, used by physicians and druggists in prescriptions. Its importance is too slight to justify teaching, although it is here inserted for those who may care to use it.

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).
12 ounces = 1 pound (lb.).

**156. Table of liquid measure:**

**4 gills (gi.) = 1 pint (pt.).**
**2 pints = 1 quart (qt.).**
**4 quarts = 1 gallon (gal.).**

A gallon contains 231 cu. in. Barrels vary in size, although in estimating the capacity of tanks and cisterns 31.5 gal. are considered a barrel, and 2 bbl. a hogshead.

There is also a table of Apothecaries' liquid measure, in which 16 fluid ounces make 1 pint.

**157.** Table of dry measure:

**2 pints (pt.) = 1 quart (qt.).**
**8 quarts = 1 peck (pk.).**
**4 pecks = 1 bushel (bu.).**

A bushel contains 2150.42 cu. in. The dry quart is more than the liquid quart, containing 67.2 cu. in., while the liquid quart contains only 57.75 cu. in.

**158.** Table of time:

**60 seconds (sec.) = 1 minute (min.).**
**60 minutes = 1 hour (hr.).**
**24 hours = 1 day (da.).**
**7 days = 1 week (wk.).**
**12 months (mo.) = 1 year (yr.).**

"Thirty days hath September,
April, June, and November."

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. The common year has 365 days, or 52 weeks and 1 day; the leap year 366 days. A century is 100 years.

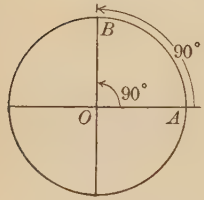
Years whose numbers are divisible by 4 but not by 100, and those divisible by both 4 and 100, are leap years. Thus, 1904, 1908, 2000, but not 2100, are leap years.

**159.** Table of value:

**10 mills = 1 cent (ct. or ¢).**
**10 cents = 1 dime (d.).**
**10 dimes = 1 dollar ($).**
**10 dollars = 1 eagle (E.).**

The term *eagle* is not used, although it commonly appears in the table of United States money. The mill is not coined, although the name is frequently used in speaking of fractional parts of a cent.

**160. Circular measure.** The circumference of a circle is measured in *degrees*.




There are 360 degrees (written 360°) in a whole circumference.

Because the angle at O, the center, increases just as fast as the part of the circumference between its arms increases, an angle is also measured in degrees.

ORAL EXERCISE

1. How many degrees in a circumference? in $\frac{1}{2}$ of a circumference? in $\frac{1}{4}$ of a circumference?

2. If one line is *perpendicular* to another, what angle do the two form? How many degrees in the angle?

3. How many degrees in a right angle? Is an angle of 100° acute, right, or obtuse? an angle of 60°? an angle of 90°? an angle of 120°? an angle of 1°?

**161. Perpendicular lines.** A line that makes a right angle with another is said to be *perpendicular* to it.

**162. Arc.** A portion of a circumference is called an *arc*.

**163. Arc and angle measure.** In measuring arcs

**360° = a circumference.**

In measuring angles

**360° = 4 right angles.**

In both cases

**60 seconds (60″) = 1 minute (1′).**
**60 minutes = 1 degree (1°).**

4. How many minutes in 5°? 7°? 11°? 25°?

5. How many seconds in 3′? in 5′? in 20′? in 1°?

6. Draw on the board an angle of about 30°; 45°; 135°.

## Reduction

**164. Denominate numbers.** Concrete numbers denoting measure (including weight) are called *denominate numbers.*

For example, \$2, 3 ft., 7 lb. 4 oz.

**165. Compound numbers.** Denominate numbers in which two or more units are expressed are called *compound numbers.*

For example, 2 ft. 3 in., 4 mi. 3 rd. 2 ft.

**166. Reduction.** We may express 2 ft. 3 in. as 27 in., or as 2.25 ft., or as $2\frac{1}{4}$ ft. This changing from one form to another, without changing the value, is called *reduction.*

**167. Reduction descending.** Reduction to a lower denomination is called *reduction descending.*

For example, 3 ft. may be reduced to 36 in.

**168. Reduction ascending.** Reduction to a higher denomination is called *reduction ascending.*

For example, 24 oz. may be reduced to 1 lb. 8 oz.

Definitions like the above have little value, the subject of compound numbers having lost most of its former importance. We speak of 2 ft. 3 in. instead of $2\frac{1}{4}$ ft., but we rarely use the pound and ounce *together*, speaking of 2 lb. 4 oz. as "2 pounds and a quarter." Hence but little time should be given to work involving more than two denominations.

### ORAL EXERCISE

1. Express 10 bu. as pecks; as quarts.
2. Express 64 qt. as pecks; as bushels.
3. Express 40 qt. as gallons; as pints.
4. Express 40 pt. as quarts; as gallons.
5. Express 20 ft. as inches; 21 ft. as yards.

## Reduction Descending

**169. Reduction descending.** Required to reduce 160 rd. to inches.

*Explanation:*

Since 1 rd. = $16\frac{1}{2}$ ft.,

160 rd. = 160 times $16\frac{1}{2}$ ft. = 2640 ft.

Since 1 ft. = 12 in.,

2640 ft. = 2640 times 12 in. = 31,680 in.

While we think of multiplying $16\frac{1}{2}$ ft. by 160, we actually multiply 160 by $16\frac{1}{2}$ because this is shorter, and so for 2640 times 12 in.

*Actual work:*

160
$16\frac{1}{2}$
80
960
160
2640 no. of ft.
12
31680 no. of in.

**170.** Required to reduce 3 T. 42 lb. to ounces.

Since 1 T. = 2000 lb.,

3 T. = 3 times 2000 lb. = 6000 lb.

6000 lb. + 42 lb. = 6042 lb.

Since 1 lb. = 16 oz.,

6042 lb. = 6042 times 16 oz. = 96,672 oz.

2000
3
6000
42
6042 no. of lb.
16
36252
6042
96672 no. of oz.

### WRITTEN EXERCISE

*Reduce:*

1. 8 bu. 2 pk. to pecks.
2. 3 mi. 75 rd. to rods.
3. 10 rd. 3 ft. to feet.
4. 2 A. to square rods.
5. 175 rd. 9 ft. to feet.
6. 5′ 6″ to inches.
7. 2 lb. 2 oz. to ounces.
8. 5 yd. 1 ft. to feet.
9. 2 gal. 3 qt. to quarts.
10. $2\frac{3}{4}$ A. to square rods.
11. 10 cd. to cubic feet.
12. $23\frac{1}{3}$ cu. yd. to cubic feet.
13. 6 cu. yd. 7 cu. ft. to cubic feet; to cubic inches.
14. 4 sq. rd. 3 sq. ft. to square feet; to square inches.

## Reduction Ascending

**171. Reduction ascending.** Required to reduce 200 pt. to gallons.

*Explanation:*

Since 1 pt. $= \frac{1}{2}$ qt.,

200 pt. $= 200$ times $\frac{1}{2}$ qt. $= 100$ qt.

Since 1 qt. $= \frac{1}{4}$ gal.,

100 qt. $= 100$ times $\frac{1}{4}$ gal. $= 25$ gal.

*Actual work:*

$2\overline{)200}$ no. of pt.

$4\overline{)100}$ " " qt.

25 " " gal.

**172.** Required to reduce 175 in. to feet and inches.

Since 1 in. $= \frac{1}{12}$ ft.,

175 in. $= 175$ times $\frac{1}{12}$ ft.

$= 14\frac{7}{12}$ ft. $= 14$ ft. 7 in.

$12\overline{)175}$ no. of in.

14 " " ft.

7 in. rem.

### WRITTEN EXERCISE

*Reduce:*

1. 960 in. to feet.
2. 492 in. to feet.
3. 480 oz. to pounds.
4. 396 in. to yards.
5. 176 oz. to pounds.
6. 7200 sec. to hours.
7. 3600 min. to hours.
8. 275 ft. to rods and feet.
9. 272 pk. to bushels.
10. 3255 lb. to tons and pounds.
11. 527 pt. to bushels, quarts, and pints.
12. 500 sq. in. to square feet and square inches.

**173.** Reduce $\frac{7}{8}$ in. to a fraction of a yard.

Since 1 in. $= \frac{1}{36}$ yd.,

$\frac{7}{8}$ in. $= \frac{7}{8}$ of $\frac{1}{36}$ yd. $= \frac{7}{288}$ yd.

If required to reduce to a decimal, $\frac{7}{288}$ yd. $= 0.024\frac{11}{36}$ yd.

13. 475 yd. 0.6 ft. to a decimal of a mile.
14. $\frac{3}{4}$ in. to a decimal of a foot; of a yard.
15. $266\frac{2}{3}$ rd. to a common fraction of a mile.

## Addition and Subtraction

### ORAL EXERCISE

*Add in Exs. 1–6:*

| | | |
|---|---|---|
| 1. 13 ft. 2 in.<br>24 8 | 2. 36 lb. 9 oz.<br>15 3 | 3. 10 rd. 4 ft.<br>25 11 |
| 4. 35 mi. 72 rd.<br>25 26 | 5. 25 yd. 15 in.<br>42 9 | 6. 42 ft. 8 in.<br>32 4 |

**174. Adding compound numbers.** Required the sum of 3 ft. 9 in., 6 ft. 7 in., and 5 ft. 10 in.

Here the sum of the inches is 26 in. = 2 ft. 2 in. The sum of the feet is 14 ft., which with the 2 ft. equals 16 ft.

3 ft. 9 in.
6 7
5 10
16 ft. 2 in.

### WRITTEN EXERCISE

| | | |
|---|---|---|
| 1. 25 lb. 12 oz.<br>24 4 | 2. 73 rd. $4\frac{1}{2}$ ft.<br>26 12 | 3. 26 mi. 300 rd.<br>23 20 |
| 4. 41 yd. 25 in.<br>41 11 | 5. 60 ft. 9 in.<br>10 4 | 6. 30 lb. 10 oz.<br>20 8 |
| 7. 30 rd. $6\frac{1}{2}$ ft.<br>40 12 | 8. 70 mi. 300 rd.<br>80 25 | 9. 30 yd. 30 in.<br>20 8 |
| 10. 3 mo. 27 da.<br>11 16 | 11. 2 wk. 6 da.<br>3 5 | 12. 44 bu. 2 pk.<br>17 3 |
| 13. 32 yd. 1 ft.<br>49 2<br>72 2<br>10 1 | 14. 94 gal. 2 qt.<br>68 1<br>29 3<br>13 2 | 15. 27 mi. 68 rd.<br>37 300<br>16 200<br>22 160 |

### ORAL EXERCISE

*Subtract in Exs. 1–9:*

| | | |
|---|---|---|
| 1. 58 ft. 10 in. | 2. 25 lb. 14 oz. | 3. 35 rd. 10 ft. |
| 9 6 | 17 6 | 26 7 |
| 4. 85 mi. 75 rd. | 5. 27 yd. 10 in. | 6. 49 ft. 7 in. |
| 66 50 | 19 5 | 36 7 |
| 7. 37 lb. 12 oz. | 8. 42 rd. 15 ft. | 9. 23 mi. 7 rd. |
| 37 7 | 27 15 | 16 |

**175.** **Subtracting compound numbers.** In subtracting 17 ft. 10 in. from 23 ft. 8 in., we have : 8 in. − 10 in., impossible ; 1 ft. 8 in., or 20 in. − 10 in. = 10 in. ; 22 ft. − 17 ft. = 5 ft.

23 ft. 8 in.
17 10
5 ft. 10 in.

### WRITTEN EXERCISE

| | | |
|---|---|---|
| 1. 49 yd. 8 in. | 2. 3 ft. 4 in. | 3. 10 lb. 3 oz. |
| 30 8 | 8 | 7 |
| 4. 6 rd. $6\frac{1}{2}$ ft. | 5. 10 mi. | 6. 10 yd. 10 in. |
| 13 | 20 rd. | 30 |
| 7. $47\frac{1}{4}$ yd. | 8. 49 ft. $4\frac{1}{2}$ in. | 9. 37 lb. $6\frac{1}{3}$ oz. |
| $38\frac{1}{2}$ | 39 $8\frac{3}{4}$ | 28 $7\frac{2}{3}$ |
| 10. 26 rd. $8\frac{1}{2}$ ft. | 11. 76 mi. $7\frac{3}{4}$ rd. | 12. 53 yd. $28\frac{1}{2}$ in. |
| 18 $9\frac{3}{4}$ | 29 8 | 47 $32\frac{3}{4}$ |

**176.** We rarely need to add or subtract compound numbers of more than two denominations. One case where three denominations are used is in finding the difference between two dates, as in Exs. 21, 22, page 100.

### WRITTEN EXERCISE

*Add in Exs. 1–11:*

1. 37 ft. 9 in.
   42 7
   67 6
   31 4

2. 23 lb. 7 oz.
   48 9
   61 4
   37 5

3. 82 rd. 5 ft.
   75 7
   62 12
   9 $3\frac{1}{2}$

4. 24 qt. 1 pt.
   3 0
   45 1
   12 1

5. 91 yd. 27 in.
   61 14
   33 31
   42 9

6. 42 bu. 3 pk.
   27 1
   63 2
   48 3

7. 23 gal. 2 qt.
   41 1
   6 0
   83 3

8. 23 mi. 57 rd.
   46 120
   29 230
   80 175

9. 32 mi. 4260 ft.
   27 2027
   33 45
   241 605

10. 92 cu. ft. 246 cu. in.
    73 487
    34 995

11. 75 sq. ft. 92 sq. in.
    86 49
    93 135

*Subtract in Exs. 12–22:*

12. 93 ft. 7 in.
    61 8

13. 47 lb. 8 oz.
    26 9

14. 32 rd. 4 ft.
    26 5

15. 81 mi. 7 rd.
    65 98

16. 87 yd. 4 in.
    69 8

17. $65\frac{1}{2}$ ft.
    48 9 in.

18. 85 lb. 6 oz.
    $36\frac{1}{2}$

19. 67 rd. $8\frac{1}{4}$ ft.
    $39\frac{1}{2}$

20. 89 mi. 80 rd.
    $69\frac{1}{2}$

21. 1910 yr. 6 mo. 8 da.
    1905 7 9

22. 1909 yr. 7 mo. 3 da.
    1905 2 15

## Multiplication

### ORAL EXERCISE

*Multiply:*

1. 3 ft. 4 in. by 2.
2. 8 lb. 2 oz. by 6.
3. 7 ft. 2 in. by 4.
4. 7 rd. 1 ft. by 8.
5. 8 ft. 1 in. by 9.
6. 11 lb. 3 oz. by 4.
7. 9 yd. 6 in. by 3.
8. 9 gal. 1 qt. by 3.
9. 8 yd. 3 in. by 8.
10. 12 gal. 1 qt. by 2.
11. 6 yd. 2 in. by 7.
12. 10 mi. 200 ft. by 5.

**177. Multiplying compound numbers.** Required to multiply 4 lb. 9 oz. by 6.

We might multiply as with other numbers and obtain 24 lb. 54 oz., and reduce this to 27 lb. 6 oz. But it is easier to say: "6 times 9 oz. = 54 oz. = 3 lb. 6 oz.; 6 times 4 lb. = 24 lb., which added to 3 lb. = 27 lb."

4 lb. 9 oz.
6
27 lb. 6 oz.

**178.** Required to multiply 243 ft. 7 in. by 27.

Multiplying as with other numbers, the result is 6561 ft. 189 in., which is reduced to 6576 ft. 9 in.

243 ft. 7 in.
27
1701 49
486 14
6561 ft. 189 in. =
6576 ft. 9 in.

### WRITTEN EXERCISE

*Multiply in Exs. 1–10:*

1. 47 ft. 6 in. by 25.
2. 27 lb. 3 oz. by 84.
3. 49 yd. 2 in. by 96.
4. 44 mi. 3 ft. by 89.
5. 109 ft. 3 in. by 47.
6. 273 ft. 2 in. by 35.
7. 121 lb. 4 oz. by 92.
8. 309 lb. 5 oz. by 67.
9. 401 yd. 3 in. by 68.
10. 370 yd. 1 in. by 83.

## Division

### ORAL EXERCISE

| | |
|---|---|
| 1. 24 ft. 6 in. ÷ 2. | 2. 48 ft. 8 in. ÷ 4. |
| 3. 36 ft. 6 in. ÷ 6. | 4. 21 ft. 7 in. ÷ 7. |
| 5. 81 lb. 9 oz. ÷ 9. | 6. 72 ft. 8 in. ÷ 8. |
| 7. 50 ft. 10 in. ÷ 5. | 8. 27 lb. 12 oz. ÷ 3. |
| 9. 56 lb. 14 oz. ÷ 7. | 10. 72 in. ÷ 9 in. |
| 11. 64 ft. ÷ 8 ft. | 12. 54 rd. ÷ 6 rd. |
| 13. 108 yd. ÷ 12 yd. | 14. 225 ft. ÷ 25 ft. |

**179. Two kinds of division.** As shown on page 20, there are two kinds of division. Because 9 times 8 ft. = 72 ft., therefore,

(1) 72 ft. ÷ 9 = 8 ft., (2) 72 ft. ÷ 8 ft. = 9.

**180. Abstract divisor.** To consider the first kind, divide 371 ft. 6 in. by 15.

Dividing as usual, 371 ft. ÷ 15 = 24 ft., and 11 ft. remainder. 11 ft. = 11 times 12 in. = 132 in., and 132 in. + 6 in. = 138 in., to be divided. 138 in. ÷ 15 = $9\frac{1}{5}$ in. Therefore the quotient is 24 ft. $9\frac{1}{5}$ in.

$$\begin{array}{r} 24 \text{ ft.} \\ 15\overline{)371 \text{ ft.}} \\ 30 \\ \hline 71 \\ 60 \\ \hline 11 \text{ ft.} = 132 \text{ in.} \\ \hline 138 \text{ in.} \end{array} \qquad \begin{array}{r} 6 \text{ in.} \\ \\ \\ \\ \\ \\ \hline \end{array} \qquad \begin{array}{r} 9\frac{1}{5} \text{ in.} \\ 15\overline{)138 \text{ in.}} \\ 135 \\ \hline 3 \end{array}$$

**181. Concrete divisor.** To consider the second kind, divide 22 ft. 9 in. by 3 ft. 3 in.

In such cases reduce dividend and divisor to the same denomination, thus:

22 ft. 9 in. = 273 in.
3 ft. 3 in. = 39 in.
Therefore 273 in. ÷ 39 in. = 7.

$$\begin{array}{r} 7 \\ 39 \text{ in.}\overline{)273 \text{ in.}} \\ 273 \\ \hline \end{array}$$

WRITTEN EXERCISE

1. 241 lb. 3 oz. ÷ 8.
2. 472 yd. 2 ft. ÷ 3.
3. 123 ft. 8 in. ÷ 11.
4. 82 yd. 18 in. ÷ 27.
5. 955 ft. 6 in. ÷ 13.
6. 423 lb. 15 oz. ÷ 21.
7. 10 yd. 20 in. ÷ 19 in.
8. 30 lb. ÷ 7 lb. 8 oz.
9. 12 ft. 10 in. ÷ 4 ft. 2 in.
10. 32 gal. 2 qt. ÷ 3 gal. 1 qt.
11. 1 cu. yd. 3 cu. ft. ÷ 10 cu. ft.
12. 10 sq. yd. 6 sq. ft. ÷ 3 sq. yd. 5 sq. ft.
13. 24 yr. 11 mo. 6 da. ÷ 2 yr. 3 mo. 6 da.
14. 2 da. 7 hr. 9 min. 18 sec. ÷ 3 hr. 27 min. 42 sec.

*Multiply in Exs. 15–20:*

15. 15 hr. 27 min. by 6.
16. 18 bu. 3 pk. by 8.
17. 621 mi. 20 ft. by 97.
18. 9 T. 1650 lb. by 4.
19. 32 min. 48 sec. by 12.
20. 404 mi. 70 ft. by 73.

21. If the average weight of the members of your class is 57 lb. 4 oz., what is the total weight of the class?

22. If the average length of your steps is 2 ft. 2 in., how far will you go in 425 steps?

23. If a wheel is 9 ft. 2 in. in circumference, how far will it travel in making 125 revolutions?

24. If one side of a square is 8 ft. 9 in., how far is it around the square?

25. If it is 18 ft. 8 in. around a square, how long is each side?

26. It is 54 ft. around a certain figure each of whose sides is 6 ft. 9 in. How many sides has the figure?

27. If a wheel is 8 ft. $9\frac{3}{5}$ in. in circumference, how many revolutions will it make in going a mile?

**182. Areas.** What is the area of a flower bed 8 ft. wide and 1 rd. long?

If it were $16\frac{1}{2}$ ft. long and 1 ft. wide, the area would be $16\frac{1}{2}$ sq. ft. Since it is 8 ft. wide, the area is 8 times $16\frac{1}{2}$ sq. ft., or 132 sq. ft.

WRITTEN EXERCISE

1. At the rate of a mile in 4 min., how far will an automobile go in 3 hr.?

2. How far will a horse go in the same time, traveling at the rate of a mile in 9 min.? How far will a train go in the same time, at the rate of 45 mi. an hour?

3. If the large wheels of an automobile are $7\frac{2}{5}$ ft. in circumference, how many times will each one turn in going 25 mi.?

4. At \$65 an acre, what is a rectangular farm worth that is 150 rd. wide and 160 rd. long?

5. At 21 ct. a foot, how much would it cost to inclose the farm of Ex. 4 by a wire fence?

6. If the man wishes to enrich the soil of a 75-acre meadow, how many tons of fertilizer would he use at 75 lb. to the acre?

7. If a farmer has cut 35 cords of 4-ft. wood from his property and piled it 4 ft. high, how long is the pile?

8. If a lot 8 rd. by 40 rd. is planted to corn, and produces $27\frac{1}{2}$ bu. to the acre, how many bushels does the owner get in all?

9. A farmer has a barn 60 ft. long, 30 ft. wide, and 15 ft. high to the eaves. The triangular gables on the ends, above the line of the eaves, have each an area of 150 sq. ft. How much will it cost to paint the barn at 20 ct. a square yard? (Illustrate the four sides by drawings.)

## II. DECIMAL FRACTIONS AND PERCENTAGE

### ADDITION AND SUBTRACTION OF DECIMALS

### Some of our Great Railways

#### WRITTEN EXERCISE

1. The lengths of the various divisions of the Atchison, Topeka, and Santa Fé road were recently as follows: 1177.81 mi., 487.37 mi., 887.99 mi., 372.48 mi., 129 mi., 60 mi., and 4818.08 mi. What was the total length?

2. On the Baltimore and Ohio the divisions were: 5.3 mi., 129.42 mi., 949.98 mi., 403.08 mi., 323 mi., 790.19 mi. What was the total length?

3. On the Boston and Maine the divisions were: 115.31 mi., 108.29 mi., 73.37 mi., 148.34 mi., 222.32 mi., 69.5 mi., 142¼ mi., 98.77 mi., 74 mi., 250.98 mi., 53.85 mi., 907.71 mi. What was the total length?

4. How much longer was the Chicago and Northwestern, when it had 8791.29 mi., than the Canadian Pacific with 8356.1 mi.?

5. How much longer was the Burlington Route, when it had 8219.87 mi., than the Denver and Rio Grande with 1708.19 mi.? than the Chicago, Milwaukee, and St. Paul with 6582.58 mi.?

6. How much shorter was the Great Northern, when it had 5240.47 mi., than the combined lengths of the Erie, 2170 mi., and the Illinois Central, 4265.5 mi.?

7. How much longer was the Pennsylvania railroad, when it had 9970 mi., than the Southern Pacific, 8206.75 mi.? than the New York Central, 3313.81 mi.? than the Wabash, 2367.3 mi.? than the Plant System, 3589 mi.?

## MULTIPLICATION OF DECIMALS

### ORAL EXERCISE

1. Multiply:

| 2 ft. | $2 | 2 tenths | 0.2 | 2 hundredths | 0.02 |
|---|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 3 | 3 |

2. What is the product of $\frac{2}{3}$ and $\frac{1}{7}$? What is the product of thirds and sevenths? What is the product of halves and fifths? What is the product of tenths and tenths?

3. What is the product of $\frac{7}{10}$ and $\frac{3}{10}$? of 0.7 and 0.3? of 0.3 and 0.3? of $\frac{3}{10}$ and $\frac{9}{10}$? of 0.3 and 0.9?

4. Multiply: $\frac{2}{10} \times \frac{5}{10}$ $\frac{6}{10} \times \frac{7}{10}$ $\frac{5}{10} \times \frac{9}{10}$

$0.4 \times 0.7$ $0.6 \times 0.8$ $0.9 \times 0.9$

**183. Multiplying by decimals.** Required to multiply 0.67 by 4.

Since 4 times hundredths is hundredths, the 268 is hundredths, and the decimal point goes before the 6. That is, 4 times $\frac{67}{100} = \frac{268}{100} = 2.68$.

$$\begin{array}{r} 0.67 \\ 4 \\ \hline 2.68 \end{array}$$

To multiply 52 by 0.03.

The product of 52 and hundredths is hundredths. Therefore the answer is hundredths and the decimal point goes before the 5. That is, $\frac{3}{100}$ of $52 = \frac{156}{100} = 1.56$.

$$\begin{array}{r} 52 \\ 0.03 \\ \hline 1.56 \end{array}$$

§§ 184 and 185 may be introduced here if desired.

### WRITTEN EXERCISE

1. $2 \times 0.79$. 2. $3 \times 0.96$. 3. $4 \times 0.78$. 4. $7 \times 0.93$.

5. $0.82 \times 8$. 6. $0.67 \times 9$. 7. $0.83 \times 4$. 8. $0.27 \times 7$.

9. $3 \times 1.75$. 10. $5 \times 2.42$. 11. $6 \times 7.23$. 12. $8 \times 9.23$.

13. At 6.95 in. to a step, how high do some stairs of 20 steps reach?

**184.** Multiply .42 by .35.

If we were to multiply 42 by 35, the result would be 1470. And since hundredths multiplied by hundredths equals ten-thousandths, the result is .1470. That is, $\frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000} = .1470$ or .147.

$$\begin{array}{r} .42 \\ .35 \\ \hline 210 \\ 126 \\ \hline .1470 \end{array}$$

**185. Decimal places in the product.** Therefore, in multiplying decimals,

1. *Multiply as with integers.*

2. *Point off as many decimal places in the product as there are decimal places in all the factors.*

It is unnecessary to remember this rule, since it is easy to see that the product of tenths and tenths is hundredths, of tenths and hundredths is thousandths, and so on. It is often helpful, however.

**186.** Multiply 6.2 by 3.5.

If we were to multiply 62 by 35, the result would be 2170. And since tenths multiplied by tenths equals hundredths, the result is 21.70 (that is, $\frac{2170}{100}$ or $21\frac{70}{100}$), or 21.7.

$$\begin{array}{r} 6.2 \\ 3.5 \\ \hline 310 \\ 186 \\ \hline 21.70 \\ \text{or } 21.7 \end{array}$$

ORAL EXERCISE

| | | | |
|---|---|---|---|
| 1. $3 \times 0.6$. | 2. $4 \times 0.7$. | 3. $5 \times 0.61$. | 4. $6 \times 0.11$. |
| 5. .5 of 2. | 6. .6 of 7. | 7. .8 of 8. | 8. .7 of 10. |
| 9. .2 of .2. | 10. .3 of .4. | 11. .5 of .7. | 12. .8 of .9. |

WRITTEN EXERCISE

| | | | |
|---|---|---|---|
| 1. .7 of 96. | 2. .8 of 74. | 3. .6 of 48. | 4. .9 of 84. |
| 5. .8 of 9.8. | 6. .7 of 4.9. | 7. .9 of 4.7. | 8. .3 of 9.9. |
| 9. $1.5 \times 67$. | 10. $1.4 \times 75$. | 11. $3.8 \times 46$. | 12. $4.7 \times 35$. |
| 13. $2.7 \times 3.4$. | 14. $1.9 \times 2.6$. | 15. $4.7 \times 3.6$. | 16. $8.1 \times 8.1$. |

17. A meter being 39.37 in. long, how many inches in 2.8 meters? in 17.6 meters?

### ORAL EXERCISE

1. Why will there be three decimal places in the product of 0.75 multiplied by 0.3? of 2.325 multiplied by 7?

2. How do you know the number of decimal places in the product of 0.62 and 2.79? of 2.756 and 3.123?

*Without multiplying, tell the order of the decimal part of each of the following:*

3. $2.8 \times 7.9$. 4. $2.007 \times 6.09$. 5. $1.203 \times 2.67$.
6. $1.7 \times 1.42$. 7. $1.234 \times 3.456$. 8. $3.421 \times 0.713$.

### WRITTEN EXERCISE

1. $43.2 \times 98.6$. 2. $89.1 \times 49.3$. 3. $68.7 \times 40.5$.
4. $.823 \times 2946$. 5. $72.9 \times 87.09$. 6. $53.6 \times 2.889$.
7. $3.49 \times 80.23$. 8. $36.4 \times 49.87$. 9. $98.9 \times 69.89$.
10. $2.99 \times 124.63$. 11. $48.7 \times 14.689$. 12. $52.9 \times 68.023$.

*Multiply the following:*

13. 234.5 by 8.001. 14. 0.500 by 0.600.
15. \$683.45 by 0.2. 16. \$281.42 by 6.50.
17. \$283.75 by 42.8. 18. \$333.33$\frac{1}{3}$ by 0.3.
19. $25 \times 9.2$. 20. $37 \times 4.6$. 21. $48 \times 0.8$. 22. $96 \times 9.6$.
23. $5.9 \times 9.5$. 24. $7.3 \times 4.8$. 25. $6.7 \times 1.8$. 26. $5.4 \times 3.5$.
27. $48.1 \times 32.5$. 28. $20.5 \times 60.7$. 29. $43.7 \times 74.1$.
30. $26.8 \times 80.8$. 31. $20.2 \times 20.2$. 32. $30.5 \times 57.2$.
33. $60.9 \times 90.6$. 34. $73.7 \times 73.7$. 35. $8.25 \times 6.37$.
36. $4.72 \times 6.93$. 37. $42.3 \times 5.37$. 38. $17.2 \times 26.25$.
39. $2.87 \times 5.492$. 40. $6.81 \times 7.92$. 41. $2.175 \times 6.73$.
42. $16.4 \times 17.25$. 43. $8.35 \times 6.423$. 44. $0.273 \times 0.625$.
45. $0.352 \times 6.725$. 46. $3.33\frac{1}{3} \times 6.963$. 47. $0.16\frac{2}{3} \times 0.41\frac{1}{5}$.

WRITTEN EXERCISE

1. At $1.75 each, what will 17 books cost?

2. What is the perimeter of a square 34.7 ft. on a side?

3. At $75.50 an acre, what will 25.5 acres of land cost?

4. At $0.75 a yard, what will 32.2 yd. of China silk cost?

5. At $125 a front foot, how much will a city lot cost having a frontage of 72.3 ft.?

6. At $65.50 an acre, how much must a man pay for 200 acres of land? for 25.4 acres? for 3.75 acres?

7. At $17.52 a front foot, what will a piece of land 28.33⅓ ft. front cost?

8. A tank when full holds 42.8 gal. How much does it contain when .37 full?

9. A certain rectangle is .78 as wide as it is long. It is 4.9 in. long. What is the perimeter?

10. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 ft. wide. What is the perimeter?

11. A city lot is 45.8 ft. wide and 76.3 ft. deep. What is the area of the lot? What is it worth at $1.50 a square foot?

12. The circumference of a circle is 3.1416 times the diameter. If the diameter is 42.27 ft., what is the circumference?

13. How many square rods of land are there in a piece of land 80 rd. wide and 114.1 rd. long? How many acres? What is it worth at $75 an acre?

14. There are three towns, A, B, C, forming the three corners of a triangle. From A to B is 27.32 mi., from B to C it is 1.4 times as far, and the perimeter of the triangle is 92.8 mi. How far is it from C to A?

## DIVISION OF DECIMALS

### ORAL EXERCISE

1. Divide 10 by 2; 100 by 20. Compare the results.

2. Divide \$2.50 by 5; \$25 by 50. Compare the results.

3. If multiplying both dividend and divisor by the same number does not change the quotient, how does the quotient of $360 \div 4$ compare with that of $36 \div 0.4$?

**187. Division of decimals.** Since both dividend and divisor may be multiplied by the same number without changing the quotient, therefore,

*In the division of decimal fractions, multiply both dividend and divisor by the power of 10 that shall make the divisor an integer, and divide as in United States money.*

**188.** Divide 6.25 by 2.5.

The result is the same if we divide 62.5 by 25, thus making the divisor an integer.

Dividing as in United States money, $62.5 \div 25 = 2$, with 12.5 still to be divided. $12.5 \div 25 = 0.5$. Therefore the quotient is 2.5.

$$\begin{array}{r} 2.5 \\ 25\overline{)62.5} \\ 50 \\ \hline 12\,5 \\ 12\,5 \\ \hline \end{array}$$

In developing this before a class, the decimal point in 12.5 should be preserved at first, and finally dropped as unnecessary.

### WRITTEN EXERCISE

1. $6.25 \div 0.25$, or $625 \div 25$.
2. $625 \div 0.25$.
3. $60 \div 0.6$, or $600 \div 6$.
4. $600 \div 0.06$.
5. $62.9 \div 3.7$.
6. $4.62 \div 2.2$.
7. $629 \div .37$.
8. $8.05 \div 3.5$.
9. $.462 \div .21$.
10. $7.14 \div 4.2$.
11. $3050 \div .61$.
12. $11.56 \div 3.4$.
13. $30.5 \div 0.61$.
14. $6.232 \div 0.82$.
15. $17.466 \div 7.1$.
16. $62.32 \div 8.2$.

WRITTEN EXERCISE

*Add in Exs. 1–8:*

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| 240.4 | 4.28 | 3.21 | 169.28 |
| 802.2 | 85.00 | 150.5 | 3.7 |
| 79.8 | 6.32 | 7.32 | 45.94 |
| 388.7 | 108.51 | 46.48 | 2.33 |
| 96.3 | 7.23 | 2.87 | 25.4 |
| 434.1 | .75 | 18.95 | 8.86 |
| 625.4 | 34.87 | 9.26 | 10.92 |

| 5. | 6. | 7 | 8. |
|---|---|---|---|
| 4.028 | .387 | .1.043 | .2909 |
| .396 | .0426 | .427 | .0628 |
| .285 | .0539 | 6.091 | .0703 |
| 4.72 | .2064 | .5 | .5076 |
| 3.942 | .0708 | .709 | .0289 |
| .409 | .5029 | 6.273 | .0079 |
| 6.029 | .0627 | .108 | .0403 |

*Subtract in Exs. 9–20:*

| 9. | 10. | 11. | 12. |
|---|---|---|---|
| 90.32 | 209.8 | 6.273 | 81.04 |
| 4.86 | 49.2 | .992 | 6.98 |

| 13. | 14. | 15. | 16. |
|---|---|---|---|
| 1.035 | .2809 | 82.481 | 64.035 |
| .876 | .1589 | 2.909 | 37.029 |

| 17. | 18. | 19. | 20. |
|---|---|---|---|
| 86.431 | 102.98 | 62.035 | 80.289 |
| 28.902 | 72.89 | 51.975 | 79.939 |

21. $1.4 \times 6.64$. 22. $1.5 \times 553.35$. 23. $1.6 \times 140.64$.

24. $3.6 \times 1972.08$. 25. $5.8 \times 439.002$. 26. $6.2 \times 53.134$.

27. $888.16 \div 2.8$. 28. $825.468 \div 4.2$. 29. $20.720 \div 0.28$.

30. $103.60 \div 0.56$. 31. $1131.264 \div 4.8$. 32. $38.8692 \div 5.4$.

**189. Illustrative problems.** 1. Divide 16.35 by 500.

Here it is better to divide both dividend and divisor by 100, canceling the zeros and moving the decimal point two places to the left.

$$5\text{ø}\text{ø}\overline{)\,.1635}\quad .0327$$

2. Divide 3.82 by 2.5, carrying the quotient to one decimal place.

Multiplying both numbers by 10, we have $38.2 \div 25 = 1.5$, with 7 remainder. If we do not write the fraction $\frac{7}{25}$ in the quotient, the quotient is written 1.5+, meaning 1.5 and a fraction less than $\frac{1}{2}$.

3. Divide .049 by .17, carrying the quotient to two decimal places.

Multiplying both numbers by 100, we have $4.9 \div 17 = 0.28$, with 14 remainder. If we do not write the fraction $\frac{14}{17}$ in the quotient, the quotient is written 0.29−, meaning 0.28 and a fraction more than $\frac{1}{2}$.

WRITTEN EXERCISE

| | |
|---|---|
| 1. $17.25 \div 500$. | 2. $3.248 \div 200$. |
| 3. $817.6 \div 700$. | 4. $82.48 \div 8000$. |
| 5. $.7364 \div 7000$. | 6. $74.34 \div 9000$. |
| 7. $15.720 \div 12{,}000$. | 8. $133.10 \div 11{,}000$. |
| 9. $404.30 \div 13{,}000$. | 10. $\$6844.50 \div 900$. |
| 11. $\$7337.75 \div 500$. | 12. $\$1247.01 \div 300$. |

*In the following divisions, carry the quotient to three decimal places:*

| | | |
|---|---|---|
| 13. $17.78 \div 1.5$. | 14. $62.23 \div .23$. | 15. $19.26 \div 3.1$. |
| 16. $423.6 \div .27$. | 17. $167.8 \div 3.7$. | 18. $8.348 \div 2.68$. |
| 19. $976.34 \div .125$. | 20. $483.62 \div 42.7$. | 21. $826.3 \div 12.5$. |

**190. Illustrative problem.** What is the quotient of 17.8 × 45.39 divided by .267?

Multiplying both dividend and divisor by 1000, to make the divisor integral, we have for the dividend 178×4539 (i.e. 10 times 17.8 and 100 times 45.39). Factoring and canceling, we have $2 \times 1513 = 3026$.

$$\frac{2 \times 89 \quad 1513}{178 \times 4539}$$
$$\frac{267}{3 \times 89}$$

WRITTEN EXERCISE

1. $\frac{3.9 \times 0.84}{15.6}$.
2. $\frac{5.5 \times 0.42}{0.77}$.
3. $\frac{2.1 \times 0.4}{0.015}$.
4. $\frac{1.4 \times 0.6}{0.63}$.
5. $\frac{4.5 \times 6.3}{8.1}$.
6. $\frac{0.21 \times 0.6}{0.126}$.
7. $\frac{2.2 \times 2.4}{4 \times 7.7}$.
8. $\frac{5.6 \times 0.3}{0.63 \times 0.8}$.
9. $\frac{1.8 \times 5.5}{9.9 \times 0.1}$.
10. $\frac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.
11. $\frac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.
12. $\frac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.
13. $\frac{6.3 \times 4.2 \times .17}{2.7 \times 3.4 \times .49}$.

14. The product of 6.3, 1.4, and 5.7 is how many times as great as the product of 17.1, 0.06, and 7?

15. The product of 0.2, 0.21, and 7.5 is how many times as great as the product of 0.14, 0.3, and 1.5?

16. How many times is the product of 1.6, 0.01, and 1.5 contained in the product of 0.032, 2.25, and 3?

17. The volume of a box whose interior dimensions are 2.4 in., 5 in., 1.7 in., is how many times as great as that of one whose interior dimensions are 3 in., 1.6 in., and .85 in.?

18. A block of marble 3.5 ft. by $4\frac{1}{4}$ ft. by 2.2 ft. weighs how many times as much as one that is $2\frac{1}{2}$ ft. by 7.7 ft. by 0.85 ft.?

## Common Business Fractions

### ORAL EXERCISE

1. Express as decimal fractions:

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | $\frac{1}{6}$ | $\frac{1}{8}$ | $\frac{3}{8}$ |
| $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{2}{5}$ | $\frac{5}{6}$ | $\frac{5}{8}$ | $\frac{7}{8}$ |

2. Express as common fractions:

| | | | | | | |
|---|---|---|---|---|---|---|
| .5 | .25 | .6 | $.16\frac{2}{3}$ | $.12\frac{1}{2}$ | .125 | $.33\frac{1}{3}$ |
| .50 | .75 | .8 | $.83\frac{1}{3}$ | $.37\frac{1}{2}$ | .625 | $.87\frac{1}{2}$ |

**191. Common business fractions.** The following are so commonly used that they should be remembered:

| | | | |
|---|---|---|---|
| $\frac{1}{4} = .25$ | $\frac{1}{16} = .06\frac{1}{4}$ | $\frac{1}{12} = .08\frac{1}{3}$ | $\frac{1}{5} = .2$ |
| $\frac{1}{2} = .5$ | $\frac{1}{8} = .12\frac{1}{2}$ | $\frac{1}{6} = .16\frac{2}{3}$ | $\frac{2}{5} = .4$ |
| $\frac{1}{2} = .50$ | $\frac{3}{8} = .37\frac{1}{2}$ | $\frac{1}{3} = .33\frac{1}{3}$ | $\frac{3}{5} = .6$ |
| $\frac{3}{4} = .75$ | $\frac{5}{8} = .62\frac{1}{2}$ | $\frac{2}{3} = .66\frac{2}{3}$ | $\frac{4}{5} = .8$ |
| | $\frac{7}{8} = .87\frac{1}{2}$ | $\frac{5}{6} = .83\frac{1}{3}$ | |

Hence to find .5 of 80, take $\frac{1}{2}$ of 80;
" " $.66\frac{2}{3}$ of 90, take $\frac{2}{3}$ of 90;
" " $.37\frac{1}{2}$ of 160, take $\frac{3}{8}$ of 160.

| | | |
|---|---|---|
| 3. .2 of 35. | 4. .4 of 50. | 5. .75 of 44. |
| 6. $.66\frac{2}{3}$ of 270. | 7. $.83\frac{1}{3}$ of 48. | 8. $.37\frac{1}{2}$ of 240. |
| 9. $.62\frac{1}{2}$ of 720. | 10. $.87\frac{1}{2}$ of 800. | 11. $.06\frac{1}{4}$ of 320. |
| 12. $.08\frac{1}{3}$ of 120. | 13. .40 of 4500. | 14. $.33\frac{1}{3}$ of 333. |
| 15. .50 of 2400. | 16. .80 of 5555. | 17. $.08\frac{1}{3}$ of 240. |
| 18. 20 times .75. | 19. 30 times $.66\frac{2}{3}$. | 20. 60 times $.83\frac{1}{3}$. |

21. At $6\frac{1}{4}$ ct. a gallon, how much will 16 gal. of oil cost?

22. At $62\frac{1}{2}$ ct. a pound, how much will 8 lb. of tea cost?

23. At $87\frac{1}{2}$ ct. a pair, how much must a dealer pay for 80 pairs of overshoes?

## HOW TO SOLVE PROBLEMS

**192. Nature of the rest of arithmetic.** Having learned how to work with integers and fractions, this work is hereafter to be applied chiefly to problems of life.

**193. Step work.** In solving a problem, or working an example as we also say, we should not only have the work in neat form on paper, but we should also have a brief statement of each operation, in numbered steps.

**194. Illustrative problem.** What is the value of a rectangular piece of land 20 rd. wide and 32 rd. long at $75 an acre?

*Work in steps:*

1. 20 times 32 sq. rd. = 640 sq. rd.
2. 640 times $\frac{1}{160}$ A. = 4 A.
3. 4 times $75 = $300.

*Actual work:*

$$\frac{\overset{4}{\cancel{20}} \times \$\overset{2}{\cancel{32}} \times \$75}{\underset{\$}{\cancel{160}}} = \$300$$

Use × for *times*, following the custom of the school as to writing 4 × $75, or $75 × 4.

*Oral Analysis*

Since a strip of land 32 rd. long and 1 rd. wide contains 32 sq. rd., a strip 20 times as wide contains 20 times 32 sq. rd., or 640 sq. rd.

Since 1 sq. rd. = $\frac{1}{160}$ A., 640 sq. rd. = 640 times $\frac{1}{160}$ A. = 4 A.

Since 1 A. is worth $75, 4 A. are worth 4 times $75 = $300.

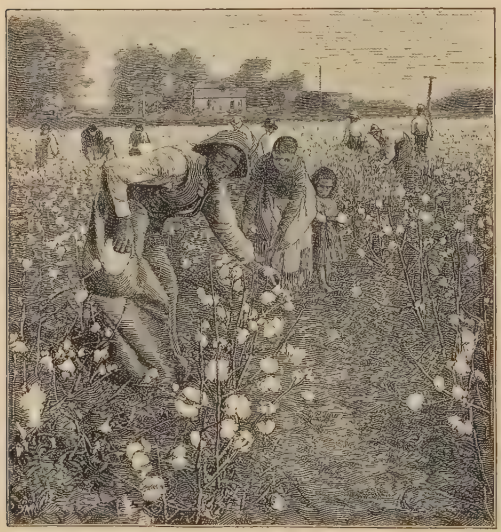
**195.** Teachers should not forget the value of each of these three phases of the work. To neglect the form of the actual computation is to place a premium upon slovenly work, always a temptation to inaccuracy; to neglect the step work is to tempt the pupil to looseness of thought as to the meaning of each operation; to neglect the analysis is to be uncertain as to the pupil's understanding of the work as a whole. The teacher's judgment must determine the emphasis to be placed at various times upon these respective phases.

## Clothing Our People

### *GROWING THE COTTON*

#### WRITTEN EXERCISE

1. This cotton field is 70 rd. long and 32 rd. wide. How many acres does it contain? What is it worth at $14.50 an acre?



2. Last year it yielded 245 lb. of cotton to the acre, net lint (without counting the seed). What was the total yield? How many bales at 490 lb. to the bale?

3. What did it cost to gin this cotton at $1.25 a bale? What to compress the bales for shipping, at 40 ct. a bale?

4. What did it cost to store it for 2 months in a warehouse, at 25 ct. a bale per month for storage and insurance?

5. The cotton was compressed in bales 5 ft. by 2 ft. by 1 ft. in size. How many cubic feet in each bale? How many cubic inches in each?

6. The 7 bales were sold by an agent. How much commission did he receive at 50 ct. a bale?

7. When cotton is worth 7 ct. a pound, what is the value of the crop produced on 14 acres averaging 245 lb. to the acre?

8. How long would it take a man to pick 3696 lb. of cotton if he averaged 154 lb. a day? It would take 4 men how long, at the same rate?

9. If this country produced 10,701,453 bales of cotton in a certain year, how many tons did it produce, estimating 500 lb. to a bale?

10. If Texas produced 2,438,555 bales in one year; Georgia, 1,345,699; Mississippi, 1,202,739; and Alabama, 1,005,313, what was the total for the four states?

11. If the Texas crop sold for $92,196,000, the Georgia crop for half as much, the Mississippi crop for $\frac{4}{5}$ as much, and the Alabama crop for $\frac{7}{8}$ as much, for how much did each sell?

12. If Texas pays $24,136,800 annually to have the cotton picked, and Georgia pays $\frac{1}{4}$ as much, and Alabama 0.9 as much as Georgia, how much does Alabama pay?

13. If there are 6,642,309 acres planted to cotton in Texas, in South Carolina $\frac{1}{3}$ as much as in Texas, and in Louisiana $\frac{1}{2}$ as much as in South Carolina, how many acres in each?

14. If our country raised $334,847,800 worth of cotton in a certain year, and paid $\frac{1}{5}$ of this amount for the labor of harvesting and marketing the crop, how much is left for the growers?

15. If we sold in one year to Great Britain $90,240,000 worth of cotton, $\frac{2}{3}$ as much to Germany, $\frac{1}{2}$ as much to France as to Germany, and $\frac{1}{6}$ as much to Italy as to Great Britain, what is the value of the cotton sold to each?

*GROWING THE WOOL*

**WRITTEN EXERCISE**

1. If the 20 sheep on a certain farm are worth on an average $3.25 apiece, what is the value of a tenant's share who owns $\frac{3}{5}$ of them?

2. How much is the wool of these 20 sheep worth, each sheep yielding $7\frac{1}{4}$ lb., and the wool selling at 35 ct. a pound?

3. In a year when 40,267,818 sheep were reported in the United States, each yielding an average of $6\frac{1}{2}$ lb. of wool, what was the total weight of the wool?

4. If this country produced 259,972,815 lb. of wool in one year, and $\frac{3}{5}$ of the weight was lost in scouring (thoroughly cleaning), how much did the scoured wool weigh?

5. In one year this country produced 302,502,000 lb. of wool, and $\frac{1}{20}$ more in the following year. How many pounds did it produce in the second year?

6. In the year when we produced the 302,502,000 lb., this was $\frac{1}{3}$ of the world's total production. How much did the world produce in that year?

7. In one year Montana, our greatest wool-growing state, produced 9,627,000 lb., Ohio 0.8 as much, and New Mexico and Wyoming each 0.9 as much as Ohio. How many pounds were produced by each of these states?

*SHIPPING THE MATERIAL*

WRITTEN EXERCISE

1. How much will it cost to ship 100 bales of cotton from New Orleans to New York, the average weight being 490 lb., and the freight being 61 ct. a hundred?

2. How much will it cost to ship a 200-lb. box of men's suits from New York to Chicago, at 75 ct. a hundred, with 75 ct. extra for cartage?

3. It costs 69 ct. a hundred pounds to ship cloth from Philadelphia to Chicago. How much would Chicago save in freight on 1000 tons of cloth, by manufacturing it at home instead of buying it in Philadelphia?

4. If 400 lb. of raw cotton are required for 100 lb. of finished clothing, and the freight on clothing from Mobile to New York is $1.18 a hundred, and on cotton 61 ct. a hundred, how much freight per 100 lb. of clothing would New York save by having clothing made in Mobile instead of at home?

5. It costs 46 ct. a hundred to ship wool in car lots from Cleveland to New York, and 53 ct. a hundred for smaller amounts. The smallest car lot of wool is 10,000 lb. If a man has 9500 lb. to ship, is it better for him to ship it as a car lot or to pay the 53-ct. rate? Give the cost in each case.

## PICTURES OF MAGNITUDES

**196. Picturing by lines.** We get a better idea of differences by illustrating the numbers.

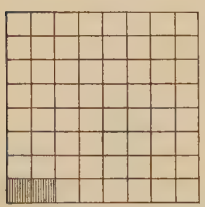
For example, the population of the United States in 1790 was 3,900,000; in 1900 it was 76,303,387. We may let a line 1 in. long represent some number, as 50 millions.

1900, 76,303,387
1790, 3,900,000
Increase, 72,403,387

Then the population in 1900, about 76 millions, will be represented by a line a little over $1\frac{1}{2}$ in. long, while that of 1790 will be represented by one a little less than $\frac{1}{12}$ in. long. The picture helps us to realize the growth of our country.

**197. Picturing by squares.** We may also represent numbers by squares or other figures.

For example, if the population of Idaho, recently about 200,000, is to be compared with that of Pennsylvania, about 6,400,000, we may let some small square, say $\frac{1}{8}$ in. on a side, represent 100,000 people. Then 64 such squares will represent the population of Pennsylvania, while 2 squares will represent that of Idaho. We then have a picture which helps us to realize the great difference in population.



Penn., 6,400,000
Idaho, 200,000
6,200,000

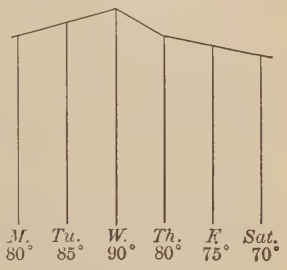
#### WRITTEN EXERCISE

The Chesapeake and Ohio Canal is 184 mi. long, the Erie 387 mi., the Lehigh 108 mi., and the Ohio 317 mi. Represent these by lines. Let 1 in. = 100 mi.

For further exercises see pages 122, 123.

**198. Number changes.** Changes in numbers may easily be shown by a picture.

For example, if the temperature at noon on Monday is 80°, and on Tuesday 85°, Wednesday 90°, Thursday 80°, Friday 75°, Saturday 70°, we may draw vertical lines to represent these temperatures. Then a line joining the tops gives us an idea of the changes in temperature.




Here $\frac{1}{8}$ in. has been taken to represent 10°. It makes no difference what distance is taken between the lines, only it must be the same because it represents 24 hours in each case.

### WRITTEN EXERCISE

1. In million bushels the production of wheat in this country in 1850 was 100; in 1860 it was 173; in 1870 it was 288; in 1880 it was 459; in 1890 it was 399; and in 1900 it was 522. Represent as above.

2. In hundred thousand bales, the production of cotton in this country for thirty years was as follows: in 1870, 31; in 1875, 38; in 1880, 58; in 1885, 57; in 1890, 73; in 1895, 99; in 1900, 94. Represent as above.

3. In hundred thousand tons, the production of sugar in the world has been as follows: in 1870, 28; in 1880, 39; in 1890, 54; in 1900, 84. Represent as above, and notice how great has been the increase as shown by the picture.

4. In hundred-thousands, the population of Illinois in 1860 was 17; in 1870, 25; in 1880, 30; in 1890, 38; in 1900, 48. Represent as above, and notice the great growth in population.

## Some Particulars in which Our Country excels Certain Others

### WRITTEN EXERCISE

1. Suppose our country should produce 302,502,328 lb. of wool in a certain year, and Spain should produce 102,600,000 lb., what would be the difference? Represent by a picture.

2. The area of Great Britain is 120,979 sq. mi., and that of the single state of Texas is 265,780 sq. mi. How much larger is Texas? Illustrate by squares.

3. The area of European Russia is 2,095,616 sq. mi., and that of the United States is 3,720,856 sq. mi. How much larger is this country? Illustrate by squares.

4. At a time when we were producing 13,689,173 tons of steel, France was producing 1,465,071 tons. What was the difference? Represent 10,000,000 tons by 1 sq. in.

5. The area of Canada is nearly as much as that of the United States, being 3,048,710 sq. mi. What is the difference in the areas of the countries? Illustrate by squares.

6. At a time when this country had $29.04 of money in circulation to each person, Italy had only $9.68. What was the difference in wealth per person? Illustrate by lines, representing $10 by 1 in.

7. Our country averages about 10,800,000 bales of cotton a year. All the rest of the world averages 2,050,000 bales. What is the difference? Represent by squares, taking a square $\frac{1}{3}$ in. on a side to represent 1,000,000 bales.

8. In a certain year we bought from Germany $101,997,523 worth of goods, and sold to that country $170,222,737 worth. How much was the balance in our favor? Illustrate by lines, representing $100,000,000 by 1 in.

## Some Particulars in which Certain Other Countries excel Ours

### WRITTEN EXERCISE

1. When the population of London was 4,536,063, that of New York City was 3,437,202. Illustrate the difference.

2. If the United States has 20,870,000 spindles in operation in the manufacture of cotton, and England has 46,100,000, what is the difference? Illustrate.

3. If our country mined 55,214,000 oz. of silver in a certain year, and Mexico mined 57,656,549 oz., how much more did Mexico mine than the United States? Illustrate.

4. The area of European Russia is 2,095,616 sq. mi., of Asiatic Russia 6,564,778 sq. mi., of the United States 3,720,856 sq. mi. How much greater is the total area of Russia than that of the United States? Illustrate.

5. The greatest wool-producing country in the western hemisphere is Argentina, which averages about 370,000,000 lb., the United States coming next with an average of about 304,500,000 lb. What was the difference? Illustrate.

6. The area of Italy is 110,646 sq. mi., and that of Nevada is 110,700 sq. mi. When the population of Italy was 32,045,404, that of Nevada was 42,335. How much greater is the area of Nevada, and how much greater was the population of Italy? Illustrate.

7. When the population of the United States was 76,303,387, that of Porto Rico was 953,243; of Hawaii, 154,001; of the Philippines and Guam, 7,909,000. The population of Russia was then 128,852,076. How much greater was the population of Russia than that of the United States and its island possessions? Illustrate.

### WRITTEN EXERCISE

In review drills like the following a time limit should be set upon the work, and checks should be required.

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| \$476.29 | \$663.48 | \$298.63 | \$348.00 |
| 36.82 | 487.29 | 42.75 | 29.02 |
| 48.19 | 23.00 | 349.62 | 76.35 |
| 293.05 | 304.09 | 79.36 | 298.42 |
| 47.26 | 62.68 | 82.59 | 109.83 |
| 83.42 | 73.48 | 682.09 | 67.08 |

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| \$1000.00 | \$824.63 | \$209.09 | \$802.35 |
| 289.00 | 981.42 | 32.78 | 608.72 |
| 40.27 | 127.69 | 45.67 | 402.81 |
| 6.35 | 842.38 | 682.73 | 96.43 |
| 42.07 | 327.29 | 102.93 | 87.29 |
| 189.27 | 420.63 | 41.52 | 289.62 |
| 96.96 | 29.00 | 647.38 | 43.59 |

9. \$640.00 − \$298.29.
10. \$702.53 − \$129.65.
11. \$902.63 − \$127.49.
12. \$1000.00 − \$298.75.
13. \$3476.42 − \$1029.29.
14. \$2025.30 − \$1627.92.
15. 27 times \$672.42.
16. 39 times \$486.39.
17. 37 times \$129.46.
18. 43 times \$298.63.
19. $4.2 \times 125.3$.
20. $3.1 \times 149.9$.
21. $4.25 \times 4864$.
22. $3.75 \times 8796$.
23. $8.9 \times 9876$.
24. $2.9 \times 8971$.
25. \$32.48 ÷ 8.
26. \$21.42 ÷ 18.
27. \$82.20 ÷ 12.
28. \$26.25 ÷ 25.
29. \$99.54 ÷ 14.
30. \$43.75 ÷ 35.
31. $\dfrac{12\frac{1}{2} + 7\frac{7}{8}}{15.75 - 5\frac{11}{16}}$.
32. $\dfrac{2.75 \div 2.5}{6.15 - 5\frac{1}{20}}$.
33. $\dfrac{7.5 \times 5.1}{.9 \times 8.5}$.
34. $\dfrac{3.45 \times 6.25}{2.3 \times 2.5}$.
35. $\dfrac{1.2 \times 93.1}{4.2 \times 3.8}$.
36. $\dfrac{2\frac{1}{2} + 3\frac{3}{4}}{2.5 \times 1.25}$.

## BILLS AND RECEIPTS

**199. Debtor and creditor.** When one man owes another he is called a *debtor* (Dr.). The man to whom he owes money is called his *creditor* (Cr.).

**200. Debit and credit.** The amount owed on a bill is called the *debit* part of the bill, and any payments that have been made form the *credit* part.

**201. Bills.** Bills usually begin like this:

Mr. A. B. Jefferson

Bought of ROE & DOE, Grocers,

No. 4 Washington Street.

They may also read: "To Roe & Doe, Dr.," this meaning that Mr. Jefferson is debtor to Roe & Doe for the goods mentioned in the bill.

The following is an example of a bill filled out, footed, and receipted.

New Orleans, La.,.....[*Date*].....

Mrs. John Doe, 47 Tenth Avenue,

To RICHARD ROE & CO., Dr.

Dry Goods Dealers, No. 10 Grand Street.

| | | | | |
|---|---|---|---|---|
| .....[*Year*]..... | | | | |
| .....[*Date*]..... | $\frac{1}{2}$ yd. Silk | .75 | | 38 |
| " | $\frac{3}{4}$ " " | 1.25 | | 94 |
| " | $7\frac{1}{2}$ " Velours | 3.50 | 26 | 25 |
| | | | $27 | 57 |

Received payment,

Richard Roe & Co.

### WRITTEN EXERCISE

*Date, fill, foot, and receipt the following bills, inserting the name and address of some purchaser and of some dealer whom you know:*

1. Bill for furniture: 6 chairs @ \$3.00; 2 armchairs @ \$6.25; 1 rocking-chair @ \$5.50; 1 table @ \$22.75.

An item like "6 chairs @ \$3.00" means \$3.00 each.

2. Bill for dry goods: 3½ yd. satin @ \$1.50; 2 yd. flannel @ 50 ct.; 1½ yd. canvas @ 20 ct.; 1½ yd. ribbon @ 23 ct.

3. Bill for meat: 2½ lb. steak @ 22 ct.; 3¼ lb. fish @ 20 ct.; 4¾ lb. roast beef @ 20 ct.; 8 lb. turkey @ 21 ct.

4. Bill for dry goods: 17½ yd. embroidery @ 10 ct.; 3 yd. lawn @ 20 ct.; 7½ yd. percale @ 25 ct.; ⅛ yd. satin @ \$1.50.

5. Bill for crockery: ½ doz. cups and saucers @ \$6.00; 3 doz. plates @ \$4.25; 1½ doz. tumblers @ 90 ct.; 2 doz. individual butter plates @ 70 ct.; ½ doz. bowls @ \$2.30.

An item like "½ doz. cups @ \$6.00" means \$6.00 a dozen.

6. Bill for groceries: 10 lb. sugar @ 8 ct.; 2 qt. berries @ 11 ct.; 4 doz. eggs @ 26 ct.; 5½ lb. butter @ 32 ct.; 1½ lb. cheese @ 22 ct.; ½ doz. bars soap @ 44 ct.

7. Bill for dry goods: 7 yd. muslin @ 18 ct.; 6 handkerchiefs @ 30 ct.; 3 pairs kid gloves @ \$1.50; 12 yd. sheeting @ 12½ ct.; 16 yd. pongee @ 70 ct.

8. Bill for stationery: 500 envelopes @ \$3.50 per M (that is, per 1000); 1 box writing paper @ \$2.25; ½ doz. pencils @ 35 ct.; 3 penholders @ 5 ct.; 1 bottle ink @ 30 ct.

9. Bill for groceries: 5 lb. coffee @ 32 ct.; 2 lb. tea @ 45 ct.; 8 lb. ham @ 15 ct.; 7 lb. lard @ 8 ct.; 3½ lb. butter @ 33 ct.; 3 qt. currants @ 7 ct.; 2 doz. eggs @ 31 ct.; 1¾ lb. cheese @ 16 ct.; ½ gal. sirup @ 65 ct.

### ORAL EXERCISE

1. Will is 10 yr. old and his father is 40 yr. old. His father is how many times as old as Will? Will's age is what part of his father's age? How many hundredths?

2. Will's age is how many fortieths less than his father's? How many fourths less? How many hundredths?

3. Will's age is what part of the sum of his age and his father's? How many hundredths? His father's age is what part of the sum? How many hundredths?

4. Will's age is what part of the difference of their ages? How many hundredths?

### WRITTEN EXERCISE

*Estimate the answers to the following, writing the estimate. Then solve, carrying the results to 2 decimal places.*

1. $\frac{22.25 + 32.75}{17.65 - 6.45}$.
2. $\frac{34.67 - 22.47}{8.72 - 2.62}$.
3. $\frac{41.73 - 12.43}{17.81 - 9.61}$.
4. $\frac{2.75 \times 3.25}{16.87 - 13.57}$.
5. $\frac{4.33\frac{1}{3} \times 666}{5680 \div 2}$.
6. $\frac{8.25 \times 9.5}{62.45 - 53.05}$.
7. $\frac{82.75 \div 0.25}{311.25 + 13.75}$.
8. $\frac{26.57 + 73.03}{37\frac{1}{2} - 12\frac{1}{4}}$.
9. $\frac{63.32 + 61.08}{3.27 + 1.73}$.

*Express the following in fractional form, like the above, and estimate the answer before solving:*

10. If a man earns \$15.50 a week, and spends \$10, how many weeks will it take him to pay for a 14-volume encyclopedia at \$2.75 a volume, with the remainder?

11. If a man earns \$1279.50 a year, on an average, and spends \$304.50, how many years will it take him to save enough to buy a 75-acre farm at \$65 an acre?

### ORAL EXERCISE

1. John is 15 yr. old and his sister is 10. His age is how many hundredths more than hers? Hers is how many hundredths less than his?

2. How many hundredths of the sum of their ages is John's age? Mary's age?

3. The difference of their ages is how many hundredths of the sum? The sum is how many hundredths more than the sister's age?

4. If a horse cost \$125 and a carriage \$150, the carriage cost how many hundredths more than the horse? The horse cost how many hundredths less than the carriage?

### WRITTEN EXERCISE

1. How many hundredths of \$150 is \$50? \$75?
2. How many fiftieths of \$250 is \$50? \$75? \$125?
3. How many thirds of 300 ft. is 100 ft.? 200 ft.?
4. How many eighths of \$400 is \$50? \$300? \$350?
5. $27.5 \times 32.6$.
6. $48.25 \times 51.75$.
7. $32.33\frac{1}{3} \times 66.6$.
8. $2.83 \times 3.82$.
9. $27.48 \times 26.35$.
10. $41.25 \times 12.75$.
11. $26.25 \div 2.5$.
12. $38.47 \div 0.33\frac{1}{3}$.
13. $685.5 \div 7.5$.
14. $3.476 \div 1.1$.
15. $1.752 \div 1.2$.
16. $4125 \div 0.15$.
17. $\dfrac{2.75 + 3.25 + 6.33\frac{1}{3}}{1.01\frac{1}{9} + 2.1 + 1}$.
18. $\dfrac{49.82 + 6.08 - 1.2}{60.1\frac{2}{3} + 0.4 - 41}$.
19. $\dfrac{3.25 + 6.25 + 4.25}{2.25 + 4\frac{7}{8} - 0.25}$.
20. $\dfrac{5.2 \times 6.3 \times 2.1}{0.7 \times 0.9 \times 2.6}$.
21. $\dfrac{4.75 + 1.75 - 2.50}{4.85 + 1.25 - 4.10}$.
22. $\dfrac{3.4 \times 4.2 \times 3.5}{0.7 \times 0.7 \times 1.7}$.

## PERCENTAGE

### ORAL EXERCISE

1. Give other names for the following: 3 ft., $\frac{1}{2}$ lb., 16 oz., $5\frac{1}{2}$ yd., 100 ct., 1 rd., 4 qt., 4 pk., $\frac{1}{2}$ qt.

2. Another name for .5 is $\frac{1}{2}$. Another way of writing $\frac{7}{10}$ is .7. Give another name for, and another way of writing $\frac{3}{4}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{1}{3}$.

**202. Per cent.** Another name for hundredths is *per cent.*

That is, 1 per cent means 1 one-hundredth, or $\frac{1}{100}$; 2 per cent means 2 one-hundredths, or $\frac{2}{100}$; $\frac{1}{2}$ per cent means $\frac{1}{2}$ of a hundredth, or $\frac{1}{200}$.

**203. Per cent sign.** There is also another way of writing per cent, or hundredths. It is by using the sign %.

That is, 1% means $\frac{1}{100}$, or .01; 6% means $\frac{6}{100}$, or .06; $\frac{1}{4}$% means $\frac{1}{4}$ of $\frac{1}{100} = \frac{1}{400}$.

**204. Reading the sign %.** The sign % may be read either *per cent* or *hundredths,* for they mean the same. It is usually read *per cent.*

If asked to write 10 per cent, you may write either 10% or .10.

### WRITTEN EXERCISE

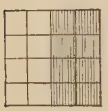
*Write in two other ways the numbers in Exs. 1–18:*

| | | |
|---|---|---|
| 1. Nine per cent. | 2. 17 per cent. | 3. 28%. |
| 4. 37 per cent. | 5. 47 hundredths. | 6. 16%. |
| 7. 63 hundredths. | 8. 87 per cent. | 9. $33\frac{1}{3}$%. |
| 10. 100 per cent. | 11. 125 per cent. | 12. $16\frac{2}{3}$%. |
| 13. 225 per cent. | 14. 300 per cent. | 15. 75%. |
| 16. One per cent. | 17. 99 per cent. | 18. 100%. |

### ORAL EXERCISE

**1.** If this square is called 100, how much is A? B? If the square is called 1, how many hundredths is A? B?

2. Read .50, using the words *per cent.* Read it using the word *hundredths.* How many halves are there in 1? Then how many times is 50% contained in 1?



3. If this square is called 16, how much is the shaded part? How many sixteenths of the square are shaded? How many hundredths of it are shaded?

4. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? The shaded part is how many hundredths of the square?

5. Read .25, using the words *per cent;* using the word *hundredths.* How many fourths in 1? Then how many times is 25% contained in 1?

6. If this square is called 1, how much is the shaded part? If it is called 100, how much is the shaded part? What per cent of the square is shaded?

7. How many eighths in 1? Then how many times is $12\frac{1}{2}\%$ contained in 1?

8. Draw a picture on the blackboard and shade 10% of it. Draw another and shade 75%.

9. How many times is $16\frac{2}{3}\%$ contained in 1? in 2?

10. How many times is $6\frac{1}{4}\%$ contained in 1? in 3?

11. How many times is 20% contained in 1? in 5?

### ORAL EXERCISE

1. Half of a square is what per cent of it?

2. A fourth of a square is what per cent of it?

3. An eighth of a square is what per cent of it?

4. A sixteenth of a square is what per cent of it?

5. How much is $\frac{1}{2}$ of $6\frac{1}{4}$? $\frac{1}{2}$ of $6\frac{1}{4}\%$? $\frac{1}{32}$ is how many per cent?

**205. Important per cents.** We have found that

$50\% = \frac{1}{2}$, $25\% = \frac{1}{4}$,

$12\frac{1}{2}\% = \frac{1}{8}$, $6\frac{1}{4}\% = \frac{1}{16}$.

6. How much is 50% of 20? of 40? of 50? of 7?

7. How much is 25% of 32? of 60? of 120? of 5?

8. How much is $12\frac{1}{2}\%$ of 40? of 72? of 88? of 100?

9. How much is $6\frac{1}{4}\%$ of 32? of 16? of 160? of 320?

10. If your class numbers 32, and 50% are boys, how many are boys?

### WRITTEN EXERCISE

1. 50% of 632.
2. 25% of 592.
3. 50% of 333.
4. 25% of \$3416.
5. 50% of \$4917.
6. 25% of 432 mi.
7. 50% of 978 bu.
8. 25% of 96 ft. 8. in.
9. $12\frac{1}{2}\%$ of \$91.28.
10. $12\frac{1}{2}\%$ of \$73.68.
11. $12\frac{1}{2}\%$ of 72 lb. 8 oz.
12. $6\frac{1}{4}\%$ of 17 lb.
13. $6\frac{1}{4}\%$ of \$43.20.
14. $6\frac{1}{4}\%$ of \$15.68.

15. Of 152 chickens a hawk captured $12\frac{1}{2}\%$. How many were captured? How many were left?

16. Of 1408 sheep owned by a farmer, $6\frac{1}{4}\%$ were sold. How many did he sell? How many remained?

### ORAL EXERCISE

1. If this circle is called 100, how much is the shaded part? the unshaded part? If the circle is called 1, what per cent is the shaded part?

2. Read .33⅓, using the words *per cent.* Read it using the word *hundredths.* How many thirds are there in 1? Then how many times is 33⅓% contained in 1?

3. If this circle is called 100, the shaded part is what part of 33⅓? How much is this? Then the shaded part is what per cent of the circle?

4. How much is ½ of 16⅔%? The shaded part of this circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?

**206. Important per cents.** We have found that

$33\frac{1}{3}\% = \frac{1}{3}$, $66\frac{2}{3}\% = \frac{2}{3}$,
$16\frac{2}{3}\% = \frac{1}{6}$, $8\frac{1}{3}\% = \frac{1}{12}$.

5. How much is 33⅓% of 90? of 120? of 45? of 60?
6. How much is 66⅔% of 30? of 66? of 90? of 300?
7. How much is 16⅔% of 36? of 24? of 66? of 72?
8. How much is 8⅓% of 24? of 48? of 60? of 120?

### WRITTEN EXERCISE

1. 33⅓% of 891.
2. 66⅔% of 711.
3. 16⅔% of 504.
4. 16⅔% of $41.10.
5. 8⅓% of $15.60.
6. 8⅓% of $159.72.
7. 66⅔% of 747 rd.
8. 16⅔% of 9774 bu.
9. 33⅓% of 222 ft. 9 in.
10. 33⅓% of 477 men.
11. 16⅔% of 726 bu. 6 qt.
12. 66⅔% of 141 lb. 15 oz.

### ORAL EXERCISE

1. How much is $\frac{300}{100}$? $\frac{200}{100}$? 200%? $\frac{100}{100}$? 100%?

2. Anything is what per cent of itself? of half itself?

3. If we call column A one, what is B? C? D? If we call A 100%, what per cent is B? C? D?

4. If we call A two, what is B? C? D? If we call A 200%, what per cent is B? C? D?

5. If we call C 100%, what per cent is D? B? A? B + D? A + B + D?

6. If we call D 100%, what per cent is C? B? A?

7. If we call B 100%, what per cent is D + C? C + A?

8. Because $1.50 = 1\frac{50}{100} = \frac{150}{100}$, 1.50 = what per cent?

**207. Reading per cents.** We have found that 2.75 may be read "two and seventy-five hundredths," or "two hundred seventy-five per cent."

9. Read, using the words *per cent:* 4.35, 0.75, 0.05, 1.00.

### WRITTEN EXERCISE

*Express as decimal fractions the numbers in Exs. 1–10:*

1. 25%. 2. $33\frac{1}{3}$%. 3. 1%. 4. $2\frac{1}{2}$%. 5. 7.5%.
6. 125%. 7. $\frac{1}{2}$%. 8. $\frac{3}{4}$%. 9. 200%. 10. 0.3%.

*Express as common fractions reduced to lowest terms the numbers in Exs. 11–20:*

11. 32%. 12. 24%. 13. 36%. 14. 17%. 15. $82\frac{1}{2}$%.
16. 68%. 17. $6\frac{1}{4}$%. 18. 125%. 19. 375%. 20. $33.3\frac{1}{3}$%.

*Express as per cents the following:*

21. $\frac{1}{2}$. 22. $\frac{4}{5}$. 23. $\frac{7}{8}$. 24. $\frac{1}{3}$. 25. $1\frac{2}{5}$.

## Important Per Cents to be remembered

### ORAL EXERCISE

1. Give the per cent equivalents of the eighths.

2. $87\frac{1}{2}\%$ is how many times $12\frac{1}{2}\%$?

3. What is the quotient of $75\% \div 62\frac{1}{2}\%$? (Divide equivalent common fractions.)

4. What is the quotient of $62\frac{1}{2}\% \div 50\%$?

5. Give the per cent equivalents of the sixths.

6. Divide $83\frac{1}{3}\%$ by $33\frac{1}{3}\%$.

7. Give the per cent equivalents of the fifths.

8. Divide 1 by 80%.

### WRITTEN EXERCISE

*Draw figures as above and write a table of equivalents of the following:*

1. The sevenths.
2. The ninths.
3. The twelfths.
4. The sixteenths.

5. Divide 20% by $16\frac{2}{3}\%$. (Divide the equivalent common fractions, and write the result both as a common fraction and as a decimal.)

*In the same way perform these operations:*

6. $50\% \div 37\frac{1}{2}\%$.
7. $33\frac{1}{3}\%$ of 75%.
8. $83\frac{1}{3}\%$ of 60%.
9. $80\% \div 50\%$.
10. $12\frac{1}{2}\%$ of 80%.
11. 60% of $33\frac{1}{3}\%$.

**208.** Since 90%, 0.90, and $\frac{90}{100}$ all mean the same, 90% of a number is the same as that number multiplied by 0.90.

Thus, 90% of 60 = .90 of 60 = 54,
85% of 70 = .85 of 70 = 59.5,
15% of 200 = .15 of 200 = 30,
12½% of 40 = .125 of 40 = ⅛ of 40 = 5.

### ORAL EXERCISE

1. If you have 50 words in a spelling test, and are marked 98%, how many words did you spell correctly?

2. If you have 10 questions in arithmetic, and are marked 80%, how many did you answer correctly?

3. If you have 12 questions in geography, and are marked 75%, how many did you answer correctly?

4. If 60% of a class of 40 have not missed a day this year, how many have been present every day?

5. If a merchant pays 2 ct. apiece for pencils, and sells them at a profit of 50%, what does he receive for each?

6. If a milk dealer pays 5 ct. a quart for milk, and sells it at a profit of 20%, what does he receive per quart?

7. If a butcher buys meat at 18 ct. a pound, and sells it at a profit of 16⅔%, at what rate does he sell it per pound?

### WRITTEN EXERCISE

| | |
|---|---|
| 1. 22% of 5475. | 2. 65% of $9200. |
| 3. 6% of $2496. | 4. 87% of 4700 ft. |
| 5. 95% of $625. | 6. 66⅔% of $4131. |
| 7. 72% of $345. | 8. 17½% of $4000. |
| 9. 35% of 5280 ft. | 10. 31% of 4920 lb. |
| 11. 62% of 3500 mi. | 12. 87½% of $14.40. |

### ORAL EXERCISE

1. From these columns show that $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, and $80\% = \frac{4}{5}$.

2. Also show that $20\%$ is $\frac{1}{2}$ of $40\%$, $\frac{1}{3}$ of $60\%$, and $\frac{1}{4}$ of $80\%$.

3. Also show that $40\%$ is $\frac{2}{3}$ of $60\%$, $\frac{1}{2}$ of $80\%$, and $\frac{2}{5}$ of $100\%$.

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 20% |
| | | | | 20% | 20% |
| | | | 20% | 20% | 20% |
| | | 20% | 20% | 20% | 20% |
| 20% | 20% | 20% | 20% | 20% | 20% |
| 20% | 40% | 60% | 80% | 100% | |

4. Also show that $60\% = 1\frac{1}{2}$ times $40\%$, $\frac{3}{4}$ of $80\%$, and $\frac{3}{5}$ of $100\%$.

These columns should be written on the board. Columns of other per cents, like $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, $25\%$, and $33\frac{1}{3}\%$ are also very useful for introducing rapid drill work.

### WRITTEN EXERCISE

1. Find $12\frac{1}{2}\%$ of 984; of 6116; of \$177.76.

2. Find $33\frac{1}{3}\%$ of 729; of \$923.22; of 5280 ft.

3. Find $16\frac{2}{3}\%$ of 5280 ft.; of \$1234.56; of 1728 cu. in.

4. A man bought a bicycle for \$40, and sold it at a loss of $35\%$. How much did he lose?

5. A man bought 83 sheep at \$6 apiece, and sold them at a profit of $15\%$. How much did he gain?

6. A dealer buys 25 cows at \$32 each, and sells them at a profit of $8\%$. How much does he gain on them all?

7. A man bought an automobile for \$700, and after using it two seasons sold it for $15\%$ less than it cost. What was the selling price?

8. A farmer bought 75 A. of land at \$65 an acre, and 12 cows at \$35 a head. He sold the land at a profit of $10\%$ and the cows at a loss of $5\%$. Did he gain or lose on the whole, and how much?

**209. Base.** The number of which some per cent is to be found is called the *base*.

**210. Rate.** The number of hundredths to be taken is called the *rate*.

Thus, in 6% of \$200, \$200 is the *base* and 6% is the *rate*. Sometimes 6 is called the *rate per cent*, but the terms *rate* and *rate per cent* are generally used to mean the same thing.

**211. Percentage.** The result found by taking a certain per cent of the base is called the *percentage*.

**212.** *The percentage is the product of the base and rate.*

Excepting *rate*, these terms are of little value in business, but in school they are occasionally convenient for designating different numbers in a problem.

WRITTEN EXERCISE

1. Given the base 725, and the rate 7%, find the percentage.

2. A man has a salary of \$1200, and saves 18% of it. How much does he save?

3. If you go to school $51\frac{7}{13}\%$ of the days of the year, how many days do you attend?

4. If a man owes \$1470, and pays $33\frac{1}{3}\%$ of his debts this year, how much remains unpaid?

5. A certain fertilizer contains $37\frac{1}{2}\%$ of lime. How many pounds of lime in 3 tons of fertilizer?

6. In a certain ore the metal is 9.8% of the total weight. How many pounds of metal in 1950 lb. of ore?

7. A dealer bought 10 automobiles at \$750 each, and sold them at a profit of 20%. How much did he gain?

8. If the value of our silk manufactures is \$107,260,000 annually, while ten years ago it was 80% as much, how much was it then?

### ORAL EXERCISE

1. 4 is what part of 32? what per cent of 32?
2. 15 is what part of 30? what per cent of 30?
3. 3 is what per cent of 36? 4 is what per cent of 48?
4. 7 is what per cent of 42? 9 is what per cent of 54?

**213.** \$17.50 is what per cent of \$250?

Because \$1 is $\frac{1}{250}$ of \$250, therefore \$17.50 is $\frac{17.5}{250}$ of \$250, and $\frac{17.5}{250} = 0.07$.

$$250\overline{)17.5}\quad .07$$
$$\underline{17.5}$$

**214.** We therefore see that *the percentage divided by the base equals the rate.*

### WRITTEN EXERCISE

1. \$130 is what per cent of \$2600? of \$3900?
2. \$11.25 is what per cent of \$375? of \$56.25?
3. \$10.12 is what per cent of \$253? of \$202.40?
4. \$29.10 is what per cent of \$485? of \$174.60?
5. A foot is what per cent of a yard? of a rod?
6. A quart is what per cent of a gallon? of 25 gallons?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. A man invested \$1750 and lost \$192.50. What per cent did he lose?
10. A school having 275 pupils graduated 33. What per cent were graduated?
11. A man had \$2500 at the beginning of a year, and \$2775 at the end. How much did he gain? What per cent did he gain?

### ORAL EXERCISE

1. If $\frac{1}{5}$ of a number is 7, what are 5 fifths of it? What is the number?

2. If 10% of a number is 30, what is 1%? What is 100% of it, or once the number?

3. If 25% of it is 7, what is the number?

*State rapidly the numbers of which*

4. 25% is 9.
5. $33\frac{1}{3}$% is 12.
6. $12\frac{1}{2}$% is 7.
7. 6% is 54.
8. 9% is 27.
9. 13% is 26.

**215.** If 28% of a number is \$12.60, what is the number?

Since 28% of the number is \$12.60,

$$1\% \text{ “ “ “ “ } \frac{\$12.60}{28},$$

$$100\% \text{ “ “ “ “ } \frac{\$1260}{28} = \$45.$$

Since $\frac{\$1260}{28} = \frac{\$12.60}{.28}$ (§ 86), we see that

**216.** *The percentage divided by the rate equals the base.*

### WRITTEN EXERCISE

1. If 20% of the people in a town are voters, and there are 236 voters, what is the population?

2. If 20% of your class are over 12 years old, and there are 8 such pupils, how many are there in your class?

3. If a farmer sold a horse for 80% of its cost, and received \$152, how much did the horse cost?

4. If a poultry raiser set some eggs, and 15% fail to hatch, how many did he set, the number failing to hatch being 33?

### ORAL EXERCISE

1. If 4 times a certain number is 36, how do we find the number? If $1\frac{1}{2}$ times a certain number is 90, how do we find the number?

2. If .50 of a certain number is 300, how do we find the number? If 25% of a certain number is 60, how do we find the number?

3. How much is $\$1 - \$\frac{1}{4}$? 1 ft. $-$ 0.25 ft.? 1 yd. $-$ 25% of 1 yd.? $1 - 75\%$? $1 - 80\%$? $1 - 30\%$?

4. What part of anything is left after taking away $\frac{1}{4}$ of it? 75% of it? 65% of it? 99% of it? 1% of it?

**217.** A man sold an automobile for \$490, which was 30% less than it cost. How much did it cost?

1. Cost $-$ .30 of cost = .70 of cost, just as $\$1 - \$0.30 = \$0.70$.
2. If .70 of cost = \$490,
3. The cost = $\$490 \div .70 = \$700$.

### WRITTEN EXERCISE

*Find the numbers of which*

| | | |
|---|---|---|
| 1. 35% is 7. | 2. 4% is 8. | 3. 75% is 75. |
| 4. 90% is 198. | 5. 10% is 100. | 6. 20% is 440. |
| 7. 13% is 22.36. | 8. 17% is 42.5. | 9. 33% is 10.89. |

10. After losing 15% of its population a town had 5950 inhabitants. How many had it before losing the 15%?

11. A dealer sold a bicycle for \$24.50, which was 30% less than the marked price. What was the marked price?

12. A man bought a set of furniture for \$52, which was 20% less than the marked price. What was the marked price?

### ORAL EXERCISE

1. If $1\frac{1}{4}$ times a number is 50, what is the number?
2. If 1.25 times a number is 25, what is the number?
3. If 125% of a number is 250, what is the number?

*Find the numbers of which*

4. 120% is 24.
5. 200% is 16.
6. 150% is 60.
7. 125% is 100.
8. $133\frac{1}{3}$% is 80.
9. 300% is 30.

**218.** A man sold a set of furniture for \$78, which was 30% more than it cost. How much did it cost?

1. Cost + .30 of cost = 1.30 times cost, just as 1 ft. + .3 ft. = 1.3 ft.
2. If 1.30 times cost = \$78,
3. The cost = \$78 ÷ 1.30 = \$60.

### WRITTEN EXERCISE

1. A dealer sold pencils at 5 ct. each, which gave him a profit of 25% How much did they cost him a dozen?

2. A farmer purchased 35 sheep, and sold them for \$252, which gave him a profit of 20%. What did they cost him?

3. A dealer sells some goods for \$31.05, thus making a profit of 15% on the cost. How much did the goods cost him?

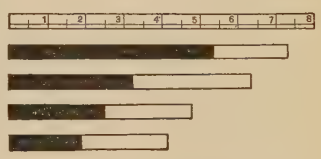
4. A manufacturer sells a suit of clothes to a merchant at a profit of $12\frac{1}{2}$%. The merchant sells it to you for \$12, and makes a profit of $33\frac{1}{3}$%. How much does it cost the merchant? the manufacturer?

5. An agent buys 100 doz. eggs, which he sells to a grocer at a profit of 20%. The grocer sells the eggs for \$22.44, and makes a profit of 10%. What did the eggs cost the grocer per dozen? the agent?

## Where Our People live

**219**. The rectangles show population in millions, as numbered on the scale above.

| *State* | *Population* | *Per cent in cities* |
|---|---|---|
| New York | 7,300,000 | 73% |
| Pennsylvania | 6,400,000 | 51% |
| Illinois | 4,900,000 | 51% |
| Ohio | 4,200,000 | 46% |




The dark part shows the numbers living in cities or villages of 4000 or over. Thus in New York state 73% of 7,300,000 people, or 5,329,000 people, lived in such cities or villages at the opening of the century.

### WRITTEN EXERCISE

1. Draw a diagram as above, on the same scale, for Missouri, the population being supposed to be 3,200,000, and the city population 35% of the whole.

2. Also for Iowa, 2,300,000, city population 21%.

3. Also for California, 1,500,000, city population 49%.

4. Of our total population of 5,308,000 in 1800, 207,012 lived in cities. What per cent lived in cities?

5. Of our total population of 76,303,000 in 1900, 28,079,504 lived in cities. What per cent lived in cities?

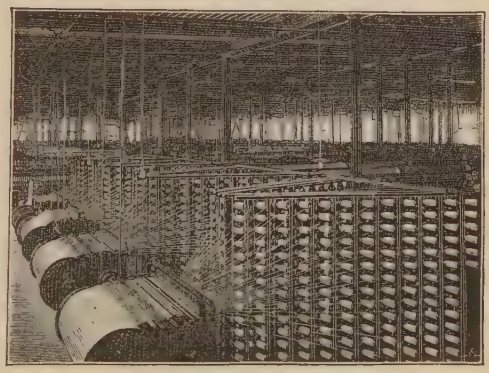
6. In 1820 our population was 9,600,000, and 124,800 lived in cities of over 100,000 inhabitants. What per cent was this?

7. In 1900 our population was 76,303,000, and 14,192,358 lived in cities of over 100,000 inhabitants. What per cent was this?

## Manufacturing the Clothing for Our People

### WRITTEN EXERCISE

1. If the United States manufactured $345,000,000 worth of cotton cloth in a certain year, and wool goods worth 90% as much, and hosiery worth $\frac{1}{3}$ as much as the wool goods, what was the value of each of these products?



2. If twenty years ago only 2% of the 18,000,000 spindles (for spinning cotton) were in the southern states, while now 42% of the 21,558,600 spindles are there, what was the number then? What is it now?

3. If in the cotton-producing states there are 500 mills, having a total of 5,001,500 spindles, how many spindles are there on an average to each mill?

4. If twenty years ago the northern mills used 1,165,000 bales a year, and the southern mills 18% as much, while now the northern mills use 2,262,000 bales and the southern mills use 85% as much, how many bales did the latter use then? How many do they use now?

5. If the value of our wool manufactures is now $316,800,000 annually, while ten years ago it was 82% as much, how much was it then?

6. In New York City 20,000 workers were employed in making men's clothes in a certain year, and the output was $103,000,000 worth a year. What was the average output per worker?

7. There were also 19,000 workers employed there in making women's clothes, and the output was $102,030,000 worth a year. What was the average output per worker?

8. In a certain coat factory 9 women make buttonholes at $2\frac{1}{2}$ ct. apiece. At 4 buttonholes to a coat, and 600 coats a week, what are the average weekly wages of each?

9. In this factory a woman receives $5\frac{1}{2}$ ct. a vest for basting, and she can baste 120 vests a week. What are her weekly wages?

10. A dressmaker uses in making a waist $2\frac{1}{2}$ yd. of white Persian lawn worth 24 ct. a yard, $4\frac{1}{4}$ yd. of embroidery worth 16 ct. a yard, 6 buttons worth 40 ct. a dozen, and 10 ct. worth of thread. How much does the material cost?

11. A working girl in a city shop pays $3 a week for board and room, 5 ct. a day 6 days in the week for car fare, and 15 ct. a day 6 days in the week for luncheon. Her incidental expenses are 60 ct. a week. If her wages are $6 a week, how much does she save a week? How much in a year of 50 working weeks?

12. Of the $39,271,900 worth of knit goods produced in this country in a certain year, New England produced $27\frac{1}{3}\%$, the middle states $66\frac{2}{3}\%$, the western states 6%, and the South practically none. Twenty years later New England produced 18% of the $101,337,000 worth manufactured, the middle states 64%, the West 12%, and the South 6%. Find the value of the product for each section for each year.

## Our Fisheries

### WRITTEN EXERCISE

1. This is part of the fleet that landed 176,774,300 lb. of fish at Boston and Gloucester in a certain year. Of this amount, 63% was landed at Boston. How many pounds were landed at Boston?

2. The value of the total catch in Ex. 1 was $4,193,600, and that of Gloucester was $2,725,840. The latter amount was what per cent of the former?

3. If $2,800,000 is invested in the Lake Erie fisheries, and the annual product is $1,176,000, this is what per cent of the amount invested?

4. If the herring fishery produces $435,120, this is what per cent of the total $1,176,000?

5. If Maryland has 42,800 persons engaged in fisheries, Virginia 65% as many, and New York 21% as many, how many have Virginia and New York together?

6. Whales are not really fish, but whaling is usually classed as a fishery. The United States recently had 48 vessels engaged in whaling. If they secured 11,904 bbl. of sperm oil in one year, valued at $49 a barrel, what was the average amount and value of oil produced by each?

7. If they also secured 3792 bbl. of whale oil, valued at $35 a barrel, what was the average for each vessel, in quantity and value?

8. If they also secured 321,600 lb. of whalebone, valued at $2.70 a pound, what was the average for each vessel, in quantity and value?

9. If Florida has 4472 men engaged in the fishing industry, and this is 40% of the total number engaged in this work in the Gulf states, what is the total number? The total number is what per cent of the number in Florida?

10. Sponge gathering, a Florida industry, is usually ranked as one of the fisheries. If it amounted in one year to 13½% of the $2,340,000 received from the Gulf-states fisheries, what was its value?

11. If the oyster industry of Louisiana and Mississippi amounted to 23½% of this same total, $2,340,000, what was the value?

12. The principal fishing industry in North Carolina is catching shad. What is the value of shad which are worth 28% of the whole annual income, in a year when the fisheries bring in $1,300,000?

13. In catching salmon in the rivers flowing into Puget Sound, a seine measuring about 100 fathoms long and 35 ft. wide is often used. A fathom being 6 ft., how many square feet in such a net?

14. The Indians there often use a seine that is only ⅓ as long and ⅓ as wide. What is its area? This area is what part of that of the large net?

15. If a boat brings back from a day's fishing 25 salmon averaging 7 lb. 8 oz. each, what is the weight of the day's catch? What is it worth at 2 ct. a pound?

## DISCOUNT

### ORAL EXERCISE

1. How much is \$200 less \$50? \$100 less 25% of itself?

2. If from 1 we take $\frac{1}{4}$, what is left? If from 1 we take 25%, what is left?

3. How much is 1 less $\frac{1}{5}$? $\frac{100}{100} - \frac{20}{100}$? $1 - .20$? $1 - 20\%$?

4. If you buy a \$1 book for 10% less than the list price, how much does it cost you?

**220. Discount.** A deduction made on a price or amount is called a *discount.*

Discounts are usually reckoned as so many per cent.

**221. Rate of Discount.** The per cent of discount is called the *rate of discount.*

**222.** What is the discount and the selling price, when the list price is \$245.50 and the rate of discount is 12%?

12% of \$245.50 = \$29.46, discount.
\$245.50 − \$29.46 = \$216.04.

\$245.50 list price
.12 rate of discount
\$29.46 discount

\$245.50
29.46
\$216.04 selling price

### WRITTEN EXERCISE

1. How much will a set of Scott's works, marked \$14, cost at 20% discount? (Also take 80% of \$14.)

2. If you can buy a \$7 suit of clothes at 15% discount, how much will you gain on the marked price?

3. If the list price of your books, pencils, and paper is \$2.50 a year, how much will they cost at 12% discount?

4. At a bargain sale an \$8 chair was sold at 30% discount. For how much did it sell?

*Find the discounts on goods of which the list price and rate of discount are given in Exs. 5–13:*

5. \$430, 15%. 6. \$281, 20%. 7. \$425.50, 10%.
8. \$1235, 17%. 9. \$2350, 14%. 10. \$43.26, $33\frac{1}{3}$%.
11. \$42.64, 25%. 12. \$6336, $6\frac{1}{4}$%. 13. \$234.54, $16\frac{2}{3}$%.

*Find the selling price, given the list price and rate of discount as follows:*

14. \$275, 20%. 15. \$575, 5%. 16. \$462, 3%.
17. \$650, $12\frac{1}{2}$%. 18. \$425, 12%. 19. \$320, 14%.
20. \$12.75, 20%. 21. \$6202, 14%. 22. \$52.84, $16\frac{2}{3}$%.
23. \$26.32, 50%. 24. \$4235, 22%. 25. \$14.43, $33\frac{1}{3}$%.

26. A \$17.50 set of china is marked down 15%. What is the selling price?

27. If you buy \$1.80 worth of fireworks at 15% discount, how much do they cost?

28. Your book dealer buys a 65-ct. arithmetic at 22% discount. How much does it cost him?

29. A \$31 overcoat is marked "25% off" at a bargain sale. What is the selling price?

30. If you buy a \$1.20 sled at 10% discount, how much does it cost? Suppose the discount is 20%? 25%?

31. How much will 2 dozen blackboard erasers cost at 45 ct. a dozen, discount 10%? discount 20%?

32. A man paid \$150 for a carriage, and after using it for a year sold it at a discount of $16\frac{2}{3}$%. What did he receive for it?

33. A man bought a \$900 automobile at 15% discount. After using it for a season he sold it for 15% less than it cost. What did he receive for it?

## REVIEW OF DENOMINATE NUMBERS

### ORAL EXERCISE

1. Repeat the table of length; of square measure; of cubic measure.

2. Repeat the table of weight.

3. Repeat the table of dry measure; of liquid measure.

Teachers should drill frequently upon the facts of these tables. The mere routine recitation of the tables is not so valuable as the frequent use of the facts in simple oral problems.

**223. Table of dozens.** In some kinds of business this table is common:

**12 units = 1 dozen.**
**12 dozen, or 144 = 1 gross (gr.).**
**12 gross, or 1728 = 1 great gross.**

The term *great gross* is going out of use.

**224.** Years ago the word *score* was often used for 20.

4. How many pens in 2 gross? in 6 gross? in 8 gross?

5. How many dozen in $\frac{2}{3}$ of a gross? in $3\frac{1}{2}$ gross?

6. What will a gross of padlocks cost at \$5 a dozen?

7. How many years old is a man who is 4 score? One who is 3 score? One who is 4 score and 10?

### WRITTEN EXERCISE

1. How many are 7 great gross? $9\frac{2}{3}$ gross?

2. What will 9 gr. of tablets cost at 25 ct. a dozen?

3. At \$7.20 a gross what will a dozen lamp chimneys cost?

4. A school bought a gross of blackboard erasers, but $8\frac{1}{3}\%$ were damaged. How many were all right?

**225. Counting sheets of paper.** Formerly, in counting sheets of paper,

**24 sheets = 1 quire.**

**20 quires = 480 sheets = 1 ream.**

Now, for convenience in counting, 500 sheets are more often called a *ream*, and the word *quire* is used only for folded note paper, other paper being usually sold by the pound.

### ORAL EXERCISE

1. How many sheets of fine writing paper in 2 quires? in 5 quires? in $3\frac{1}{2}$ quires? in 10 quires? in $12\frac{1}{2}\%$ of a quire?

2. If this paper costs 15 ct. a quire, how much will 4 quires cost? How many sheets will there be?

Notice that 4 times 24 is the same as 4 times (25 − 1), or 100 − 4.

3. At 20 ct. a quire, how much will 7 quires of paper, with \$1 worth of envelopes, cost?

4. If a dealer buys paper at 19 ct. a quire, and sells it at 25 ct. a quire, how much will he make on 10 quires?

### WRITTEN EXERCISE

1. How many sheets in 17 quires? in $33\frac{1}{3}\%$ of a quire?

2. In 7 dozen reams of 500 sheets each, how many more sheets than in a dozen reams of 480 sheets each?

3. At $16\frac{2}{3}$ ct. a quire, how much will $7\frac{1}{2}$ quires of paper cost? (Remember aliquot parts.)

4. At $12\frac{1}{2}$ ct. a quire, how much will $9\frac{1}{2}$ quires of paper cost? $6\frac{1}{2}$ quires? a dozen quires?

5. At 24 ct. a quire for paper, and 1 ct. each for envelopes, how much will it cost for paper, envelopes, and postage for eight letters from where you live to some other place in this country? Allow 1 sheet to a letter.

## MEASUREMENTS AND COMPARISONS

### ORAL EXERCISE

1. Draw on the blackboard a picture of a parallelogram; of a rectangle; of a square.

Pupils are, of course, already familiar with certain of these figures. The formal definitions are not of so much importance as the ability to recognize the figures by name.

**226. Quadrilateral.** A figure bounded by four straight lines is called a *quadrilateral.*

**227. Trapezoid.** A quadrilateral which has one pair of opposite sides parallel is called a *trapezoid.*

**228. Parallelogram.** A quadrilateral which has two pairs of opposite sides parallel is called a *parallelogram.*

**229. Rectangle.** A parallelogram whose angles are right angles is called a *rectangle.*

**230. Square.** A rectangle whose sides are all equal is called a *square.*

**231. Polygon.** A figure bounded by straight lines is called a *polygon.*

The figures pictured above are all special kinds of polygons.

2. What is the perimeter of a square 12 ft. 6 in. on a side?

3. What is the perimeter of a parallelogram two of whose sides are 10 ft. 8 in. and 9 ft. 4 in.?

4. What is the perimeter of a quadrilateral whose sides are 16 ft., 14 ft., 13 ft., 12 ft.?

**232. Equilateral triangle.** A triangle whose three sides are equal is called an *equilateral* triangle.

Triangles classified with respect to sides

**233. Isosceles triangle.** A triangle two of whose sides are equal is called an *isosceles* triangle.

**234. Scalene triangle.** A triangle whose three sides are unequal is called a *scalene* triangle.

Triangles classified with respect to angles

**235. Right-angled triangle.** A triangle having one right angle is called a *right-angled* triangle.

**236. Acute-angled triangle.** A triangle having three acute angles is called an *acute-angled* triangle.

**237. Obtuse-angled triangle.** A triangle having one obtuse angle is called an *obtuse-angled* triangle.

### WRITTEN EXERCISE

1. What is the perimeter of an equilateral triangle $275\frac{2}{3}$ ft. on a side? (Draw the pictures in all such cases.)

2. A triangle has a perimeter of 342 ft. 8 in. One side is 90 ft. and another is 50% more; how long is the third?

3. One side of an isosceles triangle is 35 ft., and one of the equal sides is 35% longer; what is the perimeter?

4. Cut off the three angles of a triangle and place them together. The sum equals how many right angles?

### ORAL EXERCISE

1. How long is the side of an equilateral triangle whose perimeter is 45 in.? 9 ft. 6 in.? $9\frac{1}{2}$ yd.? 63 ft. 9 in.?

2. How long is the side of a square whose perimeter is 84 in.? 8 ft. 4 in.? 16 yd. 8 in.? $60\frac{1}{2}$ rd.? 120 in.?

3. How long is the fourth side of a quadrilateral whose perimeter is 85 in., and three of whose sides are 10 in., 20 in., 30 in.?

4. The straight-line distance from Chicago to St. Louis is 270 mi.; from St. Louis to Cincinnati, 310 mi.; from Cincinnati to Chicago, 250 mi. What is the perimeter of this triangle?

### WRITTEN EXERCISE

*How long are the sides of the equilateral triangles whose perimeters are as follows?*

1. 113 ft. 3 in. 2. 127 yd. 3. 88 ft. 3 in.

*How long are the sides of the squares whose perimeters are as follows?*

4. 110 ft. 4 in. 5. 61 yd. 2 ft. 6. 118 ft. 8 in.

7. 42 ft. $3\frac{1}{2}$ in. 8. 3 yd. 2 ft. 4 in. 9. 227 mi. 40 rd.

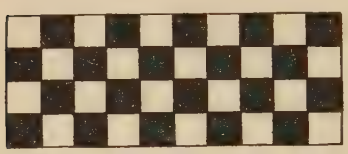
10. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. Find the length of one of the equal sides.

11. The perimeter of an isosceles triangle is 88 ft., and the unequal side is 20% of the perimeter. Required the length of each side.

12. The perimeter of a scalene triangle is 24 ft. One side is $33\frac{1}{3}\%$ of the perimeter; another is $37\frac{1}{2}\%$ of the perimeter. How many feet in each of the three sides?

### ORAL EXERCISE

1. How many square inches in a rectangle 1 in. high and 10 in. long? 4 in. high and 10 in. long?



2. How do we find the area of a rectangle? Give reasons.

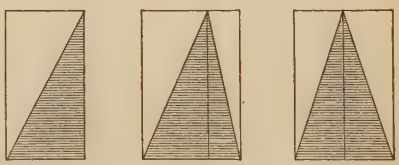
*State the areas of the following rectangles:*

3. 5 ft. by 9 ft. 4. 6 ft. by $4\frac{1}{2}$ ft. 5. 1 ft. by 8 in.

6. 8 ft. by $7\frac{1}{4}$ ft. 7. 9 in. by $8\frac{1}{3}$ in. 8. 9 in. by $7\frac{1}{3}$ in.

9. 3 yd. by $1\frac{1}{2}$ ft. 10. 1 rd. by 10 ft. 11. 10 yd. by 1 rd.

12. How do the areas of these triangles compare with the areas of the rectangles? Then a triangle is what part of a rectangle of the same base and same height?




**238. Base.** The line on which a figure appears to stand is called the *base*.

**239. Height or altitude.** The perpendicular distance from the highest point above the base to the base is called the *height* or *altitude* of a triangle or parallelogram.

**240. Area of a rectangle.** If a rectangle is 4 in. by 10 in., we find the area by taking 4 times 10 sq. in. We do not multiply 10 in. by 4 in., but 1 sq. in. by $4 \times 10$.

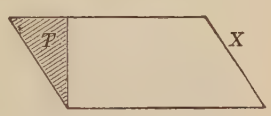
**241.** *The area of a rectangle equals the product of the base and height.*

**242. Area of a triangle.** From the above figures,

*The area of a triangle equals half the product of the base and height.*

### ORAL EXERCISE

1. If we cut triangle T from this parallelogram and place it where X is, what kind of a figure have we? Try it. What does this tell us about the area of a parallelogram?




*State the areas of the parallelograms whose bases and heights are given in Exs. 2–7:*

2. 11 ft., 9 ft.
3. 6 ft., $9\frac{1}{2}$ ft.
4. 8 ft., $10\frac{1}{4}$ ft.
5. 2 yd., $1\frac{1}{2}$ ft.
6. 4 in., $6\frac{3}{4}$ in.
7. 10 ft., $5\frac{1}{2}$ yd.

*State the areas of the triangles in Exs. 8–13:*

8. 7 ft., 6 ft.
9. 8 ft., $4\frac{1}{2}$ ft.
10. 3 ft., 6 in.
11. $3\frac{1}{2}$ ft., 4 ft.
12. $2\frac{3}{4}$ in., 8 in.
13. 10 ft., $8\frac{1}{5}$ ft.

**243. Area of a parallelogram.** *The area of a parallelogram equals the product of the base and height.*

### WRITTEN EXERCISE

*Find the areas of the parallelograms in Exs. 1–6:*

1. $19\frac{3}{4}$ in. by 16.8 in.
2. 14.7 ft. by 16.5 ft.
3. 171.4 ft. by 27.2 ft.
4. 3.75 in. by $2.33\frac{1}{3}$ in.
5. 6 ft. 4 in. by 4 ft. 6 in.
6. 16 ft. 6 in. by 8.25 ft.

**244.** What is the area of a triangle whose base is 10 rd. and height 8 rd.?

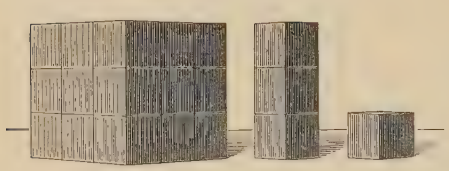
Because it is half of a rectangle 10 rd. by 8 rd., the area is $\frac{1}{2}$ of 10 times 8 sq. rd. = 40 sq. rd.

*Find the areas of the triangles in Exs. 7–10:*

7. 31.2 in. by 27.3 in.
8. 16.75 in. by 14.2 in.
9. 32 ft. 6 in. by 12 ft.
10. 17.3 ft. by 1 rd.

### ORAL EXERCISE

1. If C = 1 cu. in., what is the volume of B? of A? Explain how you would find the volume of a box 4 in. by 3 in. by 2 in.




*State the volumes of the solids whose dimensions are given in Exs. 2–7:*

2. 4 in., 5 in., 6 in.
3. 2 in., 3 in., 10 in.
4. 3 in., 3 in., 7 in.
5. 6 in., 8 in., 10 in.
6. 2 in., 3 in., 12 in.
7. 9 in., 9 in., 10 in.

**245. Volume of solid.** *The volume of a rectangular solid equals the product of its three dimensions.*

This means a solid whose faces are rectangles. We do not say 4 in. times 3 in. times 2 in., but 4 times 3 times 2 cu. in. It is customary, however, to write 4 in. × 3 in. × 2 in., or 4″ × 3″ × 2″, reading it "4 inches by 3 inches by 2 inches."

### WRITTEN EXERCISE

1. How many cubic inches in an aquarium 16 in. long, $8\frac{1}{2}$ in. wide, and $9\frac{1}{2}$ in. deep?

2. How many cubic feet of masonry in a wall 40 ft. long, $30\frac{1}{2}$ ft. high, and $1\frac{1}{2}$ ft. thick?

3. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40 ct. a load (cubic yard)?

4. The dimensions of a room are 14 ft., 12 ft., 9 ft. How many square feet of floor? of walls? of ceiling? How many cubic feet of volume?

### WRITTEN EXERCISE

**246. Measures occasionally used.** In the following problems you may refer to these measurements:

31.5 gal. = 1 barrel. $7\frac{1}{2}$ gal. = 1 cu. ft., nearly.
231 cu. in. = 1 gallon. $4\frac{1}{7}$ cu. ft. = 1 barrel, nearly.
$24\frac{3}{4}$ cu. ft. = 1 perch of stonework.
2150.42 cu. in. = nearly $1\frac{1}{4}$ cu. ft. = 1 bushel.
35 cu. ft. = the volume of 1 ton of coal, nearly.
500 cu. ft. = the volume of 1 ton of hay, nearly.
22 bricks, including the mortar = 1 cu. ft., nearly.

All measures refer to inside dimensions, in examples like the following. 1 ton of hay varies from about 450 cu. ft. to 550 cu. ft., depending on the quality and how much it has settled.

1. How many bushels in a bin 7 ft. by 6 ft. by 4 ft.? (Use $1\frac{1}{4}$ cu. ft. = 1 bu. in all such cases.)

2. How many bushels in a bin 9 ft. by 5 ft. by 3 ft.?

3. How many bushels in a box 18 in. wide, 3 ft. long, and 2 ft. deep?

4. How many gallons in a tank 6 ft. by 5 ft. by 4 ft.?

5. How many gallons in an aquarium 14 in. long, 9 in. wide, and 11 in. deep?

6. How many gallons in a tank $6\frac{1}{4}$ ft. long, $3\frac{1}{2}$ ft. wide, and $2\frac{1}{3}$ ft. deep?

7. A cistern is 6 ft. square at the bottom and 5 ft. deep. How many barrels will it hold? (Use $4\frac{1}{7}$ cu. ft. = 1 bbl. in all such cases.)

8. An iron tank is 8 ft. long, $6\frac{1}{2}$ ft. wide, and $4\frac{3}{4}$ ft. deep. How many barrels does it contain?

9. A rectangular watering trough is 6 ft. long, 18 in. wide, and 14 in. deep. How many barrels does it contain?

10. A coal bin is 22 ft. long, 13 ft. wide, and 8 ft. deep. About how many tons of coal will it hold?

11. A coal bin is 16 ft. long, $8\frac{1}{2}$ ft. wide, and $6\frac{1}{4}$ ft. deep. About how many tons of coal will it hold?

12. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. About how many tons of hay will it hold?

13. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. About how many tons of hay will both hold?

14. How many cords of wood in a pile 4 ft. wide, 7 ft. high, $32\frac{1}{2}$ ft. long?

15. How many cords of wood in a pile 4 ft. wide, $6\frac{1}{2}$ ft. high, 44 ft. long?

16. If a schoolroom is 30 ft. long, 15 ft. wide, and 12 ft. high, and 180 cu. ft. should be allowed to each person, how many will the room accommodate?

17. A tank holds 82,500 gal. About how many cubic feet does it contain?

18. An aquarium contains $3\frac{4}{5}$ cu. ft. About how many gallons does it hold?

19. An aquarium is 18 in. long, $9\frac{1}{4}$ in. wide, and $10\frac{1}{8}$ in. deep. How many gallons does it hold?

20. A can containing 1732.5 cu. in. is full of milk. What is the milk worth at 6 ct. a quart?

21. A wall is $32\frac{1}{2}$ ft. long, 2 ft. thick, and $32\frac{1}{2}$ ft. high. How many perches of masonry does it contain?

22. A brick wall is to be built 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high. About how many bricks will be needed?

23. How much will a mason charge for laying a wall 20 ft. long, $1\frac{1}{2}$ ft. thick, and $5\frac{1}{2}$ ft. high, at $\$1.87\frac{1}{2}$ a perch? at $\$1.75$ a perch?

# CHAPTER II

## I. PERCENTAGE AND ITS APPLICATIONS. SIMPLE INTEREST. MEASUREMENTS

### THE FORMAL SOLUTION OF PROBLEMS

**247. Solving problems.** We have already (page 115) learned something about stating the solutions of problems. We are now ready to consider this part of arithmetic more fully. There are three matters which are especially important.

1. *Work rapidly and accurately.* The business man demands this first of all. Rapidity not only means money in the time saved, but it usually means accuracy, for when a man works rapidly he keeps his mind on his work. To insure accuracy, *always check your work.* One of the best checks is that of estimating the answer in advance. If the answer is not about what was expected, there is probably a mistake. This has been explained on page 88.

2. *Work as the business man does.* He does not put his work in steps, with signs like ÷ and +. He works in columns because it is quicker and he can more easily check his work. He also makes his work look neat, because he finds slovenly work leads to inaccuracy.

3. *Analyze the problems.* This means that we are to state the reasons. The business man does not write out his analyses, but he thinks them. We are asked to give analyses both orally and in writing, so we will learn to think accurately as the business man does.

**248. Oral analysis.** Many problems are so easy as to be solved orally, without using pencil and paper. We usually see merchants solving problems this way, and we should do so whenever possible.

**249. Illustrative problems.** 1. If a dealer pays \$10 for 8 books of the same kind, what is the price of each?

No one would think of solving such a problem on paper. If asked to analyze (explain) it, we would say: "Since 8 books cost \$10, 1 book costs $\frac{1}{8}$ of \$10, or \$1.25."

In analyzing, speak briefly and accurately. Use your own language, not feeling that you must use the exact language of the book or the teacher.

2. If a stationer buys 100 sheets of blotting paper for \$3.50, and retails it for 5 ct. a sheet, what does he gain?

Analysis. Since he sells 1 sheet for 5 ct., he sells 100 sheets for 100 times 5 ct., or \$5. (Don't stop to say "500 ct."; reduce this to \$5 mentally.) He therefore gains the difference between \$3.50 and \$5, or \$1 50.

3. A dealer pays \$24 for 120 doz. pencils. What does each one cost?

Analysis. Since 120 doz. cost \$24, 1 doz. costs $\frac{1}{120}$ of \$24, or 20 ct. Therefore 1 pencil costs $\frac{1}{12}$ of 20 ct., or $1\frac{2}{3}$ ct.

4. If a dealer pays \$3.60 for a dozen knives, and sells them at 50 ct. each, how much does he gain? This is what part of the cost?

Analysis. Since he sells 12 at 50 ct. each, he receives 12 times 50 ct., or \$6. He therefore gains \$6 − \$3.60 = \$2.40. \$2.40 is $\frac{2.40}{3.60} = \frac{24}{36} = \frac{2}{3}$ of the cost.

If the teacher begins a year's work with Chapter II, it will be found that these oral problems form an excellent review of the old work and an introduction to the new.

### ORAL EXERCISE

*When the teacher has given the number of the problem, give (1) the result and (2) the analysis.*

1. At $12\frac{1}{2}$ ct. a pound, what will 8 lb. of cheese cost?

2. If 4 doz. oranges cost $1.80, what does 1 doz. cost?

3. At $492 a year, how much is the rent of a house for a month?

4. At $6 a ton (of 2000 lb.), how much will 3000 lb. of coal cost?

5. At $2 a ton, how many tons of soft coal can be bought for $96?

6. At 20 ct. a dozen, how many dozen eggs can be bought for $1.20?

7. At $5 a barrel, how many barrels of flour can be bought for $625?

8. At $6 a hundred, how many pounds of meat can be bought for $636?

9. At $20,000 a mile, how many miles of railroad can be built for $150,000?

10. At 15 ct. a dozen, how many dozen paper pads can be bought for $1.65? How many pads?

11. If 1 load of mortar contains 30 hods, how many loads are equal to 510 hods?

12. A jacket which cost a merchant $5.67 was sold for $7.25. How much did the merchant gain?

13. A dealer bought 12 doz. pairs of shoes for $360, and sold them at $3 a pair. How much did he gain?

14. If a bricklayer's hod contains 20 bricks, how many loads must he carry for a drain requiring 2920 bricks?

### ORAL EXERCISE

1. At 40 ct. a yard, what do 9 yd. of silk cost?

2. At $1.20 a yard, what do $5\frac{1}{2}$ yd. of silk cost?

3. At 90 ct. a yard, what do $2\frac{1}{2}$ yd. of cloth cost?

4. At 25 ct. a yard, what do $8\frac{1}{2}$ yd. of gingham cost?

5. At 35 ct. a yard, what do $10\frac{1}{2}$ yd. of madras cost?

6. At 96 ct. a yard, what do $10\frac{1}{4}$ yd. of pongee cost?

7. At $12\frac{1}{2}$ ct. a yard, what do 8 yd. of dimity cost? also 9 yd.? 10 yd.? 16 yd.? 24 yd.?

8. At 45 ct. a yard, what do 20 yd. of linen suiting cost? also $10\frac{1}{2}$ yd.? 5 yd.? 8 yd.? $2\frac{1}{3}$ yd.?

9. A lady buys a walking skirt for $7.75, and 4 collars at $12\frac{1}{2}$ ct. each. What is her bill?

10. A man buys a pair of shoes for $4.50, and a pair of slippers for $2.75. What is his bill?

11. A lady buys a pair of gloves for $1.50, and 3 handkerchiefs at 25 ct. each. What is her bill?

12. A lady bought 4 yd. of ribbon at 19 ct. a yard, a 25-ct. handkerchief, and a shirt waist for $1.50. What was her bill?

13. A lady bought a hat for $8.50, and 2 yd. of ribbon at 35 ct. a yard. How much change should she receive for a $10 bill?

14. On the way home she ordered 4 lb. of meat at 23 ct. a pound, and paid for it with a $1 bill. How much change did she receive?

15. She stopped at the grocer's and ordered 3 heads of lettuce at 9 ct. each, 2 bunches of asparagus at 6 ct. each, and $1.75 worth of preserves. What was her bill?

### ORAL EXERCISE

1. What will $2\frac{1}{2}$ gal. of milk cost at 7 ct. a quart?

2. How many yards in a coil of rope 2700 ft. long?

3. When lobsters sell at 20 ct. a pound, what will $1\frac{3}{4}$ lb. cost?

4. At 20 T. to the load, how many tons of coal in $5\frac{1}{2}$ car loads?

5. If a boy solves 3 out of 5 problems, what per cent does he solve?

6. If $2\frac{1}{2}$ lb. of porterhouse steak cost 55 ct., what is the price per pound?

7. If a man buys a horse for \$72 and pays $\frac{5}{6}$ down, how much does he still owe?

8. If it takes $3\frac{1}{2}$ yd. of gingham for a girl's dress, how much does the cloth cost at 30 ct. a yard?

9. At \$1.25 a yard, how much will three strips of carpet cost, the lengths being 13 yd., $4\frac{1}{2}$ yd., 7 ft. 6 in.?

10. A man buys 7 lb. of rice at 12 ct. a pound, and hands the grocer a dollar bill. How much change is due him?

11. I have 6 pictures to be hung, each requiring 7 ft. of picture wire. Find the cost of the wire at 1 ct. a yard.

12. What is the cost of a bunch of carrots at 13 ct., a box of strawberries at 18 ct., a quart of new potatoes at 12 ct., and a bunch of water cress at 5 ct.?

13. Sugar is 8 ct. a pound, and our recipe for candy calls for $1\frac{1}{4}$ lb. For our class we shall need 6 times the amount stated in the recipe. What will the sugar cost?

14. How much will a 9-word telegram from London to Chicago cost, at 24 ct. a word to New York, and 40 ct. for 10 words or less from New York to Chicago?

**250. Written analysis.** In the written work we wish to find out two things: (1) if we can solve problems quickly and accurately as they do in business; (2) if we clearly understand the solution. For this reason give both the analysis and the computation as in the following example, unless the teacher directs omitting one. In the simpler examples there is only one step in the analysis.

**251. Illustrative problem.** A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water in a cubical tank 6 in. on each edge?

*Analysis:*

1. 6 in. $= \frac{1}{2}$ ft.
2. $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of 1 cu. ft. $= \frac{1}{8}$ cu. ft. (Explain orally.)
3. $\frac{1}{8}$ of $62\frac{1}{2}$ lb. $= 7\frac{3}{16}$ lb. $= 7$ lb. 13 oz.

*Work:*

$8\overline{)62\frac{1}{2}}$

$7\frac{6\frac{1}{2}}{8}$

$7\frac{3}{16}$

WRITTEN EXERCISE

1. At $37\frac{1}{2}$ ct. a yard, how much will $15\frac{1}{2}$ yd. of cloth cost?
2. If 175% of a number is $194\frac{1}{4}$, what is the number?
3. At 680 bu. to a car, how many cars will it take to carry 11,560 bu.?
4. At 40,000 shingles to a car, how many cars will it take to carry 760,000 shingles?
5. The President of the United States receives $50,000 a year. How much does he receive a month?
6. A bushel of wheat weighs 60 lb., how many bushels of wheat in a car load weighing 21,240 lb.?
7. A bushel of oats weighs 32 lb., how many bushels of oats in a car load weighing 20,000 lb.?
8. How many chests of tea, weighing 25 lb. each, and costing 48 ct. a pound, can be bought for $1500?

9. If 32 bricks will pave 1 sq. yd., how many will be needed to pave a space 63 ft. by 36 ft.?

10. If a homing pigeon makes a flight of 1270 mi. in 13 da. $5\frac{1}{2}$ hr., what is the average speed per hour?

11. How many railway ties placed 2 ft. 6 in. from center to center would be required for a double line of road 13 mi. long?

12. One of the yachts in the international races made 30 mi. in 3 hr. 12 min. 30 sec. What was the average time per mile?

13. A cubic foot of pure cast gold weighs 1204 lb., and a cubic foot of granite weighs 172 lb. Gold is how many times as heavy as granite?

14. How many tons' pressure will a hurricane exert on the front of a city building 150 ft. high and 100 ft. wide, allowing 45 lb. pressure to the square foot?

15. How long would it take a line of soldiers, the line being $\frac{1}{2}$ mi. long, to pass a fixed point, if they took 75 steps a minute, and the steps averaged 2 ft. 8 in. in length?

16. In a certain arid region in the West they put in 250 mi. of main irrigating ditches at an average cost of $10,000 a mile, and 400 mi. of lateral ditches at $1250 a mile. Required the total cost.

17. By this irrigation (Ex. 16) they made 125,000 acres of desert land productive and worth $125 an acre. What was the value of the redeemed land? How much was the value above the cost of irrigating?

18. A fair breeze blows 880 ft. a minute, and a hurricane 7040 ft. a minute. In each case how many feet a second? In each case how many miles an hour? Answer to two decimal places only.

**252. Illustrative problems.** 1. If a grocer bought 200 bbl. of flour at \$3.67 a barrel, and sold it for \$4.13 a barrel, how much did he gain?

| | |
|---|---|
| | \$4.13 |
| | 3.67 |
| | .46 |
| | 200 |
| | \$92.00 |

1. \$4.13 − \$3.67 = \$0.46, the gain on 1 bbl.
2. 200 times \$0.46 = \$92, " " " 200 bbl.

2. A merchant bought 500 yd. of India silk at 49 ct. a yard. He sold 350 yd. at 60 ct. a yard, and the rest at a bargain sale at 48 ct. a yard. How much did he gain?

1. 500 times \$0.49 = \$245, the cost.
2. 350 " \$0.60 = \$210, received from 350 yd.
3. 150 " \$0.48 = \$72, " " 150 "
4. \$210 + \$72 = \$282, total receipts.
5. \$282 − \$245 = \$37, the gain.

19. If a man earns \$30 a week, and spends \$21.50, how much can he save at this rate in 48 weeks?

20. If a dozen pairs of scissors cost \$1.84, and were sold at 28 ct. a pair, how much did the seller gain?

21. If 1 gal. = 231 cu. in., what is the value of the oil in a can containing 4158 cu. in., at 7 ct. a gallon?

22. The cost of a piece of cloth is \$63, and the cost per yard is \$1.12½. How many yards are there in the piece?

23. If 35 bales of cloth contain 182,700 yd., and each piece contains 58 yd., how many pieces are there in each bale?

24. If 42 bales of cloth contain 41,412 yd., and each bale contains 34 pieces, what is the average number of yards in a piece?

25. A grocer bought 10 bu. of new potatoes at \$1.12½ a bushel, and sold them at 5 ct. a quart. How much did he gain on the lot?

26. A grain elevator contained 16,250 bu. of wheat, worth 82 ct. a bushel. If $\frac{1}{5}$ of it is destroyed by fire, what is the loss?

27. A man paid $300 for a horse and carriage, the carriage costing $50 more than the horse. How much did he pay for each?

28. A merchant bought 1200 pieces of cloth, 48 yd. to the piece, for $23,040, and sold it for 50 ct. a yard. How much did he gain?

29. A dealer buys 3000 men's collars at $1.92 a dozen, and sells them at 25 ct. each. How much does he gain on the total number?

30. A meat dealer bought 3650 lb. of beef at the average price of $12\frac{1}{2}$ ct. a pound, and sold it at the average price of 17 ct. How much did he gain by the transaction?

31. A dealer buys 50 arithmetics at 39 ct. each, and 40 readers at $37\frac{1}{2}$ ct. each. At what uniform price must he sell them in order to make $8.70 on the whole transaction?

32. A man sold a load of hay for $13.32. The wagon and hay together weighed 4160 lb., and after unloading, the wagon weighed 1200 lb. At what price per ton was the hay sold?

33. A fruit buyer receives $210 for 6 weeks' work, and spends $14.50 a week for board, $13.74 a week in traveling, and $3 a week for other purposes. What is his average gain per week?

34. A merchant bought 2000 yd. of pongee silk at $87\frac{1}{2}$ ct. a yard. He marked it $1.10 a yard and sold 950 yd. at that price. He then sold 750 yd. at 10% off, and the rest at 50% off. Did he gain or lose on the lot, and how much?

**253. Unitary analysis.** Some examples are easily solved by a particular kind of analysis called *unitary analysis.*

**254. Illustrative problems.** 1. If a man invested some money and gained 10%, and then had $275, how much did he invest?

1. The sum invested + 0.10 of itself = 1.10 times itself.
2. Since 110% of the sum invested = $275,
   1% of the sum invested = $275 ÷ 110 = $2.50.
3. Therefore 100% of the sum invested = 100 times $2.50 = $250.

Notice that the problem was solved by finding the *unit* per cent, 1% of the sum. Hence the name, *unitary analysis.*

2. If 5 tons of coal cost $27.50, what will 7 tons cost?

1. Since 5 T. cost $27.50, 1 T. costs $\frac{1}{5}$ of $27.50 = $5.50.
2. Therefore 7 T. cost 7 times $5.50 = $38.50.

Notice again the *unit,* 1 ton.

### WRITTEN EXERCISE

1. If 3000 ft. of lumber cost $96.60, what will 16,100 ft. cost at the same rate?

2. If a man gained 16% on an investment and then had $3770, how much did he invest?

3. If a man invested some money and lost 11%, and then had $1869, how much did he invest?

4. At $460 per 100 bbl., how many barrels of flour can be bought for $73.60? for $782? for $8556?

5. At $5.60 a ton of 2000 lb., what will 2775 lb. of coal cost? 1575 lb.? 1850 lb.? 3225 lb.? 6450 lb.?

6. At $12.50 per hundred pounds, what will $4\frac{1}{2}$ lb. of cheese cost? $7\frac{1}{4}$ lb.? 9 lb. 4 oz.? 12 lb. 12 oz.?

7. At $5.20 a hundred pounds, how many pounds of meat can be bought for $639.60? for $587.60? for $4004?

**255.** Occasionally examples like this are met:

If 4 loads of hay will last 6 horses 8 weeks, 8 loads will last how many horses 3 weeks?

1. If 4 loads will keep 6 horses 8 wk.,
2. 4 loads " " $8 \times 6$ horses 1 wk. Do you see why?
3. 1 load " " $\frac{1}{4}$ of $8 \times 6$ horses 1 wk.
4. 1 load " " $\frac{1}{3}$ of $\frac{1}{4}$ of $8 \times 6$ horses 3 wk.
5. 8 loads " " $8 \times \frac{1}{3}$ of $\frac{1}{4}$ of $8 \times 6$ horses 3 wk.
6. $8 \times \frac{1}{3}$ of $\frac{1}{4}$ of $8 \times 6$ horses $= 32$ horses.

Notice the two *units* in step 3, 1 load and 1 wk.

Such problems were formerly solved by the unexplained rule of compound proportion. In actual business they are but little used.

### WRITTEN EXERCISE

1. If it takes 4 boys 16 da. to build a playhouse, how long will it take 1 boy working at the same rate?

2. In Ex. 1 how long will it take 8 boys working at the same rate? also 2 boys? also 6 boys? also 10 boys?

3. If 3 reapers can cut 90 acres in 2 da., how long will it take 5 reapers to cut 3000 acres?

4. How long will it take 25 men to do a piece of work which 12 men can do in 15 days, working at the same rate?

5. If 42 bu. of oats last 16 horses a week, how many bushels will be needed for 24 horses for 3 wk., at the same rate? for 48 horses for $1\frac{1}{2}$ wk.?

6. If a man working 10 hr. a day can complete a piece of work in 12 da., in how many days can he complete it working 8 hr. a day?

7. If 702 bricks are required for a walk 39 ft. long and 4 ft. wide, how many bricks will be required for a walk half as wide again and 3 times as long?

8. If it costs \$702 to excavate a cellar $27' \times 39' \times 9'$, what will it cost to excavate a cellar $30' \times 36' \times 12'$ at the same rate?

9. A California hop picker picks 3 bags of hops a day, averaging 60 lb. to a bag. At 2700 lb. to the acre, how long will it take him to pick an acre of hops? At this rate, how many men are needed to pick an acre in 5 da.? in 3 da.? in $7\frac{1}{2}$ da.?

10. At the rate given in Ex. 9, how many men will it take to pick 75 acres in 15 days?

11. At the rate given in Ex. 9, how many days will it take 90 men to pick 60 acres?

12. At the rate given in Ex. 9, how many acres can 75 men pick in 6 days?

13. These hops are baled in bales 20 in. wide, 20 in. high, and 5 ft. long. If such a bale weighs 200 lb., how much does 1 cu. ft. of baled hops weigh?

14. A well at Madison, Wisconsin, furnishes enough water to irrigate 110 acres of land 2 in. deep every 10 days. At this rate, how many acres can it cover to a depth of 1 in. every day?

15. Another well at Chamberlain, South Dakota, furnishes enough water to irrigate 657 acres to a depth of 4 in. every 10 days. At this rate, how many acres can it cover to a depth of 2 in. every 3 days?

16. In Wisconsin experiments it was found that an 8-horse-power farm engine could draw from a lake to a height of 26 ft. sufficient water to cover $22\frac{1}{8}$ acres a day to a depth of 1 in. At these rates, how many acres could be covered half as deep by a 16-horse-power engine raising the water only 13 ft.?

### ORAL EXERCISE

1. If 3 collars cost 45 ct., what will 2 cost?

2. If 2 yd. of cloth cost \$3, what will 3 yd. cost?

3. If 4 spools of silk cost 24 ct., what will 5 cost?

4. If 2 tons of hay cost \$18, what will 11 tons cost?

5. At 40 ct. a quart, what will $\frac{7}{8}$ of a quart of cream cost? $1\frac{3}{4}$ qt.? $2\frac{1}{2}$ qt.? 3 gal.?

6. If $\frac{3}{4}$ of a barrel of beans cost \$6, what will a barrel cost? 2 bbl.? 5 bbl.? $6\frac{1}{2}$ bbl.? 10 bbl.?

7. At the rate of 72 mi. in 2 hr., how far will a train go in 3 hr.? in 5 hr.? in $1\frac{1}{2}$ hr.? in 4 hr. 30 min.?

### WRITTEN EXERCISE

1. At the rate of \$110 for 40 sheep, what will 24 cost?

2. After losing 31% of his capital a man has \$3726. What was his capital?

3. At the rate of \$216 for 300 bu. of wheat, how much will 700 bu. cost? 50 bu.? 275 bu.? 1000 bu.?

4. If 11 rd. of ditching cost \$16.50, how much will 20 rd. cost, at the same rate? 50 rd.? 75 rd.? $5\frac{1}{2}$ rd.?

5. At the rate of 980 lb. to 2 bales of cotton, how much will 3 bales weigh? 7 bales? 9 bales? 35 bales?

6. If $\frac{3}{4}$ of an acre of land costs \$90, how much will $\frac{7}{8}$ of an acre cost, at the same rate? $2\frac{1}{2}$ acres? 35 acres?

7. The Twentieth Century Express makes the 980 mi. from New York to Chicago in 20 hr. At this rate, how far does it travel in 6 hr.? in $7\frac{1}{2}$ hr.? in 9 hr. 45 min.?

8. Every 10 people in the United States consume on an average 650 lb. of sugar annually. At this rate, how much does a family of 3 consume? a family of 7? of 9?

**256. Illustrative problems.** 1. A man paid \$78.75 for the building of a stone wall $52\frac{1}{2}$ rd. long. How much would he have paid, at this rate, if it had been 75 rd. long?

$$\frac{75 \times 78.75}{52.5} = \frac{75 \times 78.75}{52.5} = 112.50$$

1. \$78.75 ÷ $52\frac{1}{2}$ = \$1.50, cost of 1 rd.
2. 75 times \$1.50 = \$112.50, cost of 75 rd.

2. Two men pasture cattle in the same field, one pasturing 10 head for 3 wk., and the other 8 head for 4 wk. They pay \$12.40 rent. What is the share of each?

Since 10 cattle in 3 wk. eat as much as 30 cattle in 1 wk., and 8 cattle in 4 wk. eat as much as 32 in 1 wk., they all eat as much as 62 in 1 wk. Hence the first man should pay $\frac{30}{62}$, and the second $\frac{32}{62}$ of the rent. Therefore we have:

1. 3 times 10 cattle = 30 cattle.
2. 4 " 8 " = 32 "
3. Total " = 62 " for 1 wk.
4. $\frac{30}{62}$ of \$12.40 = \$ 6.
5. $\frac{32}{62}$ " " = \$ 6.40
6. The sum = \$12.40, checking the work.

### WRITTEN EXERCISE

1. If 8 tons of hay cost \$76, what will 5 tons cost?

2. If 35 A. of land cost \$3220, what will 125 A. cost?

3. If sound travels 3696 ft. in 3.3 sec., how far does it travel in 7 sec.? in $9\frac{1}{2}$ sec.? in 1 min. 45 sec.?

4. If 15 bales of cotton weigh 7305 lb., what will 4 bales weigh? 7 bales? 16 bales? 25 bales?

5. If a pole 6 ft. high casts a shadow 8.3 ft. long, how high is an adjacent tree that casts a shadow 74.7 ft. long at the same time?

6. If a stack of brick $2' \times 4' \times 7'$ contains 1000 bricks, how many bricks in a stack $6' \times 10' \times 14'$?

7. If a tree 25 ft. high casts a shadow 20 ft. long, how long will be the shadow of a tree 30 ft. high at the same time?

8. If a boy 5 ft. high casts a shadow 3.2 ft. long, what is the height of a school building that casts a shadow 35.2 ft. long at the same time?

9. If it takes a boat 20 hr. to go the length of the Albemarle and Chesapeake Canal, 44 mi., how long will it take it to go 9 mi. at the same rate?

10. Three men pasture cattle in the same field, the first pasturing 8 head for 6 wk., the second 9 head for 5 wk., and the third 12 head for 4 wk. They pay \$28.20 rent. What is the share of each?

11. If a roofer can lay 2 squares (a square meaning 100 sq. ft., lapped and laid) of slate in 10 hr., how many men will it take to cover, in 5 da. of $7\frac{1}{2}$ hr. each, an A-shaped roof 100 ft. long, each slope being 30 ft. wide?

12. If 3 squares (see Ex. 11) of roofing slate weigh a ton, what is the weight of the slate on a roof 72 ft. long, and 20 ft. wide on each slope?

13. The average width of shingles being 4 in., and the shingles being laid 4 in. to the weather, how many will be required to cover a square (100 sq. ft.) of roof? How many if they were laid $4\frac{1}{2}$ in. to the weather?

14. If the total product of butter in the United States is 1,430,000,000 lb. a year, and the number of cows used for this purpose is 11,000,000, what would be the total product if there were 4,000,000 more cows, the same rate being maintained?

**257. Stating a problem in one step.** It is often a good plan to indicate the solution of a problem all in a single step, as in the following example:

If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., and gold is 19.5 times as heavy as water, how much is 1 cu. ft. of gold worth at \$18.60 an ounce avoirdupois?

1 cu. ft. of gold weighs 19.5 times $62\frac{1}{2}$ lb., or 19.5 times $62\frac{1}{2}$ times 16 oz.

Therefore the value is

$$19\tfrac{1}{2} \times 62\tfrac{1}{2} \times 16 \text{ times } \$18.60,$$

or

$$\frac{39 \times 125 \times 16 \text{ times } \$18.60}{2 \times 2}.$$

We may now cancel and work this out more easily than by multiplying step by step.

WRITTEN EXERCISE

1. Finish the solution of the above problem.

2. When sound travels 1122 ft. per second, how many miles away is a lightning flash which is seen 13.2 sec. before the thunder is heard?

The velocity of sound varies slightly with the temperature.

3. Soldiers marching quickstep take 120 paces a minute, averaging 28 in. each. At this rate, how many miles can a company march in an hour?

4. If the circumference of a wheel is $3\frac{1}{7}$ times the diameter, and the drive wheel of a locomotive is 6 ft. in diameter, how many revolutions will it make in going from New Orleans to New York, 1344 mi.?

5. What is the expense of covering a floor with plain matting 1 yd. wide, the room being 18 by $21\frac{1}{2}$ ft., and the matting being worth 30 ct. a yard, allowing 6 in. extra on each strip for turning under? Draw a plan.

### ORAL EXERCISE

1. At the rate of \$2.10 for 3 bu. of wheat, what will 11 bu. cost? 16 bu.? 25 bu.? 40 bu.?

2. At the rate of \$25 a dozen, what will 8 silver tablespoons cost? What will 16 cost? 18? 20?

3. At the rate of 38 tons of hay to 19 acres, how many tons will 49 acres produce? 57 acres? 90 acres?

4. At the rate of 75 bu. of wheat to 3 acres, how many bushels will 50 acres produce? 12 acres? 40 acres? 275 acres? 450 acres?

### WRITTEN EXERCISE

1. If a piece of cast iron $10'' \times 8'' \times 2''$ weighs 41.6 lb., what is the weight of a piece $16'' \times 10'' \times 3''$?

2. If 475 horse power from some engines costs \$13,490 a year, what will 60 horse power cost at the same rate?

3. A dealer bought 25 bu. of grass seed at \$2.90 a bushel. He paid \$1.20 for paper bags, and sold the seed in bags at 10 ct. a quart. How much did he gain?

4. Two steamers started for Europe on the same route at the same time. After 4 days one had averaged 448.7 mi. a day, and the other 502.1 mi. How far apart were they?

5. A box of arithmetics cost a dealer \$53.94, including \$1.48 freight. He sold the arithmetics for 50 ct. each, thus gaining 7 ct. on the list price of each book. How many did he buy?

6. A man bought two adjacent farms containing the same number of acres. For one he paid \$95 per acre, and for the other \$130 per acre. The two farms cost him \$31,950. How many acres did he buy?

**258. The equation form.** There is a modern and a much clearer way of solving many kinds of examples, known as the *equation method.* This will now be studied.

### ORAL EXERCISE

1. What number added to 232 makes 250? 300?
2. What number subtracted from 250 leaves 170?
3. What number multiplied by 12 equals 132? 144?
4. What number divided by 12 equals 9? 90? 9000?
5. If 180 be subtracted from a certain number, the remainder is 25. Required the number.
6. If 320 be multiplied by a certain number, the product is 960. Required the number.
7. If 440 be divided by a certain number, the quotient is 40. Required the number.
8. The square of a certain number (the product of the number multiplied by itself) is 121. Required the number.
9. What is the number whose half is 55? Whose fourth is 70? Whose tenth is 75.2? Whose third is 92?
10. Tell the missing number ($x$) in each statement: $30 + x = 95$; $125 - x = 95$; $9 \times x = 720$; $550 \div x = 11$.

### WRITTEN EXERCISE

*In these statements,* x *stands for what missing numbers?*

1. $425 + x = 973$.
2. $x + 741.3 = 963.8$.
3. $173\frac{1}{2} + x = 342\frac{3}{4}$.
4. $687.25 + x = 963.5$.
5. $x + 57.4 = 99.23$.
6. $609.37 + x = 1243.48$.
7. $427 - x = 298$.
8. $521.2 - x = 137.5$.
9. $34 \times x = 799$.
10. $448 \div x = 28$.

**259. Unknown quantity.** In an expression like $5+x=11$, $x$ is called an *unknown quantity.*

**260. Equation.** An equality that is true for some value of an unknown quantity is called an *equation.*

For example, $2+3=5$ is an equality; $2+x=5$ is an equation. The two terms are often used as meaning the same.

**261. Indicated multiplication.** Because $x$ and $\times$ look so nearly alike when written, the $\times$ is usually omitted when $x$ is used, $5\times x$ being written $5x$, and read "five $x$."

**262. To solve an equation** means to find the value of the unknown quantity.

For example, to solve the equation $x+17=26$ is to find that $x=9$. To solve the equation $6x=48$ is to find that $x=8$.

**263. How to solve.** If we have an equation like $x+5=11$, we may think of the two sides as just balancing. Any change made on one side must be made on the other, to keep the balance. If we take 5 from the left side, we must also take 5 from the right side, leaving $x=6$.

**264.** *To solve an equation, do to one side what is necessary to leave* x *by itself, and then perform the same operation on the other side.*

For example, if $3x=15$, divide both sides by 3, so as to leave $x$ by itself, whence $x=5$.

Also, if $\frac{x}{3}=7$, multiply by 3, because $3\times\frac{x}{3}=x$, thus leaving $x$ by itself, whence $x=21$.

**265.** Teachers often find it valuable to represent the required numbers by $n$, the number of dollars by $d$, the number of dollars of cost by $c$, and so on with other initial letters. But $x$ and $y$ must finally be used as the universal conventions of mathematics.

**266. Illustrative problems.** 1. I have thought of a number; I add 7 to it; then I subtract 3; the result is now 28. What is the number?

1. Let $x$ = the number.

Then $x + 7$ is the number with 7 added, and $x + 4$ is the remainder after subtracting 3, and the problem tells us that this remainder equals 28.

2. Therefore $x + 4 = 28$.

3. Subtracting 4 from both sides, $x = 24$.

To *check* this, put 24 in the statement of the problem.

2. What number is that which, being doubled, and 7 being then added, equals 29?

1. Let $x$ = the number.

Then $2x$ = double the number, and $2x + 7$ = double the number with 7 added, and the problem tells us that this equals 29.

2. Therefore $2x + 7 = 29$.

3. Subtracting 7 from both sides, $2x = 22$.

4. Dividing both sides by 2, $x = 11$.

### WRITTEN EXERCISE

1. What number multiplied by $1\frac{1}{2}$ equals 96?

2. What number multiplied by 1.04 equals 7.28?

3. If 5 times a certain number, less 70, is 80, what is the number? (Solve also without using $x$.)

4. If 72 times a number, together with 72, is 792, what is the number? (Solve also without using $x$.)

5. If to twice a number I add 15, the result is 75. What is the number? (Solve also without using $x$.)

6. I have thought of a number; I add 7; I add this result to itself; I take away 12; the result is 42. Required the number.

**267. Illustrative problems.** 1. A horse and carriage together cost \$375, and the carriage cost twice what the horse did. What did the horse cost?

1. Let $x$ = the *number of dollars* the horse cost.
Then $2x$ = " " " " " carriage cost,
and $x + 2x$ = " " " " both cost, which is 375.
2. Therefore $3x = 375$, because $x$ (that is, $1x$) and $2x$ are $3x$.
3. " $x = 125$, by dividing both sides by 3.
Therefore the horse cost \$125.

2. After increasing 6%, a certain sum amounts to \$318. What is the sum?

1. Let $x$ = the *number of dollars* in the sum.
Then $0.06x$ = " " " " increase,
and $1.06x$ = " " " " after the increase.
2. Therefore $1.06x = 318$.
3. " $x = 318 \div 1.06 = 300$.
Therefore the sum is \$300, as is easily proved.

### WRITTEN EXERCISE

1. After gaining 12%, a certain sum amounts to \$392. What is the sum?

2. If to $\frac{1}{5}$ of a certain number I add 730, the result is 855. What is the number?

3. A boy lost $\frac{2}{7}$ of his marbles and had 18 left. How many had he at first?

4. A man sold $\frac{3}{7}$ of his cattle and had 68 left. How many had he at first?

5. A certain sum was increased 50% and then amounted to \$505.50. What was the sum?

6. If your father is 3 times as old as you, and if the sum of your ages is 52 yr., how old are you?

**268. Illustrative problem.** A man sold a carriage for \$120, which was 20% less than it cost. How much did it cost?

*Complete analysis:* *Actual work:*

1. Let $x$ = the *number of dollars* of cost. $.80\ x = 120$
Then $0.20\ x =$ " " " " lost, $8\overline{)1200}$
and $x - 0.20\ x =$ " " " " received. $150$

2. Therefore $0.80\ x = 120$, because $1\ x - 0.20\ x = 0.80\ x$.

3. Therefore $x = 120 \div 0.80 = 150$.

Therefore the cost was \$150, as is easily proved.

### WRITTEN EXERCISE

1. A man sold a house for \$2250, which was 20% less than it cost. How much did it cost?

2. With the \$2250 he bought 20 acres of land from Mr. A, who thereby gained 25% on what it cost him. How much did Mr. A pay per acre for the land?

3. A man sold a farm of 100 acres at the rate of \$56.25 an acre. He lost 10% on the cost of the farm. How much did the farm cost him?

4. A dealer bought two horses at the same price. He sold one, at a profit of 20%, for \$102. The other he sold at a loss of 10%. How much did he receive for the latter?

5. A certain village, after gaining 10% on its population in 1900, had 10% less population than another village of 4180 inhabitants. What was the population of the first village in 1900?

6. A furniture dealer bought 5 sets of furniture at \$65 a set, 3 at \$90, and 2 at \$40. The \$40 sets he sold at a profit of 30%, the \$65 sets at a profit of 20%, but on the \$90 sets he lost 10%. How much did he gain on the lot?

## PERCENTAGE

### ORAL EXERCISE

1. Express as per cents: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$.
2. Express as per cents: $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, 1.
3. Express as per cents: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$.
4. If $\frac{3}{8}$ of a number is 9, what is the number?
5. If $37\frac{1}{2}\%$ of a number is 6, what is the number?
6. If $62\frac{1}{2}\%$ of a number is 15, what is the number?
7. If $75\%$ of a number is 66, what is the number?
8. Express as common fractions: $20\%$, $33\frac{1}{3}\%$, $87\frac{1}{2}\%$.
9. Express as common fractions: $6\frac{1}{4}\%$, $8\frac{1}{3}\%$, $12\frac{1}{2}\%$.
10. How much is $25\%$ of \$8000? $33\frac{1}{3}\%$ of \$9000?
11. How much is $16\frac{2}{3}\%$ of \$120? $10\%$ of \$225.50?
12. If $87\frac{1}{2}\%$ of a number is 1400, what is the number?
13. 30 is $16\frac{2}{3}\%$ of what number? $33\frac{1}{3}\%$ of what number?
14. 40 is $66\frac{2}{3}\%$ of what number? $83\frac{1}{3}\%$ of what number?
15. Tell the short way of finding $12\frac{1}{2}\%$ of a number; of finding $66\frac{2}{3}\%$ of a number.
16. Tell the short way of finding $6\frac{1}{4}\%$ of a number; of finding $16\frac{2}{3}\%$ of a number.

### WRITTEN EXERCISE

1. $37\frac{1}{2}\%$ of \$164.
2. $6\frac{1}{4}\%$ of \$33.60.
3. $83\frac{1}{3}\%$ of \$1110.
4. $66\frac{2}{3}\%$ of \$72.30.
5. $8\frac{1}{3}\%$ of \$145.20.
6. $16\frac{2}{3}\%$ of \$417.30.
7. $125\%$ of \$4020.
8. $183\frac{1}{3}\%$ of \$2166.
9. $150\%$ of \$2178.
10. $160\%$ of \$475.50.
11. $175\%$ of \$5260.
12. $133\frac{1}{3}\%$ of \$286.41.

**269. Percentage.** That part of Arithmetic that treats of per cents is often called *percentage.*

**270. To find some per cent of a number.** The most important problem in percentage is the one already studied, of finding some per cent of a number.

**271. Illustrative problems.** 1. What is 4% of \$62.25?

Here we may multiply \$62.25 by .04, as in the first process below; or we may take 1%, by moving the decimal point two places to the left, and then multiply by 4. The second process is considered the simpler by some teachers.

| *First process:* | *Second process:* |
|---|---|
| \$62.25 | \$0.6225 = 1% of \$62.25 |
| .04 | 4 |
| \$2.4900 | \$2.4900 |

Result = \$2.49.

2. How much is 0.50% of \$1000?

This means $\frac{1}{2}$% of \$1000, or $\frac{1}{2}$ of \$10, or \$5.

WRITTEN EXERCISE

1. 22% of \$4870.
2. 35% of \$64.80.
3. 620% of \$4964.
4. 750% of \$4780.
5. 0.75% of \$2000.
6. 500% of \$987.25.
7. $33\frac{1}{3}$% of \$1773.30.
8. $0.33\frac{1}{3}$% of \$6312.

9. A man whose income was \$1500 a year spent 48% of it. How much did he save?

10. In a city having 3210 school children, $16\frac{2}{3}$% never missed a day at school last year. How many had perfect attendance?

11. A man bought 240 barrels of apples, and 5% of them were spoiled during the winter. He sold 50% of the remainder. How many had he left?

**272. To find what per cent one number is of another.** 25 is what per cent of 225?

Since 1 is $\frac{1}{225}$ of 225, therefore 25 is $\frac{25}{225}$ of 225, or $\frac{1}{9}$ of 225, or $11\frac{1}{9}\%$ of 225.

$$9\overline{)1.00}$$
$$.11\frac{1}{9}$$

We might also say: "$x\%$ of 225 = 25.

Hence $x\% = 25 \div 225 = 11\frac{1}{9}\%$."

Hence, as found on page 138,

*The percentage divided by the base equals the rate.*

WRITTEN EXERCISE

*Find what per cent the first number is of the second:*

1. 25, 750.
2. $1\frac{1}{2}$, $7\frac{1}{2}$.
3. 37, 148.
4. \$1.80, \$3.
5. 5 ft., $37\frac{1}{2}$ ft.
6. 7 ft., 7 yd.
7. 528 ft., 2 mi.
8. \$4.90, \$29.40.
9. \$8.60, \$77.40.
10. 73 ft., 194 yd. 2 ft.

11. A man's income is \$1650 a year, and he spends \$693. What per cent of his income does he save?

12. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? girls?

13. The purity of gold is measured in carats, or 24ths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

14. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine? in a ring that is 14 carats fine?

15. If $6\frac{1}{4}\%$ of a class of 32 have been neither tardy nor absent during the year, and $12\frac{1}{2}\%$ have not been absent, how many have been neither tardy nor absent? How many have not been absent?

**273. To find a number when a certain per cent of it is given.** \$8.40 is 12% of what amount?

$$0.12\,x = \$8.40$$
$$x = \$8.40 \div 0.12 = \$70.$$

.12)8.40
12)840
70

We therefore find, in a way different from that on page 139, that

*The percentage divided by the rate equals the base.*

WRITTEN EXERCISE

1. \$1.05 is 15% of what amount?
2. \$2.08 is 32% of what amount?
3. 6 ft. 2.8 in. is 22% of what length?
4. 9.9 in. is 22% of how many yards?
5. 63 lb. 7 oz. is 35% of what weight?
6. 112 ft. 6 in. is 45% of what length?
7. A man saves \$675.20 a year, which is 32% of his income. How much is his income?
8. A school has 20% of its pupils in the sixth grade, which numbers 29. How many are there in the school?
9. If a school is in session 50% of the days of a certain year, and is in session 183 days, is that a leap year or not?
10. A man buys some lumber, and sells it for \$60, thereby gaining 20% on the cost. The \$60 is what per cent of the cost? What is the cost?
11. A certain school has 119 boys, which is 85% of the number of girls. How many girls are there in the school? What is the total number of pupils?
12. A man sells 41 sheep, which is $33\frac{1}{3}$% of all he owned. He received \$5.50 a head. At this rate, what is the value of the sheep remaining in his flock?

### ORAL EXERCISE

1. If $1\frac{1}{2}$ times a number is 6, what is the number?
2. If $1\frac{1}{3}$ times a number is 16, what is the number?
3. If 1.50 times a number is 12, what is the number?
4. If $1.33\frac{1}{3}$ times a number is 8, what is the number?
5. If 125% of a number is 250, what is the number?

**274. To find a number when the number plus a certain per cent is given.**

What amount increased by 50% equals \$900?

If $x$ is the amount,

then $x + .50\,x = \$900$,

or $1.50\,x = \$900$,

and $x = \$900 \div 1.50 = \$600$.

*Check.* \$600 + 50% of \$600 = \$600 + \$300 = \$900.

Compare this with the method on page 141.

### WRITTEN EXERCISE

1. A man's salary has been increased 22% this year. It is now \$1525. What was it last year?

2. If I have added 60% to the size of my farm, and now have 240 acres, how many did I have before the increase?

3. If you weigh 78.1 lb., and have increased 42% in weight in the last 5 years, how much did you weigh five years ago?

4. If our regular army has been increased 21%, and now numbers 73,810, how many did it number before the increase?

5. If a library has increased in size $16\frac{2}{3}$% in the past five years, and now has 8323 volumes, how many volumes had it before the increase?

### ORAL EXERCISE

1. If $\frac{3}{4}$ of a number is 90, what is the number?

2. If 75% of a number is 60, what is the number?

3. If I spend 25% of my money, and have \$30 left, how much had I in the first place?

4. If a number is decreased by $\frac{1}{3}$ of itself, and then equals 80, what is the number? If it is decreased by $33\frac{1}{3}\%$ of itself, and then equals 40, what is the number?

**275. To find a number when the number minus a certain per cent of itself is given.** What number decreased by 42% of itself equals 406?

If $x$ is the number,

then $x - .42\,x = 406,$

or $.58\,x = 406,$

and $x = 406 \div .58 = 700.$

*Check.* $700 - 42\%$ of $700 = 700 - 294 = 406.$

### WRITTEN EXERCISE

1. What number decreased by $2\frac{1}{2}\%$ of itself equals 11.7?

2. If a man sells 12% of his hens, and has 66 left, how many had he at first?

3. If a man spends 5% of his salary for clothes, and then has \$855 left, what is his salary?

4. If you have read all but 40% of a book, and have read 234 pages, how many pages has the book?

5. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

6. If you lose 10% of a kite string, and have 585 ft. left, how long was the string? How many feet did you lose?

### WRITTEN EXERCISE

1. If 45% of the 240 pupils in a school are boys, how many are girls?

2. A man purchased 530 head of cattle and sold 30% of them. How many had he left?

3. If a man pays 12% of his income for house rent, which is $15 a month, how much is his income?

4. If an automobile going 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

5. If the boys in a certain school are 45% of the total, and there are 66 girls, how many boys are there?

6. What sum increased by $33\frac{1}{3}\%$ of itself becomes $600? What sum decreased by $33\frac{1}{3}\%$ of itself becomes $600?

7. A stenographer can write 105 words a minute, which is 25% more than she could write three months ago. What was her rate then?

8. A stenographer can write 95 words a minute. Three months ago she wrote 20% less than now. What was her rate then?

9. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than gold?

10. A bank averaged $1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

11. A clerk's salary is $12.00 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in a year?

12. The championship record in baseball in a certain year was 52 games won, 14 lost. What per cent of the games were won?

13. One of the best records of baseball in recent years was when Brooklyn won the pennant with a total of 82 games won, 54 lost. What was the per cent of games won?

14. Two of the best running averages in baseball were 597 times at the bat, 139 runs, and 543 times at the bat, 145 runs. What was the per cent of runs in each case?

15. The area of the Atlantic Ocean is 24,651,410 sq. mi., and this is 49% of the area of the Pacific Ocean. What is the area of the latter?

16. A boy last year weighed 63.9 lb., which was 10% less than he weighs now. The 63.9 lb. is what per cent of his present weight? How much does he weigh now?

17. The population of the earth at the beginning of the Christian era is estimated at 54,000,000. At the opening of this century it was estimated at 1,487,900,000. The latter population is what per cent of the former?

18. Of the 161,800,000 people who spoke European languages in 1800, 31,450,000 spoke French. Of the 401,700,000 in 1900, 51,200,000 spoke French. What per cent spoke French in each of these years?

19. Of the same (Ex. 18) totals, 20,520,000 spoke English in 1800, and 111,100,000 in 1900. What per cent spoke English in each of these years?

20. I own a half interest in a farm worth $6000. The annual incomes from my share for the last five years have been $120, $90, $178, $210, and $152. What has been the average annual rate of income?

21. A bar of iron 1 in. in diameter and 1 ft. long weighs 3.368 lb. What is the weight when the length is increased 25%?

22. A man bought a house and lot in a village for $3480, and sold it at an advance of 12½%. What was the selling price?

23. If the weight of an iron plate 1 ft. square and 1 in. thick is 40.42 lb., what is the weight when the thickness is decreased 12½%?

24. A factory sells $133,833.48 worth of goods in a year, the cost of manufacturing being $101,389. The gain is what per cent of the cost?

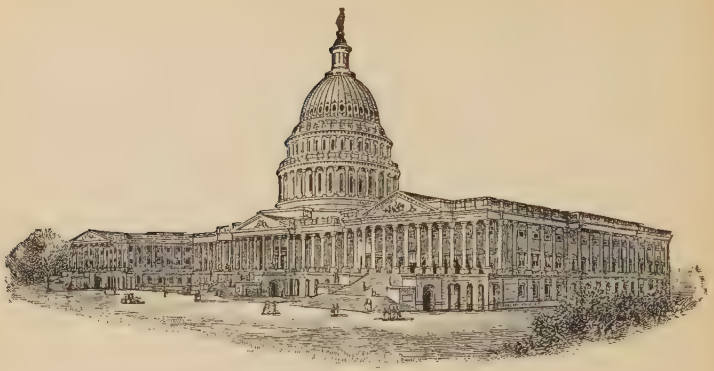
25. A factory sells $100,606.41 worth of goods in a year, the cost of manufacturing being $91,878. The gain is what per cent of the cost?

26. If 14 cu. ft. of chalk weighs 1 ton, and clay is 30% lighter than chalk, and sand is 20% lighter than clay, how many cubic feet of clay to a ton? How many cubic feet of sand?

27. A grain dealer bought 9000 bu. of wheat at 85 ct. a bushel. He sold a third of it at an advance of 33⅓%, a fourth of it at an advance of 25%, and the rest at 90 ct. a bushel. How much did he gain?

28. A dealer bought 2000 tons of coal for $9000. He sold 600 tons at $5.25 a ton. A strike then occurring, he was able to sell the rest at $6.75 a ton. What per cent did he make on his investment?

29. How much is 25% of $400? $400 is 25% of what number? $400 increased by 25% of itself is how much? $400 decreased by 25% of itself is how much? What sum increased by 25% of itself becomes $400?



SOME OF THE LARGE BUILDINGS OF THE WORLD

WRITTEN EXERCISE

1. The rotunda of the capitol at Washington is 95 ft. 6 in. in diameter, and the interior is 185 ft. 3 in. high. The diameter is what per cent of the height?

2. The tallest church spire in the world is that of the Cologne cathedral, 501 ft. The dome of the capitol at Washington, 287.5 ft., is what per cent as high?

3. The tallest column in the world is the Washington monument, 555 ft. high. This is what per cent of the height of the Alexander column at St. Petersburg, 175 ft.?

4. The greatest altitude above the sea, in the state of Delaware, is 280 ft. The height of a certain office building in New York is 322 ft. The office building is what per cent higher than the Delaware altitude?

5. The largest cathedral in the world is St. Peter's at Rome. It has a standing capacity of 54,000 persons. A school hall with a standing capacity of 405 persons has what per cent of floor space as compared with St. Peter's?

**276. Per cents off.** Sometimes merchants have bargain sales, to dispose of odd pieces or of goods that are going out of style. At such times they usually sell goods at a certain "per cent off."

For example, goods marked \$2.50, but sold 10% off, sell at \$2.50 − 10% of \$2.50, or \$2.25.

Merchants who sell at wholesale, that is, in large quantities to dealers, sell at a certain per cent off.

\$2.50
.25
\$2.25

ORAL EXERCISE

*Find the price that must be paid for goods marked as stated in these examples, and discounted at the rates given:*

1. \$2, 10% off.
2. \$6, 20% off.
3. \$5, 20% off.
4. \$4, 25% off.
5. \$3, 33⅓% off.
6. \$10, 30% off.
7. \$25, 40% off.
8. \$30, 50% off.
9. \$50, 30% off.
10. \$60, 33⅓% off.
11. \$1.60, 10% off.
12. \$100, 30% off.
13. \$3.50, 20% off.
14. \$3.50, 10% off.
15. \$4.50, 20% off.
16. \$4.50, 33⅓% off.

WRITTEN EXERCISE

1. \$635, 6% off.
2. \$350, 15% off.
3. \$275, 15% off.
4. \$425, 35% off.
5. \$21.25, 8% off.
6. \$45.50, 20% off.
7. \$375.25, 4% off.
8. \$175.50, 20% off.
9. \$225.75, 20% off.
10. \$975.70, 10% off.
11. \$1527.50, 4% off.
12. \$310.00, 40% off.
13. \$33.33, 33⅓% off.
14. \$2736.40, 3% off.

**277. Several discounts.** In some kinds of business two or more discounts are allowed. For example, a dealer may buy \$200 worth of hardware with discounts of 20%, 10% (20% and 10%). This means that 20% is to be deducted, and then 10% from the remainder.

The marked price is \$200.
This less 20% is \$160.
The \$160 less 10% is \$144, the cost, or *net price.*

ORAL EXERCISE

1. From \$50 take 10%, and 10% from the remainder.
2. From \$200 take 20%, and 1% from the remainder.
3. From \$100 take 25%, and 10% from the remainder.
4. From \$500 take 20%, and 25% from the remainder.
5. A man bought some paper at a marked price of \$80, with discounts of 25%, 10%. How much did he pay?

WRITTEN EXERCISE

*How much must be paid for goods marked and discounted as stated in Exs. 1–10?*

1. \$200, 30%, 20%.
2. \$700, 35%, 5%.
3. \$500, 25%, 15%.
4. \$500, 12%, 2%.
5. \$650, 10%, 10%.
6. \$400, 30%, 10%.
7. \$300, 15%, 10%.
8. \$275, 10%, 10%.
9. \$240, $12\frac{1}{2}$%, $33\frac{1}{3}$%.
10. \$800, 25%, 25%.

11. What is the difference between a discount of 50% on \$800, and the two discounts of 25%, 25%?

12. Is there any difference between a discount of 10%, 5% on \$900, and that of 5%, 10%? How is it on \$400? on \$200? on \$1200?

**278. Bills discounted.** Here is a bill for some goods purchased by a retail merchant from a wholesale dealer.

*Chicago, December 17, 19.....*

*Mr. A. M. Nourse, Joliet, Ill.*

*Bought of Starr & Tiffany, Jewelers*
*378 Wabash Ave.*

Terms cash in 60 da. *2% and 1%*

| | | |
|---|---|---|
| *Dec. 2.* | *3 doz. Silver Forks @ $22* | *$66.* |
| *"* | *1/6 doz. Salad Forks @ $24* | *4.* |
| | | *$70.* |
| | *Less 2%, 1%* | *2.09* |
| | | *$67.91* |

*Received payment*
*Starr & Tiffany*

**WRITTEN EXERCISE**

*Make out bills for the following, as on page 125:*

1. Bought 40 arithmetics @ 70 ct., 65 geographies @ 90 ct., 25 readers @ 35 ct. Discount 12%, 5%.

2. Bought 3 doz. locks at $4.25, 4 doz. bolts @ $1.10, 2½ doz. No. 2478 hinges @ $2.35. Discount 30%, 5%.

3. Bought 4 doz. spoons @ $15, 8 doz. plated tablespoons @ $5, 6½ doz. teaspoons @ $10.50. Discount 8%, 5%.

4. Bought 3 doz. pencils @ at 25 ct., ½ doz. rulers @ $1.00, 1 pt. ink @ 25 ct., 4 lb. paper @ 30 ct. Discount 10%.

## PROFIT AND LOSS ON PURCHASES

### ORAL EXERCISE

1. What is \$2.50 with 10% added? with 10% subtracted?

2. What is \$1200 with 20% added? with 25% subtracted?

3. A dealer buys some books for \$100, and sells them at a profit of 20%. How much does he gain?

4. A dealer buys some cloth for \$50, and sells it at a profit of 20%. How much does he gain? What is the selling price?

5. A grocer bought \$15 worth of berries, but left them in the sun so long that he had to sell them at a loss of 10%. How much did he lose?

6. A man bought some goods for \$100, and sold them for \$125. He gained what per cent on the cost? He gained what per cent on the selling price?

7. If a man sells goods at a profit of 25%, the selling price is what per cent of the cost? If at a loss of 25%, the selling price is what per cent of the cost?

**279. Computing profit or loss.** The per cent of profit or loss is always reckoned on the *cost* of the goods.

**280. Illustrative problems.** 1. A man buys goods at \$40 and sells them at a profit of 20%. What is the selling price?

1. 20% of \$40 = \$8, profit.
2. \$40 + \$8 = \$48, selling price.

2. A man buys goods at \$60 and sells them at \$40. He loses what per cent?

1. \$60 − \$40 = \$20, loss.
2. Since $x$% of \$60 = \$20,
   $x$% = \$20 ÷ \$60 = $33\frac{1}{3}$%.

### ORAL EXERCISE

1. A hat dealer buys hats at $48 a dozen and sells them at $5 each. What is his per cent of profit?

2. A grocer buys starch at 50 ct. a box and sells it at 10% profit. What is the selling price?

3. A dealer buys stove polish at 3 ct. a cake and sells it at 4 ct. What is his per cent of profit?

4. A grocer buys matches at 48 ct. a dozen boxes and sells them at 54 ct. What is his per cent of profit?

5. A druggist buys some fine soap at the rate of $2.40 a dozen cakes, and sells it at 25 ct. a cake. What is his per cent of profit?

6. Some soap that a druggist bought at $1.50 a dozen cakes is damaged by dampness, so he sells it at 10 ct. a cake. What is his per cent of loss?

### WRITTEN EXERCISE

1. A grocer pays $12 a gross for boxes of starch. He sells the starch at 10 ct. a box. What is his per cent of profit?

2. He buys washing powder in 1-lb. packages at $5.88 a gross, and sells it at 5 ct. a package. What is his per cent of profit?

3. Some of this washing powder that he bought at $5.88 a gross was damaged and had to be sold at 4 ct. a package. What was the per cent of loss?

4. A dealer bought 150 bags of salt, 56 lb. to the bag, for $63. He sold $\frac{2}{3}$ of the salt at the rate of 1 ct. a pound, and the rest at the rate of 2 lb. for a cent. Did he gain or lose, and what per cent?

## Some Problems of the Grocer

### ORAL EXERCISE

1. If I buy olives at $2.50 a dozen bottles, at what rate must I sell them to gain 20%? 30%?

2. If I buy mustard in bottles at $14.40 a gross, how much do I pay a bottle? If I sell it at 15 ct., what per cent do I gain? Suppose I sell it at 20 ct.?

3. If I buy pickles at $1.80 a dozen bottles, and wish to sell them so as to gain $33\frac{1}{3}\%$, at what price per bottle must I sell them? Suppose I wish to gain 20%?

4. Boneless codfish costs me 36 ct. a box. Some of it is damaged so that I have to sell it at a loss of 25%. At what price per box do I sell it? Suppose the loss were 50%?

### WRITTEN EXERCISE

1. Macaroni costs me, in packages, $15 a gross. I sell it at 12 ct. a package. What per cent do I gain?

2. Gelatin costs me $11.52 a gross, in packages. At what rate per package must I sell it to gain 25%?

3. I buy allspice at the rate of $9 for 60 lb. I sell it at a profit of $33\frac{1}{3}\%$. At what price per pound do I sell it?

4. I pay $40.32 a gross for 1-lb. boxes of pepper, and I sell it at 35 ct. a box. What per cent do I gain?

5. Some packages of oatmeal crackers that cost me $10.80 a gross are damaged, and I have to sell them at 6 ct. a package. What per cent do I lose?

6. I buy 500 lb. of California prunes for $45, and sell 400 lb. at a profit of $33\frac{1}{3}\%$, and the rest at a loss of 50%. Do I gain or lose, and how much?

## Some Problems of the Dry Goods Merchant

### ORAL EXERCISE

1. Silk that costs me 90 ct. a yard must be sold at what price to allow me 30% profit?

2. If I sell at $1.75 a yard lace that cost me $1.50 a yard, what per cent do I gain?

3. Some flannel that cost me 48 ct. a yard I am selling at 60 ct. a yard. What per cent do I gain?

4. Gloves that cost me $160 per 100 pairs I am selling at $1.80 a pair. What per cent do I gain?

5. If I pay $500 for 100 pieces of ribbon, 10 yd. in a piece, and sell it at 65 ct. a yard, what per cent do I gain?

### WRITTEN EXERCISE

1. Some buttons that cost me $1.08 a gross I am selling at 12 ct. a dozen. What per cent do I gain?

2. I buy 525 yd. of lawn for $94.50, and sell it at a profit of $33\frac{1}{3}\%$. At what price per yard do I sell it?

3. I buy hooks and eyes at $10 a gross of papers, and sell them at 9 ct. a paper. What per cent do I gain?

4. I pay $288 for 480 yd. of taffeta silk, and I sell it at 75 ct. a yard. How much do I gain, and what per cent?

5. In order to close out some gloves that cost me $9.60 a dozen pairs, I am selling them at 75 ct. a pair. What per cent do I lose?

6. I buy at an auction sale 252 yd. of embroidery for $20.16. I mark it 25% above cost, but because there is no sale for it I take off 10% from this price and thus sell it. Do I gain or lose, and how much?

**281. Discounts with a profit.** Dealers often allow a discount on goods, but usually so as still to allow a profit.

For example, if a hat cost a dealer \$1.50, and it is marked \$2, but is sold for \$1.75, the dealer makes 25 ct.

The 25 ct. profit is $16\frac{2}{3}\%$ of the \$1.50 of cost, and the 25 ct. discount is $12\frac{1}{2}\%$ of the \$2 marked price.

**282. Illustrative problem.** How much above cost must a man mark goods in order to take off 25% from the marked price and still make a profit of 20%?

He sells for marked price − 25% of it, or 75% of it.

He also sells for cost + 20% of cost, or 1.20 of cost.

Therefore 75% of marked price = 1.20 of cost,

and therefore " " = 1.20 of cost ÷ 75% = 1.60 of cost.

That is, the marked price is 160% of cost, or 60% above cost.

Some prefer to solve the problem like this:

Let \$1 = the cost.

Then \$1.20 = selling price, which is 25% below the marked price.

Therefore 75% of marked price = \$1.20,

and 1% " " " = \$1.20 ÷ 75 = \$0.016,

and 100% " " " = 100 × \$0.016 = \$1.60.

WRITTEN EXERCISE

*How must a man mark goods in order to take off the rate named, and make the profit or loss indicated?*

| *Discount* | *Profit* | *Discount* | *Profit* |
|---|---|---|---|
| 1. 20%, | 20%. | 2. 25%, | 25%. |
| 3. 30%, | 30%. | 4. 30%, | 25%. |
| 5. $33\frac{1}{3}\%$, | $33\frac{1}{3}\%$. | 6. 50%, | 10%. |
| 7. 0%, | $16\frac{2}{3}\%$. | 8. $22\frac{1}{2}\%$, | 0%. |

9. Discount 30% and *loss* 9%; 50% and *loss* 10%.

10. Discount 40% and *loss* 10%; $33\frac{1}{3}\%$ and *loss* $33\frac{1}{3}\%$.

## COMMISSION

### ORAL EXERCISE

1. How much is \$200 plus 10% of itself? plus 1%?

2. How much is \$3 plus 30% of itself? plus 6%?

3. How much is \$150 plus 1% of itself? plus $33\frac{1}{3}$%?

4. If you bought \$15 worth of eggs for a dealer, receiving 5% for your work, what would you make?

5. If you bought \$2 worth of eggs for a dealer, and he paid you 10% more than cost, what would you make?

**283. Commission.** The pay received by one person for buying or selling for another is often reckoned at a certain per cent of the price, and is called a *commission.*

For example, if a man buys \$1000 worth of potatoes for another man, at 5% commission, he would receive 5% of \$1000, or \$50.

### WRITTEN EXERCISE

1. Required the commission on an \$850 sale, at $2\frac{1}{2}$%.

2. Required the commission on a \$1200 sale, at $5\frac{1}{2}$%.

3. A grocer buys \$300 worth of apples, but loses 10% of them by decay. The rest he sells for 23% more than they cost. How much does he gain?

4. A man sells at an auction \$292 worth of property for another man, and receives 10% commission. How much does he get? How much is left for the other man?

5. A dealer in fruit in a city sends a man to the fruit region to buy for him. He pays him 2% commission. The buyer purchases \$2500 worth in one place, \$3200 worth in another, and \$4100 worth in another. What is his total commission?

### ORAL EXERCISE

1. How much is $2\frac{1}{2}\%$ commission on \$400? on \$8000?
2. How much is $3\frac{1}{3}\%$ commission on \$600? on \$9000?
3. How much is $12\frac{1}{2}\%$ commission on \$240? on \$6400?
4. How much is $16\frac{2}{3}\%$ commission on \$366? on \$6240?

**284. Commission on sales.** In large city stores salesmen often receive a salary, and also a commission on the amount of their sales, to induce them to sell all the goods possible.

### WRITTEN EXERCISE

1. A clerk receives \$4 a week, and $2\%$ commission on goods sold. If he sells \$372 worth of goods in a week, what is his income for the week?

2. A clerk receives \$10 a week, and $1\frac{1}{2}\%$ commission on goods sold. If he sells \$492 worth of goods in a week, how much is his income for the week?

3. A city grocer sends a man out to his customers' residences every morning to take orders. If the man receives $8\%$ commission on all orders, and if he takes orders to the amount of \$9800 a year, how much does he earn?

4. A saleswoman is offered her choice of \$8.50 a week, \$5 a week with $1\%$ on all sales, or $4\%$ on all sales. She takes the last plan, and her sales for a year average \$265 a week. How much does she gain over each of the other two plans?

5. In a large dry goods store in San Francisco a woman is employed to "shop" for out-of-town customers. She receives from the dealers a commission of $5\%$ on all purchases. Her purchases amount to \$422 a month, on the average. How much is her commission during the year?

### ORAL EXERCISE

1. At 30%, what is the commission on a $120 sale?

2. At 10% commission, what does a collection agent receive for collecting a debt of $95.70?

3. An agent sells 100 school desks at $3.75 each, and receives a commission of 10%. What does he receive?

4. A commercial traveler, whose sales average $52,000 a year, receives a commission of 5%. How much is this?

### WRITTEN EXERCISE

1. What is an architect's commission on a house costing $9472, at 5%?

2. What is an auctioneer's commission for selling $358 worth of furniture, at 6%?

3. A real estate dealer sells 168 acres of land at $40 an acre. How much is his commission at $2\frac{1}{2}\%$?

4. A lawyer charged $40 for collecting a debt of $750. This was the same as what per cent commission?

5. An agent sells our school $65 worth of maps, and his firm pays him $7\frac{1}{2}\%$ commission. How much is this?

6. A farmer sends some butter and eggs to the city, and they are sold for $72.50. He is charged 8% by his agent. What is the net amount received by the farmer?

7. A real estate dealer rents 21 city apartments for the owner, at $1500 each, and charges a commission of $2\frac{1}{2}\%$. How much is the commission?

8. A salesman's sales in a department store in a city amounted, during the six days of one week, to $257.70, $103.10, $175.02, $132.30, $302.25, $50.63. He receives $2\frac{1}{2}\%$ on his sales. How much did he receive that week?

## INTEREST

### ORAL EXERCISE

1. If one man borrows money of another, how does he pay him for the loan, besides returning the money?

2. If you put money in a savings bank and leave it 6 mo. or 1 yr., will they pay you back only what you put in?

3. If any one where you live wished to borrow $100 for a year, what interest would he have to pay?

**285. Interest.** Money paid for the use of money is called *interest.*

For example, if I borrow $50 for a year, and pay $3 for the use of it, the $3 is interest.

**286. Reckoning interest.** Interest is reckoned as a certain per cent of the sum borrowed.

If the interest is 6% of the sum borrowed, and the sum is $200, the interest for 1 yr. is 6% of $200, or $12.

**287. Principal.** The sum borrowed is called the *principal.*

**288. Rate.** The per cent of interest is called the *rate.*

That is, if the interest is 6% of the principal, the rate is 6%.

### WRITTEN EXERCISE

1. What is the interest on $375 for 1 yr. at 6%? at 4%?

2. What is the interest on $950 for 1 yr. at 5%? at 3%?

3. What is the interest on $235 for 1 yr. at 4%? at 5%?

4. If you had $27 in a bank that paid 3% a year, how much interest would you receive for a year?

5. How much are the interest and principal together of $475 at the end of 1 yr., the rate of interest being 6%?

### ORAL EXERCISE

*Find the interest for 1 yr. on the sums given in Exs. 1–12, at the rates specified:*

| | | |
|---|---|---|
| 1. \$100 at 4%. | 2. \$300 at 6%. | 3. \$60 at 3%. |
| 4. \$500 at 5%. | 5. \$900 at 3%. | 6. \$50 at 4%. |
| 7. \$400 at 6%. | 8. \$1200 at 3%. | 9. \$70 at 5%. |
| 10. \$1000 at 4%. | 11. \$3000 at 4%. | 12. \$90 at 6%. |

*Find the interest on the following for 6 mo., first finding the interest for 1 yr., and then dividing by 2:*

| | | |
|---|---|---|
| 13. \$100 at 6%. | 14. \$200 at 5%. | 15. \$40 at 6%. |
| 16. \$400 at 3%. | 17. \$250 at 4%. | 18. \$50 at 4%. |
| 19. \$600 at 5%. | 20. \$500 at 5%. | 21. \$70 at 3%. |
| 22. \$800 at 4%. | 23. \$1000 at 3%. | 24. \$80 at 5%. |

### WRITTEN EXERCISE

*Find the interest for 1 yr. on the sums given in Exs. 1–15, at the rates specified:*

| | | |
|---|---|---|
| 1. \$652 at 4%. | 2. \$375 at 5%. | 3. \$27 at $3\frac{1}{2}$%. |
| 4. \$325 at 3%. | 5. \$$433\frac{1}{3}$ at 3%. | 6. \$42 at $4\frac{1}{2}$%. |
| 7. \$275 at 4%. | 8. \$450.25 at 4%. | 9. \$75 at $2\frac{1}{2}$%. |
| 10. \$725 at 6%. | 11. \$275.20 at 5%. | 12. \$84 at $4\frac{1}{2}$%. |
| 13. \$535 at 5%. | 14. \$625.20 at 5%. | 15. \$72 at $3\frac{1}{3}$%. |

*Find the interest on the following for 6 mo.:*

| | | |
|---|---|---|
| 16. \$82 at 5%. | 17. \$175 at 4%. | 18. \$350 at $2\frac{1}{2}$%. |
| 19. \$75 at 6%. | 20. \$375 at 5%. | 21. \$675 at $3\frac{1}{3}$%. |
| 22. \$45 at 4%. | 23. \$450 at 6%. | 24. \$450 at $4\frac{1}{2}$%. |
| 25. \$35 at 2%. | 26. \$325 at 3%. | 27. \$864 at $3\frac{1}{3}$%. |

## CHECKS AND RECEIPTS

**289. Bank checks.** When a man pays money he often does it by an order on the bank where he keeps his money.

An order on a bank calling for the payment of money is called a *check*.

No. *1476* San Francisco, *January 7*, 19.....

The Fifteenth National Bank of San Francisco

Pay to the order of ..... *John Roberts* ..... $*27.50*

*Twenty-seven and* $\frac{50}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars

*Charles Fredericks*

**290. Indorsing.** This check is payable to the *order* of John Roberts. He must therefore order it paid by writing his name across the back, *indorsing* the check.

### WRITTEN EXERCISE

1. Make out three checks payable to the order of members of the class. Hand them the checks for indorsement.

2. John borrowed $4 six months ago at 6% a year. Make out a check to pay the note and interest.

3. A man deposited $125 in a bank. He drew out by checks the following amounts: $14.62, $3.75, $21.50, $35, $2.17. How much had he left? Write the checks.

4. A man borrowed $75 a year ago at 6% interest, and $130 six months ago at 5% interest. How large a check must he draw to-day to pay all the principal and interest?

**291. Receipts.** When I owe you any money and pay you, I ask for a *receipt*, so there may be no misunderstanding about it. A receipt for money paid is as follows:

New Orleans, *February 2*, 19.....

Received from.....*Frank Franklin*.....

*Fourteen and* $\frac{75}{100}$ ~~~~~~~~~~~~~~~~~~~~~~~~~~~~Dollars

for *rent to date*

*$14.75* *Joseph Joslin*

WRITTEN EXERCISE

1. Robert rents a house of you at $18.50 a month. Write a receipt for $3\frac{1}{2}$ months' rent to date.

2. I borrowed $125 a year ago at 6%. I have paid you the principal and interest to-day. Write a receipt for me.

3. I borrowed $65 a year ago at 5%. I have paid you the principal and interest to-day. Write a receipt for me.

4. Write receipts for the principal and interest on the following, the money having been borrowed just a year ago: $140 at 4%; $420 at 6%; $375 at 5%.

5. Mary has bought these things at your store: $2\frac{1}{2}$ yd. ribbon @ 20 ct., 2 pr. gloves @ $1.25, 3 collars @ 15 ct. Make out a bill for the goods, and receipt it.

6. Write receipts for the principal and interest on the following, the money having been borrowed six months ago: $250, 6%; $300, 5%; $150, 4%; $375, 6%.

7. Write a receipt covering the following payments made to-day: $372 and interest for 6 mo. at 5%; $280 and interest for a year at 6%; $1000 and interest for $1\frac{1}{2}$ yr. at 5%.

## DENOMINATE NUMBERS REVIEWED

### ORAL EXERCISE

1. Repeat the tables of dry measure and liquid measure.

2. Repeat the tables of length, square measure, and cubic measure.

3. How many things make a dozen? a gross? a great gross? a score? a quire? When are these used?

4. Draw on the blackboard the different kinds of triangles studied by you and name them. (See page 152.)

5. If a rectangle is 60 ft. long and 40 ft. wide, what is the area? Explain your work. (See page 154.)

6. If a triangle has a base 40 ft. and a height 40 ft., what is the area? Explain your work. (See page 154.)

7. If a parallelogram has a base 20 ft. and a height 15 ft., what is the area? Explain your work. (See page 155.)

8. If a rectangular solid is 7 ft. long, 6 ft. wide, and 2 ft. high, what is the volume? Explain your work. (See page 156.)

### WRITTEN EXERCISE

1. What is the volume of a room 16 ft. by 18 ft., and 10 ft. high? $22\frac{1}{2}$ ft. by $24\frac{1}{2}$ ft., and 9 ft. 10 in. high?

2. What is the area of the floors of the same rooms?

3. What is the area of the walls of the same rooms, no allowance being made for openings?

4. Some 4-ft. wood is piled 5 ft. high. The pile is 2 rd. long. How many cords are there? Suppose it is 7 rd. long?

5. How many gallons in a tank 12 ft. 3 in. long, 4 ft. 8 in. wide, and 4 ft. 2 in. deep, allowing 231 cu. in. to a gallon? Suppose it is $13\frac{1}{2}$ ft. $\times$ $6\frac{1}{3}$ ft. $\times$ $3\frac{1}{6}$ ft.?

### ORAL EXERCISE

1. A bin holds 385 cu. ft. of coal. At 35 cu. ft. to the ton, how many tons does it hold? how many pounds?

2. A brick wall contains 300 cu. ft. At 22 bricks to the cubic foot, how many bricks are in the wall?

3. A bin is 4 ft. by 3 ft., and 1 ft. deep. How many cubic feet does it contain? At $1\frac{1}{4}$ cu. ft. to the bushel, how many bushels? How many if it were 5 ft. deep?

**292. Certain equivalents.** The following, already referred to, are often considered important enough to be memorized:

**2150.42 cu. in. = 1 bushel.**
**231 cu. in. = 1 gallon.** (Page 157.)
**7000 grains = 1 avoirdupois pound.**
**5760 grains = 1 troy pound.** (Page 92.)

**293.** Teachers are advised to require memorizing the above equivalents only when their course of study so directs. In certain parts of the country other measures may be demanded by local conditions, as in those states which still make use of the Mexican measures: 1 vara = $33\frac{1}{3}$ in., 1900.8 varas = 1 mi., 5645.376 square varas = 1 acre. Such local demands should always be met.

### WRITTEN EXERCISE

1. How many cubic inches in 1645 bu.? in $2743\frac{1}{3}$ bu.?

2. A grain elevator has a capacity of 812,000 bu. At $1\frac{1}{4}$ cu. ft. per bushel, what is its capacity in cubic feet?

3. A grain elevator has a capacity of 1,500,000 cu. ft. It charges $1\frac{1}{4}$ ct. per bushel for storing grain for the first 10 da. and $\frac{1}{2}$ ct. per bushel for each succeeding period of 10 da. If it is $66\frac{2}{3}\%$ full for exactly 30 da., what is its income for that time? (Allow $1\frac{1}{4}$ cu. ft. to a bushel.)

4. The avoirdupois pound is what per cent heavier than the troy pound? The troy pound is what per cent lighter than the avoirdupois?

5. Bran weighs 20 lb. to the bushel, and oats are 60% heavier. Timothy seed is $40\frac{5}{8}\%$ heavier than oats. How much does timothy seed weigh per bushel?

6. A bale of cotton averages 477 lb. in the United States, 719 lb. in Egypt, and 220 lb. in Brazil. The Egyptian bale is what per cent heavier than ours? The Brazilian is what per cent lighter than ours?

7. A barrel of flour weighs 196 lb., and a barrel of beef or pork weighs 200 lb. A barrel of flour is what per cent lighter than a barrel of beef? A barrel of beef is what per cent heavier than a barrel of flour?

8. Corn meal, barley, and buckwheat weigh 48 lb. to the bushel, while wheat, clover seed, peas, and white beans weigh 60 lb. Wheat is what per cent heavier than barley? Barley is what per cent lighter than wheat?

9. A rectangular tank is $6\frac{1}{4}' \times 4\frac{1}{2}' \times 2\frac{1}{2}'$. How many gallons of water does it hold?

10. A wagon box is 6 ft. long and 3 ft. 6 in. wide. How deep must it be to hold 1 cu. yd.?

11. At $\$1.12\frac{1}{2}$ a load (cubic yard), how long a ditch $1\frac{1}{2}$ ft. wide and 3 ft. deep can be excavated for \$13.50?

12. A 240-gal. tank is to be placed in an attic 4 ft. high. What length and width may the tank have?

13. What will it cost to plaster a room $16' \times 14'$, $9'6''$ high, at 18 ct. a square yard, allowing 96 sq. ft. for doors, windows, and baseboard?

14. The sum of all the edges of a certain cube is 138 in. What is the volume of the cube? the area of each face?

*Add in Exs. 1–6:*

1. 127 ft. 4 in.
   286 3
   491 7
   302 5
   27 2

2. 329 ft. 7 in.
   206 8
   129 11
   43 5
   209 10

3. 635 ft. 7 in.
   82 6
   39 10
   241 11
   43 8

4. 278 yd. 10 in.
   63 13
   49 17
   326 5
   42

5. 27 gal. 1 qt.
   92 3
   167 1
   43 2
   286 3

6. 427 bu. 1 pk.
   63
   429 3
   28 2
   65 3

7. 480 ft. 2 in. − 197 ft. 10 in.
8. $9\frac{1}{2}$ lb. + 9 oz.
9. 293 lb. 5 oz. − 169 lb. 12 oz.
10. $3\frac{2}{3}$ ft. + 8 in.
11. 402 yd. 5 in. − 327 yd. 12 in.
12. $8\frac{2}{3}$ rd. + 5 yd.
13. 572 bu. 1 pk. − 296 bu. 3 pk.
14. $7\frac{1}{2}$ bu. + 3 pk.
15. 235 gal. 1 qt. − 186 gal. 3 qt.
16. $6\frac{2}{3}$ mi. + 90 rd.
17. 42 times 102 lb. 1 oz.
18. 27 times 63 lb. 2 oz.
19. 32 " 249 lb. 2 oz.
20. 16 " 78 lb. 9 oz.
21. 75 " 243 ft. 2 in.
22. 96 " 329 ft. 7 in.
23. 35 " 127 yd. 1 ft.
24. 82 " 306 yd. 2 ft.
25. 48 " 206 gal. 1 qt.
26. 32 " 142 bu. 1 pk.
27. 87 lb. 8 oz. ÷ 4.
28. 95 lb. 2 oz. ÷ 3.
29. 648 gal. ÷ 4 gal.
30. 981 gal. ÷ 9 gal.
31. 302 lb. 8 oz. ÷ 2.
32. 428 lb. 6 oz. ÷ 4.
33. 125 ft. 10 in. ÷ 5.
34. 425 ft. 5 in. ÷ 25.
35. 275 lb. 15 oz. ÷ 25.
36. 625 lb. 5 oz. ÷ 25.
37. 13,328 bu. ÷ 28 bu.
38. 38,208 bu. ÷ 36 bu.
39. 20,720 mi. ÷ 56 mi.
40. 41,440 yd. ÷ 56 yd.

*Find the difference between the following dates:*

41. April 6, 1905, May 1, 1907.

42. July 4, 1909, May 1, 1911.

43. May 24, 1905, April 2, 1910.

44. July 4, 1776, January 1, 1920.

45. September 6, 1906, April 4, 1908.

46. June 7, 1907, September 2, 1909.

47. September 4, 1906, April 2, 1908.

48. August 2, 1908, February 1, 1910.

49. If a watch gains 13 sec. a day, how much will it gain in January?

50. July 4, 1912, falls on Thursday. On what day does April 1 of that year fall?

51. How many hours from 7 P.M., February 28, to 9 A.M., March 1, in a leap year?

52. If a watch loses 17 sec. a day, and is set correctly at noon July 1, what time will it indicate at noon August 1?

53. A piece of oak $12' \times 9'' \times 6''$ weighs how many times as much as one $8'\ 3'' \times 8'' \times 4''$?

54. A man can row across a lake, $2\frac{3}{4}$ mi., in 28 min. At this rate, how long will it take him to row the length of the lake, $6\frac{1}{4}$ mi.?

55. How many days from April 17 to July 16? from January 23 to September 10? from August 27 to December 5? (Give the exact number.)

56. Make a diagram of the parts of a room 18 ft. long, 15 ft. wide, and 9 ft. 6 in. high, that are usually plastered. Not allowing for openings and baseboard, how many square yards of plaster are needed?

## II. SIMPLE INTEREST. RATIO AND PROPORTION

### SIMPLE INTEREST

**294. Finding interest.** We have found the interest on money, at various rates, for one year, and also for half of a year. We shall now find it for other lengths of time.

#### ORAL EXERCISE

1. If the interest for 1 yr. is \$16, how much is it for $\frac{1}{2}$ yr.? for $\frac{1}{4}$ yr.? for $\frac{1}{8}$ yr.?

2. If the interest for 1 yr. is \$24, how much is it for 1 mo.? for 3 mo.? for 11 mo.?

3. If the interest for 1 yr. is \$8, how much is it for 3 yr.? for 5 yr.? for 7 yr.? for 8 yr.?

4. At 6%, how much is the interest on \$500 for 1 yr.? for 4 mo.? for 2 yr.? for 6 mo.? for 3 yr.?

#### WRITTEN EXERCISE

1. What is the interest on \$436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

2. What is the interest on \$360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

3. If the interest on a certain sum is \$93.84 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 9 mo.?

4. If the interest on a certain sum is \$36.48 for 1 yr., how much is it for 1 mo.? for 5 mo.? for 10 mo.?

5. What is the interest on \$540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 5 da.? for 1 da.? for 1 wk.?

6. What is the interest on \$175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for 10 da.?

### ORAL EXERCISE

*What is the interest on the following sums?*

1. \$100, 5%, 1 yr.
2. \$300, 6%, 1 yr.
3. \$500, 4%, 1 yr.
4. \$800, 5%, 1 yr.
5. \$200, 6%, 2 yr.
6. \$500, 5%, 2 yr.
7. \$120, 5%, 1 mo.
8. \$300, 5%, 6 mo.
9. \$100, 6%, 6 mo.
10. \$600, 3%, 6 mo.
11. \$700, 4%, 6 mo.
12. \$500, 3%, 4 mo.
13. \$200, 6%, 4 mo.
14. \$400, 3%, 8 mo.
15. \$300, 4%, 5 mo.
16. \$200, $3\frac{1}{2}$%, 6 mo.
17. \$100, $4\frac{1}{2}$%, 6 mo.
18. \$240, 5%, 11 mo.
19. \$200, 4%, 2 yr. 6 mo.
20. \$300, 4%, 2 yr. 1 mo.

**295. Interest for years and months.** We have seen that to find the interest for 2 yr. 6 mo., we find the interest for 1 yr., and multiply by $2\frac{1}{2}$. That is,

*Find the interest for 1 year, and multiply by the number of years.*

Thus, to find the interest on \$125 for 2 yr. 3 mo. at 5%.

1. 5% of \$125 = \$6.25, int. for 1 yr.
2. $2\frac{1}{4}$ times \$6.25 = \$14.06, " " 2 yr. 3 mo.

| | |
|---|---|
| | \$125 |
| | .05 |
| | \$6.25 |
| | $2\frac{1}{4}$ |
| | 1 56$\frac{1}{4}$ |
| | 12 50 |
| | \$14.06 |

### WRITTEN EXERCISE

1. \$250, 6%, 6 mo.
2. \$300, $5\frac{1}{2}$%, 8 mo.
3. \$8200, $5\frac{1}{2}$%, $2\frac{1}{2}$ yr.
4. \$625, 4%, 3 yr. 6 mo.
5. \$35, 6%, 1 yr. 6 mo.
6. \$45.50, 4%, 2 yr. 3 mo.
7. \$750, 5%, 2 yr. 4 mo.
8. \$75.50, 6%, 2 yr. 6 mo.
9. \$225, 6%, 3 yr. 6 mo.
10. \$4250, $4\frac{1}{2}$%, 1 yr. 6 mo.

**296. Interest for years, months, and days.** Suppose one man borrows from another $400 on September 10, 1904, at 6%. What will the interest amount to on August 7, 1907?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 6% of $400 = $24, | int. for | 1 yr. | 1907 | 8 mo. | 7 da. |
| 2. | 2 times $24 = $48, | " | 2 yr. | 1904 | 9 | 10 |
| 3. | $\frac{1}{12}$ of $24 = $20, | " | 10 mo. | 2 | 10 | 27 |
| 4. | $\frac{27}{30}$ of $\frac{1}{12}$ of $24 = $1.80, | " | 27 da., or $\frac{27}{30}$ of $\frac{1}{12}$ of a year. | | | |
| 5. | Total int. = $69.80, | " | 2 yr. 10 mo. 27 da. | | | |

WRITTEN EXERCISE

*Find the interest as directed in Exs. 1–12:*

1. $325, 1 yr. 8 mo. 5 da., 5%.
2. $300, 1 yr. 4 mo. 15 da., 6%.
3. $200, 3 yr. 6 mo. 10 da., 6%.
4. $150, 1 yr. 3 mo. 20 da., 5%.
5. $250, 2 yr. 3 mo. 10 da., 4%.
6. $475, 3 yr. 6 mo. 15 da., 6%.
7. $850, 2 yr. 7 mo. 6 da., $4\frac{1}{2}$%.
8. $2500, 3 yr. 3 mo. 3 da., 3%.
9. $775, 1 yr. 6 mo. 24 da., $3\frac{1}{2}$%.
10. $75, 4%, June 9, 1906 to May 2, 1908.
11. $225, 5%, July 8, 1908 to April 3, 1910.
12. $2000, 3%, May 7, 1907 to March 4, 1909.
13. If I borrow $50, at 6%, on February 8 of this year, how much will be due on May 2 of next year?
14. If Mr. Roberts borrows $350 from Mr. Bugbee, at 4%, on September 25 of this year, how much principal and interest will be due Mr. Bugbee on April 10 of next year?

**297. Promissory note.** A paper signed by a borrower, agreeing to repay money, is called a *promissory note.*

**298. Face of note.** The sum borrowed is called the *principal*, or, if a note is given, the *face of the note.*

**299. Amount of note.** The sum of the principal (or face) and interest is called the *amount.*

Notes should state value received, the date, face, rate, person to whom payable, and time of payment.

$75.00 BUFFALO, N.Y., February 7, 1907

Six months after date, for value received, I promise to pay to ..... John Johnson ..... or order, Seventy-five and $\frac{00}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars, with interest at 5 %

Frank Francis

**300. Rate of interest.** The rate of interest varies in different parts of the country. The United States has to pay about 2%. Savings banks pay 3% or 4%. People borrowing money in cities, on good security, pay from 4% to 6%.

### WRITTEN EXERCISE

1. Compute the amount of the above note.

2. Write a promissory note, signed by X, payable to Y, for $25, due in 1 yr., at 6%. Find the amount.

3. Write a promissory note as before, for $250, due in 10 mo., at 5%. Find the amount.

4. Write a promissory note as before, for $400, due in 1 yr. 6 mo., at 4%. Find the amount.

**301. Bank discount.** When a man borrows from a bank he pays the interest in advance. Interest is not mentioned in the note, because it has already been paid.

Interest paid in advance on a note is called *discount.*

Teachers should call attention to the fact that the same word is used for bank and commercial (trade) discount, explaining that the process is the same in both cases, — finding some per cent of a number.

The technicalities of interest-bearing notes are explained later. In all work at present call 30 days a month.

**302. Proceeds.** The face of a note less the discount is called the *proceeds.*

**303. Illustrative problem.** What is the discount on a \$200 note for 6 mo. at 5%? What are the proceeds?

1. The discount (interest) for 1 yr. is 5% of \$200, or \$10.
2. For $\frac{1}{2}$ yr. it is $\frac{1}{2}$ of \$10, or \$5, discount.
3. \$200 − \$5 = \$195, proceeds.

WRITTEN EXERCISE

*Find the discount and proceeds:*

1. \$75, 1 mo., 6%.
2. \$90, 30 da., 6%.
3. \$250, 3 mo., 5%.
4. \$275, 3 mo., 5%.
5. \$425, 4 mo., 5%.
6. \$300, 90 da., 6%.
7. \$375, 6 mo., 6%.
8. \$400, 90 da., 6%.
9. \$150, 45 da., 6%.
10. \$400, 1 mo., $5\frac{1}{2}$%.
11. \$200, 60 da., 6%.
12. \$175, 2 mo., $4\frac{1}{2}$%.
13. \$300, 4 mo., $5\frac{1}{2}$%.
14. \$500, 2 mo., $3\frac{1}{2}$%.
15. \$2975, 63 da., 5%.
16. \$4250, 93 da., 6%.
17. \$1250, 10 da., 6%.
18. \$2500, 15 da., 6%.
19. \$1500, 20 da., 6%.
20. \$1250, 45 da., 5%.
21. \$2750, 30 da., $5\frac{1}{2}$%.
22. \$3520, 90 da., $4\frac{1}{2}$%.

**304. Payee of a note.** The person named in the note as the one to whom it is payable is called the *payee*.

**305. Maker of a note.** The one who signs the note is called the *maker*.

**306. Indorsing notes.** If the payee wishes to sell the note, he must, when it is payable to himself or *order*, indorse it.

A note is indorsed by the payee writing his name across the back. The indorser must pay the note if the maker does not.

If he wishes it payable to the order of John Roberts, he writes across the back, "Pay to the order of John Roberts," and signs his name.

A note payable to John Johnson or *bearer* may be sold without indorsement. Such notes are not common.

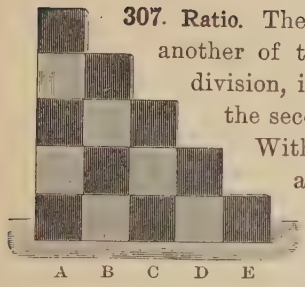
### WRITTEN EXERCISE

1. F. H. Holmes borrows \$350, at 6%, for 1 yr., from A. B. Seel. He gives a note payable to Mr. Seel or order. Mr. Seel sells the note to B. C. Dean. Make out the note, indorse it, and find the amount due at the end of the year.

*Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due at the end of the time stated:*

| | *Maker* | *Payee* | *Buyer* | *Face* | *Rate* | *Time* |
|---|---|---|---|---|---|---|
| 2. | J. R. Roberts | A. N. Cole | P. R. Jones | \$375 | 6% | 6 mo. |
| 3. | M. L. King | A. R. Owen | E. F. Pease | \$750 | 5% | 2 yr. |
| 4. | A. K. Jewett | S. M. Wood | E. L. Brown | \$75 | 6% | 10 mo. |
| 5. | B. S. Stevens | O. N. House | B. R. Lodge | \$250 | 5% | 3 yr. 6 mo. |
| 6. | O. M. Richards | D. E. Dodge | C. King | \$125 | 6% | 1 yr. 2 mo. |
| 7. | A. R. Castle | A. J. Jackson | G. F. Ray | \$250 | $5\frac{1}{2}$% | 2 yr. 4 mo. |
| 8. | F. E. Taylor | A. B. Baker | G. F. Coe | \$350 | $4\frac{1}{2}$% | 1 yr. 3 mo. |
| 9. | F. E. Glover | M. T. Thorn | F. Taylor | \$250 | 6% | 4 yr. 1 mo. |

## RATIO




**307. Ratio.** The relation of one quantity to another of the same kind, as expressed by division, is called the *ratio* of the first to the second.

Without using the name, we studied about ratio in studying fractions. The ratio of \$2 to \$4 is $\frac{2}{4}$ or $\frac{1}{2}$; the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$. In this picture the ratio of E to A is $\frac{1}{5}$; of D to C, $\frac{2}{3}$; of C to B, $\frac{3}{4}$; of D to B, $\frac{1}{2}$.

### ORAL EXERCISE

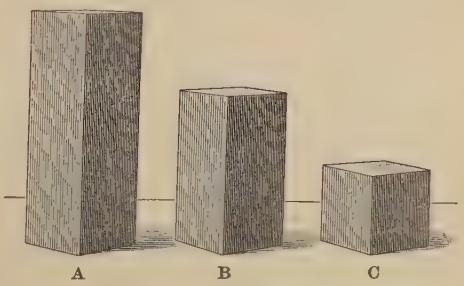
1. In the picture above, what is the ratio of B to A?

2. Also, what is the ratio of A to B? of E to C?

3. In the picture above, if A represents \$15, what values must be given to B, C, D, and E?

4. If each cube weighs 6 oz., give the ratios of A to B; B to C; C to D; E to D.

5. In the picture below, what is the ratio of C to B? of C to A? of B to A?




6. What is the ratio of A to C? of B to C? of A to B + C?

7. What is the ratio of any two equal numbers? of the lengths of $16\frac{1}{2}$ ft. and 1 rd.? of 1 mi. and 5280 ft.?

**308. Writing ratios.** A ratio may be written like a fraction, $\frac{2 \text{ ft.}}{3 \text{ ft.}}$, or with a sign of ratio (:), as 2 ft. : 3 ft.

We think of the ratio of 20 ft. to 4 ft. as $\frac{20 \text{ ft.}}{4 \text{ ft.}}$, 20 ft. : 4 ft., $\frac{20}{4}$, 20 : 4, or simply as 5. The word *ratio* is used for all.

**309. Antecedent.** The first term is called the *antecedent.*

**310. Consequent.** The second term is called the *consequent.*

$$\frac{2 \text{ ft.}}{3 \text{ ft.}} = \frac{2}{3} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}.$$

**311. Ratios always abstract.** Because any number divided by a like number, as feet by feet, has an abstract quotient,

*A ratio is always abstract, and its terms may be written as abstract numbers.*

Teachers should use the fractional form at first, because the pupils are already familiar with it.

### ORAL EXERCISE

*Find the value of* x *in Exs. 1–12:*

1. $\frac{x}{2} = 10.$
2. $\frac{x}{3} = 3.$
3. $\frac{x}{4} = 2.$
4. $\frac{x}{5} = 3.$
5. $\frac{x}{10} = 1.$
6. $\frac{x}{10} = \frac{1}{2}.$
7. $\frac{x}{7} = \frac{3}{7}.$
8. $\frac{x}{6} = \frac{2}{3}.$
9. $\frac{x}{5} = \frac{4}{5}.$
10. $\frac{2}{x} = \frac{1}{3}.$
11. $\frac{20}{x} = .4.$
12. $\frac{10}{x} = 1.$

13. What is the number whose ratio to 5 is 6?

14. What is the number to which 75 has the ratio 3?

15. The ratio of the height of a certain door to its width is $3\frac{1}{2}$. It is 7 ft. high. How wide is it?

16. The ratio of Fred's father's height to Fred's height is $\frac{3}{2}$. Fred's father is 6 ft. tall. How tall is Fred?

### ORAL EXERCISE

1. Separate 9 in the ratio of 2 to 1.

2. What is the ratio of the number of boys to the number of girls in your class?

3. What is the ratio of the time you are in school to the whole 24 hours of the day?

4. What is the ratio of the length of summer, 3 months, to the whole year? to the rest of the year?

5. There were 6 cloudy days in September, the rest being clear. What was the ratio of cloudy to clear days?

6. In playing ringtoss, Will has a score of 15 and Mollie has one of 18. What is the ratio of Will's score to Mollie's?

**312. Separating in a given ratio.** Separate 25 ct. in the ratio of 2 to 3.

1. Since there are 2 parts in one group and 3 in the other, there are 5 parts in all.
2. Since one group has $\frac{2}{5}$, it has $\frac{2}{5}$ of 25 ct., or 10 ct.
3. The other has $\frac{3}{5}$ of 25 ct., or 15 ct.

*Checks.* 10 ct. + 15 ct. = 25 ct.; 10 : 15 = 2 : 3.

### WRITTEN EXERCISE

1. Divide \$100 in the ratio of 4 : 1; of 7 : 3; of 1 : 9.

2. In a year having 165 cloudy and 200 clear days, what is the ratio of cloudy days to clear?

3. The ratio of clear to cloudy days in November was 3 : 2. How many were there of each?

4. Taking the area of the United States as 3,600,000 sq. mi., and the ratio of farm land to other land as 7 to 18, find the area of the farm land.

## Miscellaneous Problems of the Farmer

### WRITTEN EXERCISE

1. A farmer puts 5 acres into celery, setting out 20,000 plants to the acre. The yield being 1500 doz. heads to the acre, what is the ratio of the plants matured to the others?

2. He pays $95 an acre for seeds, fertilizers, labor, and other expenses, and sells the crop at 15 ct. a dozen heads. What is his profit on the 5 acres?

3. Another farmer tries setting out 30,000 plants to the acre, but only 80% mature, and these are so small that he has to put 16 in a bunch to sell for a dozen, and then gets only 14 ct. a bunch. His expenses are $100 an acre. At this rate, what is his profit on 5 acres?

4. A farmer has a 30-acre meadow yielding $1\frac{1}{2}$ tons of hay to the acre. If by spending $300 a year for fertilizers, he can bring the yield to 4 tons to the acre, how much more will he make a year, hay being worth $8 a ton?

5. A farmer reads that a good mixture of seed for his meadow is, by weight, as follows: timothy 40%, redtop 40%, red clover making up the rest. At 40 lb. of seed to the acre, how many pounds of each should he sow?

6. The following is, by weight, a good mixture of seed for a pasture: Kentucky blue grass 25%, white clover $12\frac{1}{2}$%, perennial rye $28\frac{1}{8}$%, red fescue $9\frac{3}{8}$%, redtop 25%. At 32 lb. to the acre, how many pounds of each are used?

7. A cow weighing 1000 lb. consumes the equivalent of $4\frac{1}{4}$ tons (2000 lb. to the ton) of dry food a year; a 100-lb. sheep, 770 lb.; every ton of live pork, 12 tons; and every ton of live horseflesh, 8.4 tons. Each class of animals consumes what per cent of its own weight of dry food a year?

8. For spraying potato vines 1 lb. of Paris green is used to 100 gal. of water. How much water should be used with an ounce of Paris green?

9. If hops lose 72% of their weight in drying, 100 lb. of dry hops represent how many pounds of green hops? How many pounds are lost in drying?

10. To make grafting wax a man takes 60% resin, 22% beeswax, and the rest linseed oil, measuring all by weight. How many pounds of each will he need for 15 lb. of wax?

11. He can also make grafting wax by taking equal parts of beeswax and linseed oil, and 3 times as much resin as of beeswax and oil together. Each is then what per cent of the total?

12. What per cent less rice is yielded by a field that produces 32 bu. to the acre than by one that produces 10 barrels to the acre, counting 45 lb. to the bushel and 162 lb. to the barrel?

13. How much will the material cost for spraying an orchard of 1000 trees twice during a season, allowing 5 gal. to a tree for each spraying, every 150 gal. containing 1 lb. of Paris green worth 20 ct. a pound?

14. A farmer wishes to spray his orchard. He can make lead poison by using white sugar of lead at $7\frac{1}{2}$ ct. a pound, and arsenate of soda at 5 ct. a pound, in the ratio of 7 : 3, or he can buy Paris green at 20 ct. a pound. Which is the cheaper by the 100 lb., and how much?

15. For protection against the cotton worm a grower uses $1\frac{1}{2}$ lb. of dry Paris green to the acre. A man can cover 15 acres a day, with the help of a horse. At 20 ct. a pound for Paris green, and \$2.25 a day for a man and horse, how much will it cost to dust 45 acres?

16. A Tennessee farmer has 85 acres planted to peanuts. The average yield is 42 bu. to the acre, but by using a better fertilizer he increases the yield $33\frac{1}{3}\%$. How many bushels does the entire farm then yield?

17. In feeding 750 lb. of dry cornstalks to cattle it was found that 225 lb. were wasted, but that when the green stalks were put into a silo and then fed to the cattle, only 32 lb. were wasted out of 640 lb. What was the ratio of the per cent of waste in the latter case to that in the former?

18. A farmer paid out the following amounts for a 50-acre wheatfield: plowing, 9 da. @ \$2.75; harrowing and rolling, 6 da. @ \$2.50; 100 bu. seed @ \$1.15; drilling, 6 da. @ \$2.75; harvesting, \$40; threshing, \$75. The yield was 25 bu. to the acre, and he sold the crop at 73 ct. a bushel. How much did he make?

19. A man in Florida went into the business of raising pineapples. He paid \$250 for 5 acres of land, and up to the sale of his first crop had spent 50% of that amount for clearing, \$2000 for plants, 3 times as much for fertilizer as for clearing, \$200 for freight, express, and packages, \$1800 for a shed, and as much for labor as the land cost. His first crop averaged \$500 to the acre. His receipts were what per cent of his expenses?

20. A farmer asked his son to experiment on the cost of feeding chickens and ducks. The boy took 10 ducks and 10 chickens, and kept a record of their weight and the cost of their feed for 5 weeks. When he began, the 10 ducks together weighed $3\frac{3}{4}$ lb., and the chickens 1 lb. 14 oz. After 5 weeks the ducks weighed $30\frac{1}{4}$ lb., and the chickens $7\frac{5}{8}$ lb. The feed for the ducks cost 53 ct., and for the chickens 26 ct. What was the cost of each pound of increase in the ducks? in the chickens?

## PROPORTION

**313. Proportion.** An expression of the equality of two ratios is called a *proportion.*

For example, the ratio \$2 : \$3 equals the ratio 10 ft. : 15 ft. Therefore \$2 : \$3 = 10 ft. : 15 ft. is a proportion. It is read, "\$2 is to \$3 as 10 ft. is to 15 ft." It may, of course, be written simply $2 : 3 = 10 : 15$, or $\frac{2}{3} = \frac{10}{15}$.

**314. Illustrative problems.** 1. If $x : 3 = 8 : 12$, we may find $x$ as follows:

$\frac{x}{3} = \frac{8}{12}$. Multiplying by 3, as in § 264, $x = \frac{\cancel{3} \times \overset{2}{\cancel{8}}}{\underset{4}{\cancel{12}}} = 2$.

2. If $5 : x = 15 : 18$, we may find $x$ as follows:

1. Because $5 : x = 15 : 18$, we must have $x : 5 = 18 : 15$.

2. That is, $\frac{x}{5} = \frac{18}{15}$, and $x = \frac{\cancel{5} \times \overset{6}{\cancel{18}}}{\underset{3}{\cancel{15}}} = 6$.

### WRITTEN EXERCISE

*Find the value of* x *in Exs. 1–10:*

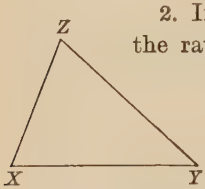
1. $x : 51 = 12 : 17$.
2. $x : 17 = 54 : 51$.
3. $x : 22 = 41 : 11$.
4. $x : 250 = 35 : 50$.
5. $x : 27 = 36 : 243$.
6. $x : 1001 = 8 : 143$.
7. $24 : x = 3 : 20$.
8. $35 : x = 7 : 91$.
9. $111 : x = 37 : 38$.
10. $19 : x = 209 : 143$.

11. I am thinking of a number which has to 17 the ratio 35 : 119. What is the number?

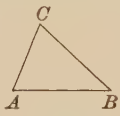
12. In a class in our school the ratio of boys to girls is 3 : 4, and there are 20 girls. How many boys are there?

### ORAL EXERCISE

1. These two triangles have the same shape. If $XY$ is twice $AB$, how does $ZX$ compare with $CA$?

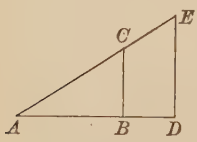



2. If the ratio of $ZX$ to $XY$ is $\frac{3}{4}$, what is the ratio of $CA$ to $AB$?




3. What is the proportion existing between the sides $AB$, $BC$, $XY$, and $YZ$? State the proportion in more than one way.

4. If $XY = 1$ in., and if $ZX : XY = 3 : 4$, and if $AB$ is half as long as $XY$, how long is $CA$?




5. In these triangles, what proportion exists between $AB$, $AD$, $AC$, and $AE$? If $AB = \frac{2}{3}\ AD$, what is the ratio of $AC$ to $AE$?

6. If $AD = 1\frac{3}{16}$ in., $AB = \frac{9}{16}$ in., and $AC = 1\frac{9}{16}$, or $\frac{5}{8}$ in., how can we find the length of $AE$?

7. If $DE$ represents the height of a man 6 ft. tall, $BC$ the height of a boy, $AD$ the shadow cast by the man, and $AB$ the shadow cast by the boy, show how to find the height of the boy from measuring the shadows.

**315. Proportional quantities.** Quantities that are in proportion are said to be *proportional.*

**316. Similar figures.** Figures that are of exactly the same shape are said to be *similar.*

For example, the triangles in Ex. 1.

**317. Proportional sides.** *The corresponding sides of similar figures are proportional.*

For example, $DE$, $BC$, $AD$, $AB$, in the figures of Ex. 5.

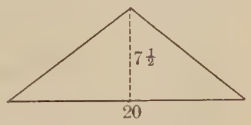
### WRITTEN EXERCISE

1. The ratio of children in school to our total population is 1 : 5. If we have 85,000,000 population, how many children have we in school?

2. The ratio of the width of our schoolroom to the length is 7 : 9. The room is 21 ft. wide. How long is it?

3. If the United States has a standing army of 50,000 soldiers, and an army of 8,000,000 schoolboys, the ratio of soldiers to schoolboys is 1 to how many? State this fact as a proportion.

4. A carpenter wishes to construct a roof as shown here, the house being 20 ft. wide and the gable $7\frac{1}{2}$ ft. high. He wishes to know the length of slope. He draws a plan, representing 20 ft. by 8 in. How many inches will represent the $7\frac{1}{2}$ ft.?




5. He found this was not convenient, so he drew a right-angled triangle representing half the plan, and represented the 10 ft. by 10 in. Draw such a plan, representing the $7\frac{1}{2}$ ft. correctly, and measure the slope. How long did the carpenter find the slope of the roof to be?

6. Edward and Henry wished to find the height of the school building. Edward suggested standing a yardstick upright, measuring its shadow, and at the same time measuring the shadow of the school. The yardstick shadow was 4 ft., and the shadow of the building was 36 ft. How high was the school building? Draw a picture.

It is very easy to measure the height of trees and buildings in this way, or by sighting from the ground across the top of a pole to the top of a building, and pupils will find such work interesting and profitable.

## Proportion used in Agriculture

### WRITTEN EXERCISE

1. In a pound of grass there are 10.8 oz. of water. How many pounds of water in a ton of grass?

2. In 3 lb. of dried corn there are 6 oz. of liquid. How many pounds of liquid in 100 lb. of dried corn?

3. In 20 lb. of potatoes it is found that there are 4.2 lb. of solid matter. How much is there in 17.5 lb.?

4. In 25 lb. of oats it is found that there are 1.9 lb. of fat-producing matter. How much is there in 30 lb.?

5. In 15 lb. of corn it is found that there is 0.135 lb. of muscle-producing matter. How much is there in 20 lb.?

6. A farmer found that he needed 26 lb. of seed for $\frac{1}{2}$ acre. How many pounds does he need for 19.9 acres?

7. A Louisiana farmer obtained 9 hogsheads of molasses from 5 acres. At this rate, how many would he get from 93 acres?

8. In 35 lb. of cabbages there are $5\frac{1}{4}$ lb. of solid matter, the rest being liquids. How many pounds of solid matter in a ton of cabbages?

9. The insects eaten by a woodpecker one morning were found to number 3270, of which 1 out of 3 were harmless to man. How many harmful insects were destroyed?

10. To irrigate a farm at the rate of 2 in. in depth every 10 days requires for each acre a flow of 3.77 gal. a minute. At this rate, what flow is necessary for 3 in. in depth?

11. The water necessary to irrigate properly an acre of land can be stored in a circular reservoir 3.1 ft. deep and 55.5 ft. in diameter. Keeping the same diameter, how deep would the reservoir have to be for $3\frac{1}{2}$ acres?

### WRITTEN EXERCISE

1. Blue jays' food consists of 1 lb. 3 oz. of noxious insects to 5 oz. of other animal food and 4 lb. 12 oz. of vegetable food. What per cent of this food is injurious to man?

This means, injurious to his agricultural interests.

2. Out of every 5 lb. of food taken by red-winged blackbirds, all but 10 oz. is made up of weed seed or of insects injurious to man. What per cent of this bird's food is injurious to man?

3. Out of every $7\frac{1}{2}$ lb. of food taken by phœbes, $8\frac{2}{3}$ oz. is vegetable. The rest consists chiefly of noxious insects, such as wasps and flies. What per cent of this bird's food consists of such insects?

4. In August, when grasshoppers and crickets are most harmful, 69% of a meadow lark's food is composed of these enemies of the farmer. Out of 10 lb. of food taken by these birds, how many ounces are of this kind?

5. Out of every 15 lb. of food taken by a kingbird, 1 lb. 8 oz. is vegetable, consisting almost entirely of wild fruits. The rest consists of insects injurious to man. What per cent of this bird's food is injurious to man?

6. Of 155 cuckoos observed for 3 hours, 46 black-billed cuckoos ate 1196 tree caterpillars and other pests, and 109 yellow-billed ones ate 2725. At this rate, the black-billed cuckoo destroys what per cent more than the yellow-billed?

7. A tree sparrow eats $\frac{1}{4}$ oz. of weed seed a day. Allowing 10 birds to 1 sq. mi., and supposing them to average 200 days a year in the state of Iowa, which has an area of 56,000 sq. mi., how many tons of weed seed are consumed a year by these birds alone?

Such reviews may be taken at the proper season of the year.

## MEASURES

**318. Capacity related to weight.** The laws of the various states usually specify the weight of a bushel, requiring it to weigh at least the amount specified below. This table should not be committed to memory, but used for reference.

| | | | |
|---|---|---|---|
| Corn (in the ear), | 70 lb. | Oats, | 32 lb. |
| Buckwheat, Barley, | 48 " | Dried Apples, | 26 " |
| Corn (shelled), Rye, | 56 " | Timothy Seed, | 45 " |
| Wheat, Clover Seed, Peas, | | Dried Peaches, | 33 " |
| Beans, White Potatoes, | 60 " | Sweet Potatoes, | 55 " |

### WRITTEN EXERCISE

1. How many tons to 1000 bu. of wheat? of oats?

2. How many bushels of white potatoes to the ton?

3. What number is to 12 as the bushel weight of sweet potatoes to that of white potatoes?

4. The weight of shelled corn is what per cent of the weight of the same amount of corn in the ear?

5. Barley is what per cent heavier than timothy seed? Timothy seed is what per cent lighter than barley?

6. What per cent more does 1 bu. of rye weigh than 1 bu. of barley? than 1 bu. of timothy seed? than 1 bu. of oats?

7. Clover seed is what per cent heavier than timothy seed? Timothy seed is what per cent lighter than clover?

8. In the above table it is possible to select four numbers so as to form a proportion. Do this, telling the products to which they refer.

9. State the ratios of the bushel weights of dried peaches to dried apples; of dried apples to dried peaches; of peas to beans; of sweet potatoes to white potatoes.

**319. Weight of water.** It is sometimes convenient to know that, practically, 1 cu. ft. of water weighs $62\frac{1}{2}$ lb.

### ORAL EXERCISE

1. What is the weight of 2 cu. ft. of water? of 10 cu. ft.?

2. What is the weight of $\frac{1}{2}$ cu. ft. of water? of $\frac{1}{10}$ cu. ft.?

3. What part of 1 cu. ft. of water weighs $12\frac{1}{2}$ lb.? 25 lb.?

4. Silver is about ten times as heavy as water. About how much does 1 cu. ft. of silver weigh?

### WRITTEN EXERCISE

1. What part of 1 oz. does 1 cu. in. of water weigh?

2. Water is half as heavy as sulphur. How much does 1 cu. ft. of sulphur weigh?

3. The ratio of the weight of cork to that of water is 6 : 25. How much does 1 cu. ft. of cork weigh?

4. A certain quality of gold is 19.3 times as heavy as water. How much does 1 cu. ft. weigh?

5. If silver is $10\frac{1}{2}$ times as heavy as water, how much is 1 cu. ft. of silver worth at 50 ct. an ounce (avoirdupois)?

6. If good milk is 3% heavier than water, what is the weight of a can of such milk containing 1 cu. ft., the can itself weighing 5 lb. 10 oz.?

7. Ice is 92% as heavy as water. I wish 100 lb. of ice to fill my refrigerator. The iceman sends me 2 cu. ft. He sends me how many pounds too much?

8. A man building a house wishes to put a tank for water in his attic. The tank is to be 5 ft. long, 4 ft. wide, and is to allow for $3\frac{1}{2}$ ft. depth of water. Allowing 600 lb. for the weight of the tank, how much will it weigh when full?

**320. Weights compared with weight of water.** We have found that 1 gal. contains 231 cu. in., and that 1 cu. ft. of water weighs nearly 62½ lb. Therefore

1 cu. in. of water weighs nearly $\frac{1}{1728}$ of 62½ lb.,
and 1 gal. " " " " 231 × $\frac{1}{1728}$ of 62½ lb.,
or nearly 8.3 lb., which is near enough for practical purposes.

### ORAL EXERCISE

1. How much do 2 gal. of water weigh? 10 gal.?

2. How much does ½ gal. of water weigh? 100 gal.?

3. If a 3-gal. pail weighs 3 lb., how much will it weigh when full of water? when half full?

### WRITTEN EXERCISE

1. How much does 1 bbl. (of 31½ gal.) of water weigh?

2. Milk being 3% heavier than water, how much does it weigh to the gallon? the quart? the pint?

3. Alcohol being 79% as heavy as water, how much does it weigh per gallon? per barrel of 31½ gal.?

4. Petroleum being 70% as heavy as water, how much does it weigh per gallon? per barrel of 31½ gal.?

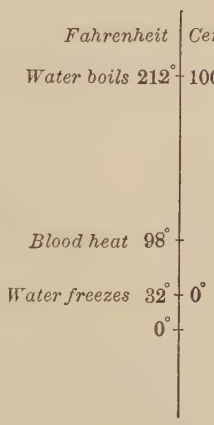
5. Copper being 8.95 times as heavy as water, how much does 1 cu. ft. of copper weigh? also 231 cu. in.?

6. Turpentine being 1% lighter than water, how much does it weigh to the gallon? the barrel of 31½ gal.?

7. A tank 7 ft. long by 5½ ft. wide is filled with water to the depth of 3 ft. How many gallons of water in it?

8. The water of the Dead Sea is so salty that the ratio of its weight to that of fresh water is 31 : 25. How much does 1 gal of Dead-Sea water weigh?

**321. Measuring temperature.** We commonly use a thermometer known as Fahrenheit's. In scientific work the centigrade (a word meaning "100 degrees") is used. Because there are just 100° between the freezing and boiling points of water, on the centigrade thermometer, and the freezing point is at 0°, that would be easier than the Fahrenheit to work with if we were used to it.




The exercises refer only to the Fahrenheit.

ORAL EXERCISE

1. When the temperature is 5° below 0, how many degrees is it below the freezing point of water?

2. How many degrees between the freezing and boiling points of water? between blood heat and the freezing point?

WRITTEN EXERCISE

1. In shipping meat from Chicago and Kansas City, refrigerator cars reduce the temperature 64° below animal (blood) heat. What temperature is used?

2. In freight cars fruits can be shipped short distances when the outside temperature is 25°. If the inside temperature is 9½° higher, how much is it above freezing?

3. The lowest temperature at which bananas should be shipped is 13½° above freezing, lemons 3° above freezing, cheese 20° below freezing. Express these temperatures in the ordinary way.

## Our Forests

**322. Our timber industry.** Our timber product is very great. Most states now have an Arbor Day to interest the people in preserving our forests.

### WRITTEN EXERCISE

1. If there are 672,000,000 acres of woodland in the United States, how many square miles are there?

2. If there are $600,000,000 invested in lumbering, and the annual product amounts to $566,520,000, this is what per cent of the amount invested?

3. If 283,260 men are engaged in the business, and are paid $116,136,600 annually, what are the average wages of each?

4. If these 283,260 men cut and saw $566,520,000 worth of lumber annually, what is the average per man?

5. The loss from forest fires annually is about $3\frac{1}{2}\%$ as much as the value of the lumber produced. If the value of the lumber is $566,520,000, how much is the fire loss?

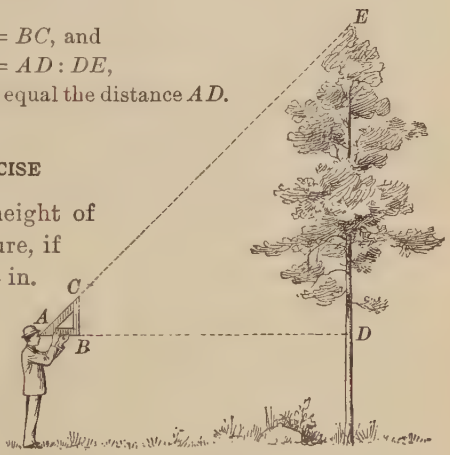
6. The average spruce tree supplies wood pulp for 500 lb. of paper. Taking the average weight of a book as 20 oz., how many trees had to be cut down to supply the paper for a million books?

**323. Measuring trees.** The height of a tree is often measured by proportion, as here shown.

This man holds a right-angled triangle in which $AB = BC$. He holds $AB$ level and walks towards the tree until he just sees the top along $AC$. Then because

$$AB = BC, \text{ and}$$
$$AB : BC = AD : DE,$$

the height *above* $D$ will equal the distance $AD$.




WRITTEN EXERCISE

1. What is the height of the tree in the picture, if the triangle is 5 ft. 4 in. from the ground, and $AD$ is 23 ft. 8 in.?

2. Suppose a triangle were used which had $AB =$ twice $BC$. What would be the height of the tree if $AD = 75$ ft., and if $D$ were 5 ft. 8 in. above the ground?

3. Suppose the triangle used had $AB = 2$ ft., $BC = 1$ ft. 6 in. What would be the height of the tree if $AD = 4$ rd., and if $D$ were 5 ft. 6 in. above the ground? if $AD = 61$ ft., and if $D$ were 5 ft. 3 in. above the ground?

4. What is the height of a tree that casts a shadow 88 ft. long at the same time that a man 5 ft. 9 in. high casts a shadow 7 ft. 8 in. long?

5. A boy whose eye is 5 ft. above the ground sights over the top of a 10-ft. pole and just sees the top of a tree. The boy is 6 ft. from the pole and 42 ft. from the tree. How high is the tree? Draw the picture.

**324. Board measure.** One foot of lumber, board measure (1 ft. B.M.), means a piece having an area of 1 sq. ft. on one surface, and being 1 in. or less in thickness.

A board 18′ long, 8″ wide, and 1″ or less thick contains $18 \times \frac{8}{12}$ ft. B.M., or 12′ B.M. A joist $12' \times 6'' \times 2\frac{1}{2}''$ (the sign × here meaning "by") contains $2\frac{1}{2} \times 12 \times \frac{1}{2}$ ft. B.M., or 15′ B.M.

### ORAL EXERCISE

*Find the number of feet B.M. in boards 1 in. or less in thickness, with dimensions as given:*

| | | | |
|---|---|---|---|
| 1. $8' \times 3''$. | 2. $16' \times 4''$. | 3. $12' \times 4''$. | 4. $8' \times 4''$. |
| 5. $16' \times 6''$. | 6. $10' \times 6''$. | 7. $16' \times 3''$. | 8. $12' \times 3''$. |
| 9. $18' \times 12''$. | 10. $16' \times 12''$. | 11. $10' \times 12''$. | 12. $18' \times 6''$ |

### WRITTEN EXERCISE

*Find the number of feet B.M., the thickness being 1″:*

| | | | |
|---|---|---|---|
| 1. $9' \times 18''$. | 2. $10' \times 7''$. | 3. $7' \times 16''$. | 4. $22' \times 9''$. |
| 5. $23' \times 10''$. | 6. $17' \times 13''$. | 7. $13' \times 9''$. | 8. $8' \times 18''$. |
| 9. $14' \times 10''$. | 10. $28' \times 14''$. | 11. $18' \times 14''$. | 12. $16' \times 11''$ |

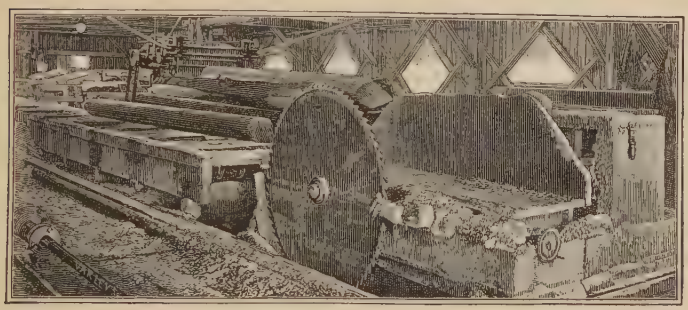
*Also in the scantling in Exs. 13–18, to the nearest inch:*

| | |
|---|---|
| 13. 6 ft. long, $1'' \times 2''$. | 14. 8 ft. long, $2'' \times 2''$. |
| 15. 8 ft. long, $2'' \times 3''$. | 16. 8 ft. long, $2'' \times 4''$. |
| 17. 10 ft. long, $2'' \times 5''$. | 18. 10 ft. long, $2'' \times 6''$. |

*Also in the planks in Exs. 19–24, to the nearest foot:*

| | |
|---|---|
| 19. 10 ft. long, $6'' \times 1\frac{1}{4}''$. | 20. 24 ft. long, $7'' \times 2''$. |
| 21. 16 ft. long, $9'' \times 1\frac{1}{4}''$. | 22. 12 ft. long, $7'' \times 1\frac{1}{4}''$. |
| 23. 16 ft. long, $15'' \times 1\frac{1}{2}''$. | 24. 14 ft. long, $10'' \times 1\frac{1}{4}''$. |

**325. Bills of lumber.** Lumber is sold at so much "per M," meaning per 1000 ft. B.M.



WRITTEN EXERCISE

1. A man wishes to build a house $28' \times 32'$. He needs 4 sills, each $6'' \times 8''$, to put under the walls. How much will they cost at \$18 per M?

2. If he buys 34 sticks of timber, $2'' \times 6''$ and $28'$ long, how much will these cost at \$18 per M?

3. If he buys 60 joists, $2'' \times 4''$ and $18'$ long, how much will these cost at \$18 per M?

4. How much will the flooring for the house cost, taking the measurement of each floor as $28' \times 32'$, and allowing for two floors, the lumber costing \$40 per M?

5. How many feet of siding are necessary for this house, supposing it to be $28' \times 32'$ and $18'$ high, the siding being $5''$ wide and laid $4''$ to the weather, no allowance being made for gables or doors and windows?

6. If he puts on a sloping roof, each slope being $34' \times 18'$, how much will the shingles cost at \$4.75 per 1000, allowing 1000 shingles to 125 sq. ft.?

### WRITTEN EXERCISE

1. How many feet (B.M.) of 2″ planking are needed for a barn floor 18′ × 25′?

2. Make out a bill for 15 planks 18 ft. long, 10″ × 2½″, and 12 joists 16 ft. long, 2″ × 4″, at $22 per M.

3. How many feet of lumber are needed for a floor 18′ × 14′, 1″ thick, and ⅛ of the floor area being added for matching?

4. An A-shaped roof is 25′ long, the rafters being 15′ long. Allowing 4½ shingles to the square foot, how many shingles are required to cover both slopes?

5. Not allowing for overlapping or for joists, how much lumber is required for a bin 8′ × 6′ and 5′ deep, open at the top, 1″ boards being used? What will it cost at $30 per M?

A man thinks of building this house. It is to be 34′ × 27½′. The walls are to be 14′ to the eaves, the ceilings in the first floor 9′ and in the second floor 8′. A builder estimates the cost of materials and labor. Part of the computation is given in Exs. 6–9; the rest is left for the class to work out.

6. Sills: 3 pieces, 6″ × 8″, 16′ long, 192′ B.M.
4 " 6″ × 8″, 14′ " (?)′ "
6 " 6″ × 8″, 12′ " (?)′ "
1 piece, 2″ × 8″, 14′ " 19′ "
1 " 2″ × 9″, 8′ " (?)′ "

| | | | | | | |
|---|---|---|---|---|---|---|
| 7. Joists: | 30 | pieces, | 2″ × 9″, | 16′ | long, (?)′ | B.M. |
| | 18 | " | 2″ × 8″, | 12′ | " (?)′ | " |
| | 13 | " | 2″ × 8″, | 6′ | " (?)′ | " |
| | 27 | " | 2″ × 8″, | 16′ | " (?)′ | " |
| | 27 | " | 2″ × 8″, | 12′ | " (?)′ | " |
| | 27 | " | 2″ × 6″, | 16′ | " (?)′ | " |
| 8. Studding, | 5 | " | 4″ × 6″, | 14′ | " (?)′ | " |
| etc.: | 100 | " | 2″ × 4″, | 14′ | " 933′ | " |
| | 35 | " | 2″ × 4″, | 10′ | " 233′ | " |
| | 25 | " | 2″ × 4″, | 18′ | " (?)′ | " |
| | 45 | " | 2″ × 5″, | 12′ | " (?)′ | " |
| | 50 | " | 2″ × 4″, | 8′ | " 267′ | " |
| | 50 | " | 2″ × 4″, | 14′ | " 467′ | " |
| | 40 | " | 2″ × 6″, | 18′ | " (?)′ | " |
| Bridging, etc.: | | | | | 175′ | " |
| Roof sheathing, etc.: | | | | | 875′ | " |

9. The total in Exs. 6–8 should be 7847′ B.M. How much will this lumber cost at $19 per M?

10. The man also needs 2000 ft. B.M. of siding @ $45 per M; 1950 ft. of flooring @ $22; 125 ft. of flooring for the porch @ $40; 1500 shingles for the gables @ $4.25 (that is, per 1000 shingles); 6200 shingles for the roof @ $6.50. What will this material cost?

11. He also needs 28 foundation posts @ 30 ct. each; water tables, cornice, corner boards, etc., $25; porch columns, $5; 16 doors at an average cost of $5.20 each; 16 windows at an average cost of $5.50 each; hardware, $40; chimney and plastering, $200; painting, $148; tin work, $15.50; the work of 4 carpenters for 30 days @ $3 a day each. What is the total cost of these items?

12. Allowing $75 for extras, what is the total cost of the house?

## Plastering

### ORAL EXERCISE

1. What is the perimeter of a room 14 ft. by 16 ft.?

2. If the room is 9 ft. high, what is the area of the four walls, not allowing for openings?

3. What is the area of the ceiling of a room 18 ft. by 20 ft.? How many square yards?

4. What will it cost to plaster 60 sq. yd. at 30 ct. a square yard? at 33 ct. a square yard? at 35 ct. a square yard?

5. A corridor is 30 ft. long, 4 ft. wide, and 9 ft. high (above the baseboard). How many square feet of plaster are needed for the two sides and top, not allowing for openings?

### WRITTEN EXERCISE

1. How many square yards of plastering in the walls and ceiling of a room 16 ft. by 20 ft., and 9 ft. high, deducting 14 sq. yd. for openings and baseboard?

2. How many square yards of plastering in the walls and ceiling of a hall 60 ft. long, 40 ft. wide, and 28 ft. high, deducting 36 sq. yd. for openings and baseboard?

3. What will it cost to plaster a schoolroom 32 ft. long, 18 ft. wide, and 12 ft. high, at 32 ct. a square yard, deducting 50% of the area of the walls on account of blackboards, baseboard, and openings?

4. On account of the danger of falling plaster, some school trustees decided to use pressed steel ceilings. There were 742 sq. yd. of ceiling in the building, and they let a contract for putting up the steel ceilings at 10 ct. a square foot. How much did the ceilings cost?

## Carpeting

### ORAL EXERCISE

1. How many strips of $\frac{3}{4}$ yd. carpet are needed for a room 18 ft. wide, not allowing for matching the pattern?

2. A carpet has an 18-in. pattern? Will there be any loss in matching if the strips are 15 ft. long? 14 ft. long?

### WRITTEN EXERCISE

1. How many yards of carpet, 9-in. pattern, are needed for a room 11 ft. 3 in. wide and 12 ft. long, the carpet being 27 in. wide? Draw a plan showing how the strips lie.

2. How many yards of carpet, $\frac{1}{2}$-yd. pattern, are needed for a room 13 ft. 6 in. wide and 16 ft. long, the carpet being 27 in. wide? How much is wasted on each strip (except the first, with which the others must match)? Draw a plan.

Show that no allowance is ever necessary for the first strip.

*Find how many yards of carpet are needed for the following, allowing for matching each strip (except the first), as indicated:*

3. 18 ft. by 20 ft., 27-in. carpet, allowing 9 in.
4. 16 ft. by 18 ft., 27-in. carpet, allowing $\frac{1}{2}$ yd.
5. 14 ft. by 15 ft., 36-in. carpet, allowing $\frac{1}{4}$ yd.

*Find the cost for carpeting these rooms with 27-in. carpet, allowing 4 inches on each strip (except the first), @ $1.20 a yard:*

6. 18 ft. by 15 ft.
7. 18 ft. by 17 ft. 6 in.
8. 14 ft. 3 in. by 18 ft.
9. $13\frac{1}{2}$ ft. by 16 ft. 3 in.
10. 12 ft. by 13 ft. 6 in.
11. 4 ft. 6 in. by 23 ft. 8 in.

**326. Papering.** The standard width for estimating wall paper is 18 in., although it varies greatly. A single roll is considered to contain 8 yd. of paper. In estimating, paper hangers usually count 3 rolls to 100 sq. ft. Fractional parts of rolls cannot be bought.

### ORAL EXERCISE

1. What will the border for a room 16 ft. by 20 ft. cost, at 25 ct. a yard?

2. Estimate the cost of the paper for a ceiling 15 ft. by 20 ft., at 40 ct. a roll.

3. A room has nearly 500 sq. ft. of wall to be papered. Estimate the cost of the paper at 50 ct. a roll.

4. A plain wall 14 ft. long and 9 ft. high above the baseboard is to be papered. How many rolls should you allow?

### WRITTEN EXERCISE

1. The walls of a room are 8 ft. 6 in. above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40 ct. a roll?

2. How many rolls of plain paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, the paper being 24 in. wide, allowing $1\frac{1}{2}$ rolls for windows and doors? At 35 ct. a roll, what will it cost?

3. The walls of a room are to be papered with plain paper worth 30 ct. a roll. They are 9 ft. high above the baseboard, and the total length of the four walls, allowing for windows and doors, is 60 ft. At the usual estimate, what will the paper cost?

The cost of papering rooms at home should be considered.

## Laying out a Garden and Grounds

### WRITTEN EXERCISE

1. The house described on page 236 was built on a lot 100 ft. wide and 200 ft. deep. The owner took a piece 40 ft. by 80 ft. for vegetables, and one 60 ft. by 80 ft. for fruit. How many square feet in each?

2. He planted a row of grapevines around the garden and four corners, 10 ft. apart. How many did he plant?

3. Across the fruit garden he set out 3 rows, 60 ft. long, of dwarf pears, 1 row of dwarf cherries, and 1 row of dwarf plums, putting the trees 10 ft. apart and 10 ft. from his neighbor's line. How many trees did he set out?

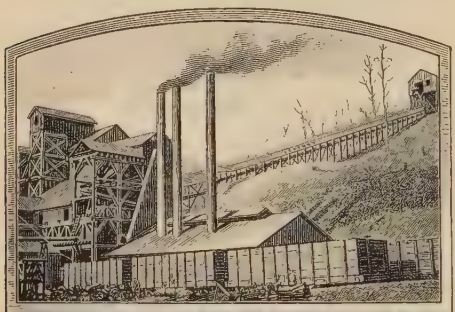
4. He set out a 60-ft. row of blackberry bushes, 3 ft. apart and 3 ft. from the fence; also 2 such rows of blackcaps and 2 of red raspberries. How many bushes did he set out?

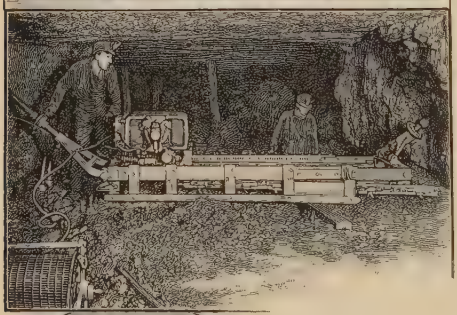
5. He planted a bed of asparagus 4 ft. wide and 36 ft. long; 2 beds of parsnips and 2 of beets, each $1\frac{1}{2}$ ft. wide and 36 ft. long; 2 beds of tomatoes, and 2 of squash, each 3 ft. wide and 36 ft. long. What area did he give to each?

6. He planted shrubs about the lawn. These, with the walks, occupied 2200 sq. ft. How many square feet were left for the lawn? See if you can improve this design.

## GENERAL BUSINESS APPLICATIONS

**327. Our mining interests.** We have frequently studied the greatest source of our national wealth, our farms.





Another source we have seen to be our forests. Still another important one is our mines. The first (raw) product of our mines was, at the beginning of the century, worth about $800,000,000 a year, the metals produced were worth about $500,000,000, and the petroleum products alone were worth over $70,000,000. We think of our gold output as valuable, but our copper is more so, our iron is three times as valuable, and our coal is worth four times as much.

The most valuable product of our mines is coal, from which coke and gas are made, and which furnishes the chief power for our manufacturing, for heating, and for general electricity.

### WRITTEN EXERCISE

1. A short ton of bituminous (soft) coal contains 76.05% of carbon and 2.32% of ash. How many pounds are there of each?

2. If 2000 lb. of anthracite (hard) coal contain 1828 lb. of carbon, how much will 100 lb. contain? What is the per cent of carbon?

3. We mined 268 million tons of coal in a certain year when Great Britain mined 252 million tons. Our output was what per cent greater than theirs?

4. If we manufacture $75,700,000 worth of gas a year, the coal and other material used costing 27% of this amount, how much does the material cost?

5. If Maryland produces 5.1 million tons of soft coal, worth at the mines $5,000,000, what is the value of Pennsylvania's output of 81.6 million tons, at the same average price per ton?

6. If we produce 226 million tons of bituminous coal, worth $1.05 a ton at the mines, and 30% as much anthracite, worth 60% more per ton, what is the total value of our annual coal product?

7. A Wilkesbarre (Pennsylvania) coal shaft is 1042 ft. deep, and the opening is 12 ft. by 51 ft. Supposing it to be rectangular, it represents the excavating of how many loads of earth, 1 cu. yd. to a load?

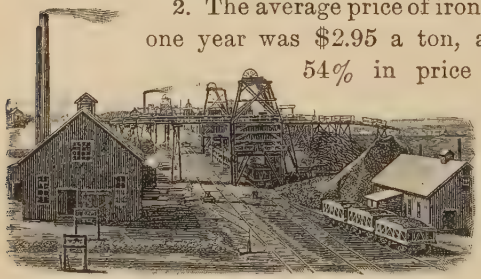
8. Near the bituminous mines are coke ovens where the coal is converted into coke by driving off the gas and moisture. We use 30 million tons of coal a year to make 19 million tons of coke. What is the loss per cent?

9. Of the 19 million tons of coke produced, Pennsylvania produced 67.4%. How many tons did she produce?

## Our Iron Mines

### WRITTEN EXERCISE

1. Iron ore is sold by the long ton. This is what per cent more than the short ton?



2. The average price of iron ore at the mines one year was \$2.95 a ton, and it decreased 54% in price the next year. What was then the average price?

3. In one year we made 2,106,500 kegs of cut nails, 100 lb. to the keg, and 300% more of wire nails. How many pounds of nails did we make in that year?

4. If we should make 2,100,000 short tons of steel rails a year, averaging 70 lb. to the yard, how many miles of rails would we make?

5. Iron melted from the ore is molded into short bars called pigs. Pig iron was worth \$16 a ton in a certain year. The next year it increased 10% in value, but the year following it decreased 10%. What was its value then?

6. At the opening of this century 39,758 wage earners in this country produced 14,432,154 tons of pig iron, while 10 years before that 33,415 men produced 8,854,975 tons. What was the average number of tons per man in each year, and the per cent of increase per man?

7. If we produced 17,160,632 tons of pig iron in one year, an increase of 13% on the year before, how many tons did we produce the year before?

**328. Some of our metals.** From the ores brought from our mines metals are extracted.

WRITTEN EXERCISE

1. When 700 lb. of copper was worth \$73.50, what was 250 lb. of the same quality worth?

2. A dealer sold 800 lb. of nickel at 55 ct. a pound, thus gaining 10% on the cost. How much did it cost him?

3. If a dealer buys 900 lb. of zinc at \$88 a short ton, at what rate per pound must he sell it to gain 50%?

4. In one year silver averaged 54.98 ct. an ounce, which was 102.8% of the average for the preceding year. What was the average then?

5. Silver is sold by the troy ounce. This is what per cent of the avoirdupois ounce (7000 gr. = 16 av. oz., 5760 gr. = 12 troy oz.)?

6. Quicksilver (mercury) is sold by the flask of 76½ lb. If a dealer buys it at \$38.25 a flask, at how much a pound must he sell it to gain 20%?

7. If a short ton of lead is worth \$96, how much is 750 lb. worth? If a dealer bought it at this rate and sold it at 8 ct. a pound, what was his per cent of profit?

## Manufactured Products

### WRITTEN EXERCISE

1. We made 50,168,000 gross of pins in a certain year, and 2% as many needles. How many needles and pins together did we make?

2. The value of the sewing machines manufactured in the first year of this century was \$21,603,200, or 143⅓% of the value ten years before. What was the value then?

3. If we manufactured 1,825,769 watch movements in a certain year, and sold them for \$2,756,911.19, and the dealers sold them at an average price of \$3.02 each, what per cent profit did the dealers make?

4. We made 144,875 typewriters in a certain year, worth on an average \$38.50 each. We sold 40% of these to other countries. How much did we receive from the foreign sales?

5. If we made 2775 locomotives in a certain year, averaging \$10,000 each, and sold 20% of them abroad, how much did we receive from our foreign trade and how many locomotives did we sell abroad?

6. If, when our population was 82,155,000, we used \$2,738,500 worth of pins and needles a year, what will they cost for a population of 100,000,000, at the same rate? How much does this average a person?

7. It was recently estimated that the farmers in this country owned \$800,000,000 worth of agricultural implements, and that if it were possible to get help to do the harvesting without machinery it would cost \$684,000,000 a year more than it did. This gain is what per cent of the cost of the implements?

## Some Problems of a Manufacturer

**329. Capital.** The money that a company has invested in such an enterprise as manufacturing is called its *capital.*

### WRITTEN EXERCISE

1. The capital of a certain company is $300,000, and its annual business is $3\frac{1}{2}$ times its capital. What is its annual business?

2. Its profits are 6% of its annual business. How much is this? This is what per cent of its capital?

3. The company carries $325,000 worth of insurance, paying $6 on each $1000 per year. What is the cost?

4. One of the articles manufactured is sold at retail for $3.50, the dealer making a profit of 40%. How much does the dealer pay for the article?

5. The manufacturer of the article mentioned in Ex. 4 pays for material $1.30, labor 45¢, expenses and advertising 45¢, selling 15¢. How much does he make on each article? What per cent?

6. On which will the manufacturer make the greater rate of profit, — an article which costs $2.35 to make, and which he sells at $2.50 less 2%, or one costing $1.84 and selling at $2 less $3\frac{1}{2}$%?

7. The company sells on 30 da. credit, or at 3% discount if paid within 10 da. About 60% of the amount of the sales are discounted. What is the rate of discount on the total sales?

8. The company pays one of its traveling salesmen $1500 a year, and allows him $1500 a year additional for expenses. He sells $75,000 worth of goods. His salary and expenses together are what per cent of his sales?

9. The buildings of the company are worth \$62,000, and the company allows 9% a year for repairs and depreciation in value. This means what annual expense?

10. In a year when the business amounted to \$954,000 the company invested $\frac{3}{4}$% of this sum in advertising in trade journals. How much did it pay for advertising?

11. The next year it invested \$51,350 in "national advertising" (in the popular magazines), which was 5% of the amount of its business for that year. What was the amount of its business? Taking the increase over the preceding year (see Ex. 10), did the advertising pay?

12. This company employs 200 girls at \$7.50 a week, 30 men at \$14.50 a week, 30 boys at \$5.75 a week. What is the weekly pay roll for this labor? How much does an increase of 10% in all the wages add to the weekly salary of one of the girls? to the total weekly pay roll?

13. The company employs great care in its sales, but its loss by poor debts last year, when its sales were \$1,125,000, was $1\frac{1}{3}$% of the sales. How many dollars did the company lose through poor debts?

14. The company employs four men on a certain kind of work, at 28 ct. an hour. The pay roll shows the number of hours each man works. Fill in this pay roll. How will you check the work?

| | Mon. | Tu. | Wed. | Th. | Fri. | Sat. | Wages | |
|---|---|---|---|---|---|---|---|---|
| E. Glover . . . . | 8 | 8 | 8 | 8 | 8 | 8 | | |
| J. Thornton . . . | 8 | 7 | 8 | $7\frac{3}{4}$ | 8 | 8 | | |
| O. M. Gooley . . . | $7\frac{1}{2}$ | $7\frac{1}{2}$ | 0 | 8 | 8 | 0 | | |
| Wm. Thompson . . | 4 | 8 | 8 | 8 | 8 | 4 | | |

## Some of Our Shipping Interests

### WRITTEN EXERCISE

1. The tonnage (weight) of the *Cedric* is 21,000 tons, which is $\frac{1}{17\frac{1}{4}}$ more than that of the *Celtic.* What is the tonnage of the *Celtic?*

2. If a steamer's hull is 40 ft. 4 in. deep and 700 ft. long, the depth is what per cent of the length? (Think of these dimensions compared with those of your school building.)

3. One of the steamship lines landed in New York on an average 1206 passengers per ship in a certain year, which was 90% of the average the year before. What was the average then?

4. One of the largest steamers will carry 80 barge loads of grain, at 8000 bu. to a barge. A freight car 40 ft. long will carry 1280 bu. What is the total length of a train carrying enough grain to load this steamer?

5. One of our American steamers pays $4760 in monthly wages to the 136 stokers who keep up the fires for its great engines. A German steamer pays $2864 to its 179 men who do this work. Each American stoker receives what per cent more monthly wages than the German?

**330. The produce buyer, the shipper, and the freight bills.** The word *produce* is commonly applied to the vegetable product of farms, as grain, hops, cabbages, and beans. If a man sends produce to some distant place, he is then a *shipper*. He may ship by cars or by boat. The buyer often puts produce in a large storage house, where it is kept so as not to be damaged, until he is ready to sell it. If he sells it delivered on the cars where he lives, he is said to sell *f.o.b.* (free on board).

WRITTEN EXERCISE

1. A produce buyer bought 250 bbl. of apples at $\$2.12\frac{1}{2}$ per barrel and sold them at a profit of $12\frac{1}{2}\%$. What did he gain?

2. A produce buyer bought 75 tons of cabbage at \$5.25 per ton. He paid 75 ct. per ton for storage and sold them at a profit of 15% on the total cost. What did he gain?

3. A produce buyer bought 50 tons of cabbage at \$5 per ton and put them in storage. He paid \$2 per ton for storage when he put them in. When he took them out he found that they had shrunk 25% while in storage. He then paid \$1 per ton for trimming and loading into cars, and sold them for \$10 per ton f.o.b. What was his gain or loss on the deal?

4. A buyer bought 1000 bbl. of apples in the fall, at \$2 per barrel. He decided to put them in storage for 3 months, which cost 40 ct. per barrel. At the end of that time he found that 6% of the apples had decayed. He then repacked them, paying 10 ct. per barrel of sound apples for repacking, and sold them at \$2.80 per barrel. What was his gain or loss on the deal?

5. A shipper sent 200 bbl. of fruit to a city, paying $2.30 per barrel f.o.b., and 20 ct. per cwt. (hundredweight) for freight. Allowing 160 lb. to the bbl., what did the shipment cost delivered at the city freight house?

6. A shipper's freight bills for 3 days were as follows:

| Freight to | Totals | May 3 | May 4 | May 5 |
|---|---|---|---|---|
| Columbus . . . . | $17.08 | $3.40 | $7.90 | $5.78 |
| Saint Joseph . . . | $x$ | 9.62 | 23.50 | 16.92 |
| Cortland . . . . | $x$ | 8.27 | 16.75 | 31.75 |
| Little River . . . | $x$ | 4.25 | 9.62 | 4.27 |
| | $y$ | $z$ | $z$ | $z$ |

Find the totals marked $x$, $y$, and $z$. The totals of the $z$'s should equal what other total?

7. A shipper in Wilson, New York, sends 27,642 lb. of cabbages to Memphis. The rate is 47¢ per 100. What freight does he pay? This freight goes over three roads, the Rome and Watertown, the Wabash, and the Illinois Central, and these roads share according to the distances they carry the freight. The Rome and Watertown gets 21.99%, the Wabash 58.16%, and the Illinois Central the rest. How much does each road get?

8. A shipper sends 150 bbl. of apples to Kansas City. He pays 44¢ per cwt. At 160 lb. to the barrel, what freight does he pay? The freight agent allows 40¢ per cwt. to the following roads: Lackawanna Railroad, 22.62%; Michigan Central, 49.18%; Rock Island, 28.20%; and allows Mississippi bridge tolls 3¢ per cwt. and terminal switch charges 20¢ per ton. How much of the total freight charge did the roads, the bridge, and the terminal station receive?

## Elementary Problems about Weather

### WRITTEN EXERCISE

1. What is the difference in temperature between 4° below zero and the freezing point of water?

2. The average January temperature in Washington is 33°, in New York it is 10% less, and in Norfolk, Virginia, it is 20% more. What is it in each of these places?

3. The coldest temperature recorded by our weather bureau is in Poplar River, Montana, 63° below zero, while the lowest recorded at Key West, Florida, is 41° above zero. What is the average of these two?

4. The highest temperature recorded at any station of our national weather bureau is 118°, at Yuma. Red Bluff, California, has come within 2.95% of it, and Boisé City, Idaho, within 7%. What is the highest temperature of these places?

5. The average velocity of the wind in Buffalo is 11 mi. per hour throughout the year. It is $18\frac{2}{11}\%$ less in Chicago, and is $22\frac{2}{9}\%$ less in Denver than in Chicago. What is the average velocity in each of these places?

6. The highest wind velocity ever known in San Francisco was 60 mi. per hour. Such a wind blows with a force of 17.712 lb. per square foot. What is its pressure on the side of a school building 100 ft. long and 40 ft. high?

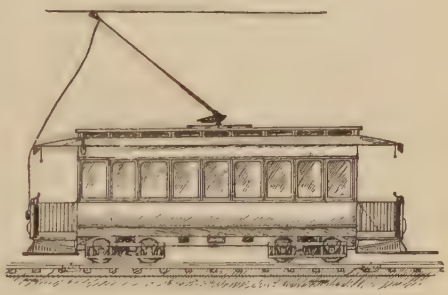
7. The average annual precipitation (rainfall) in Vicksburg is 55.7 in., in Nashville it is 5.7 in. less, in Yuma, Arizona, it is only 6% as much as in Nashville, and in Duluth it is 1000% as much as in Yuma. What is the precipitation in these various places?

It is often found to be helpful to keep records of the weather, illustrating the temperature by a curve as shown on page 121.

## Our Electricity Interests

### WRITTEN EXERCISE

1. If the number of miles of electric street railways increased in 12 years recently from 1250 to 22,500, what was the per cent of increase?



2. If the money invested in manufacturing electrical apparatus increased in the ratio of 2 to 9 in a certain ten years from \$19,000,000, how much was it at the end of the ten years?

3. If the value of the electrical apparatus produced in a certain year was \$96,000,000, an increase in the ratio of 5 to 24 in ten years, what was it ten years before?

4. If the cost of running street cars in a city is 19¢ per mile by horse power, $17\frac{3}{4}$¢ by cable, and 13¢ by electricity, what per cent cheaper is it to run them by electricity than by horse power? than by cable?

5. If the average number of telephone calls per day in this country was 9,387,600 in a certain year, an increase of 20% over the preceding year, what was the total number of calls for all of the preceding year?

6. If it is 25,000 mi. around the earth, what was the ratio of the distance around the earth to the length of wire of the Bell Telephone Company when this company had 3,250,000 miles of wire in its system?

## MISCELLANEOUS PROBLEMS

### ORAL EXERCISE

1. 15 is $\frac{1}{2}$ of what number? $\frac{1}{4}$ of what number?

2. 25 is $\frac{1}{3}$ of what number? $\frac{5}{7}$ of what number?

3. Express as decimals: $\frac{2}{5}$, $\frac{3}{4}$, $\frac{1}{25}$, $\frac{3}{50}$, $\frac{7}{25}$, $\frac{9}{50}$, $\frac{21}{50}$.

4. Express as common fractions: 0.5, 0.6, 0.25, 0.40.

5. Express as an integer and a common fraction: $1.33\frac{1}{3}$, $3.66\frac{2}{3}$, 5.75, 17.400, 32.125.

6. State the sums: $0.7 + 0.8$, $0.6 + 0.4$, $0.9 + 0.7$, $1.6 + 2.4$, $3.8 + 0.2$, $5.9 + 4.1$, $3.25 + 6.75$.

7. Multiply: $0.5 \times 0.5$, $4 \times 0.5$, $0.3 \times 6$, $0.25 \times 4$, $0.01 \times 50$, $7 \times 0.7$, $0.01 \times 0.01$, $0.05 \times 0.09$.

8. Divide: $2.5 \div 5$, $3.5 \div 7$, $0.49 \div 7$, $6.4 \div 8$, $0.72 \div 9$, $1.44 \div 12$, $14.4 \div 12$, $12.5 \div 5$, $1.25 \div 5$.

9. A foot is what decimal fraction of a yard? A quart is what decimal part of a gallon? of a peck?

10. Reduce 2 ft. 8 in. to inches; 2 lb. 8 oz. to ounces; 2 dollars and 8 cents to cents; 2 min. 8 sec. to seconds.

11. Reduce 36 in. to feet; 19 in. to feet and inches; 42 in. to yards and inches; 40 oz. to pounds and ounces.

12. State the sums: 3 ft. 8 in. + 4 ft. 4 in.; 2 ft. 9 in. + 3 ft. 9 in.; 3 yd. 20 in. + 5 yd. 20 in.; 7 lb. 10 oz. + 2 lb. 6 oz.; 4 bu. 2 pk. + 8 bu. 3 pk.

13. Subtract: 8 yd. 4 in. − 1 yd. 30 in.; 2 lb. 10 oz. − 15 oz.; 4 bu. 2 pk. − 3 pk.; 5 rd. 2 ft. − $3\frac{1}{2}$ ft.; 2 yd. 1 ft. − 1 yd. 2 ft.; 5 rd. $6\frac{1}{2}$ ft. − 2 rd. 13 ft.

14. Multiply: 2 ft. 6 in. by 2; 3 ft. 4 in. by 3; 4 ft. 3 in. by 4; 1 ft. 8 in. by 2; 2 lb. 2 oz. by 8; 1 lb. 4 oz. by 5.

15. Divide: 8 lb. 4 oz. by 4; 10 lb. 10 oz. by 5; 9 ft. 6 in. by 3; 1 lb. 10 oz. by 2; 1 lb. 8 oz. by 8; 3 yd. 4 in. by 10.

### WRITTEN EXERCISE

*First estimate the answer and write it down; then solve the problem and compare the answers* (*see page 88*):

1. $\frac{5}{7} + \frac{16}{23}$.
2. $\frac{7}{8} + \frac{12}{13}$.
3. $\frac{6}{11} + \frac{17}{21}$.
4. $\frac{12}{17} + \frac{13}{20}$.
5. $\frac{11}{21} + \frac{13}{23}$.
6. $\frac{19}{23} - \frac{11}{20}$.
7. $\frac{16}{33} - \frac{5}{21}$.
8. $\frac{32}{36} - \frac{7}{15}$.
9. $\frac{27}{28} - \frac{13}{30}$.
10. At 23¢ a yard, what will $8\frac{3}{4}$ yd. of ribbon cost?
11. At 18¢ a pound, what will $7\frac{3}{4}$ lb. of meat cost?
12. At $12\frac{1}{2}$¢ a yard, what will 18 yd. of cotton cost?
13. At \$1.30 a yard, what will 27 yd. of cheviot cost?
14. At \$2.30 a yard, what will $15\frac{1}{2}$ yd. of velvet cost?
15. At 24¢ a yard, what will $14\frac{3}{4}$ yd. of gingham cost?
16. If $7\frac{3}{4}$ yd. of velvet cost \$12.40, what is the price per yard?
17. If a dealer pays \$18.75 for 150 lb. of figs, what does he pay per pound?
18. A woman buys $3\frac{1}{4}$ lb. of coffee for \$1.04. What does she pay per pound?
19. A train of cars goes 63.9 mi. in an hour and a half. What is its rate per hour?
20. A man pays \$216 a year for the rent of his house. What is the rate per month?
21. A laundryman pays \$5.40 for 160 cakes of soap. What does he pay per cake?
22. A dealer pays \$33 a dozen for hats. At this rate, what does he pay for 50 hats?
23. If a dealer pays 28¢ a pound for a certain grade of cinnamon, at what price per ounce must he sell it to gain 0.75¢ an ounce?

### ORAL EXERCISE

*Express as per cents:*

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{3}{4}$. 4. $\frac{1}{8}$. 5. $\frac{3}{8}$. 6. $\frac{5}{8}$.

7. $\frac{1}{16}$. 8. $\frac{3}{16}$. 9. $\frac{5}{16}$. 10. $\frac{4}{5}$. 11. $\frac{7}{25}$. 12. $\frac{9}{50}$.

*Express as common fractions:*

13. 25%. 14. $33\frac{1}{3}$%. 15. 20%.

16. $12\frac{1}{2}$%. 17. $16\frac{2}{3}$%. 18. $8\frac{1}{3}$%.

19. How much is $16\frac{2}{3}$% of 6? of 54? of 48? of 60?

20. \$15 is what per cent of \$45? of \$75? of \$150?

21. A quart is what per cent of a peck? of a gallon?

22. How much is $66\frac{2}{3}$% of 3? of 60? of 90? of 120?

23. How much is $6\frac{1}{4}$% of 16? of 32? of 64? of 160?

24. How much is $33\frac{1}{3}$% of 9? of 3? of 675? of 1233?

25. How much is 50% of 96? of 120? of 480? of 15?

26. How much is 25% of 60? of 88? of 500? of 1200?

27. How much is $12\frac{1}{2}$% of 40? of 64? of 888? of 16?

28. A man pays \$2.50 a year interest on \$50. What is the rate?

29. A school having 240 pupils graduated 30. What per cent were graduated?

30. A merchant gained \$360 on \$3000 worth of goods. What was his rate of gain?

31. A man pays \$24 interest for 6 mo. on a loan of \$1200. What is the rate per year?

32. How much is $8\frac{1}{3}$% of 120? of 240? of 600? of 2400? of 4800? of 6000? of 12,000?

33. 25 is 1% of what number? 10% of what number? 50% of what number? $33\frac{1}{3}$% of what number?

### WRITTEN EXERCISE

1. $\frac{22}{23} \times \frac{161}{154}$.
2. $\frac{33}{35} \times \frac{63}{55}$.
3. $\frac{18}{17} \times \frac{102}{126}$.
4. $\frac{28}{29} \times \frac{261}{196}$.
5. $\frac{42}{43} \times \frac{344}{189}$.
6. $\frac{13}{44} \times \frac{121}{117}$.
7. $\frac{52}{53} \div \frac{364}{371}$.
8. $\frac{23}{40} \div \frac{161}{104}$.
9. $\frac{15}{37} \div \frac{105}{185}$.
10. $\frac{27}{32} \div \frac{135}{128}$.
11. $\frac{31}{41} \div \frac{62}{123}$.
12. $\frac{51}{64} \div \frac{17}{128}$.
13. $\frac{21}{22} + \frac{5}{11} + \frac{3}{44}$.
14. $\frac{32}{35} + \frac{1}{5} + \frac{5}{7}$.
15. $\frac{8}{9} + \frac{15}{16} + 1\frac{51}{144}$.

16. On a shipment of 525 bbl. of apples 4% were frozen. How many were not frozen?

17. A bookseller pays 50¢ less $\frac{1}{6}$ for a book, and sells it for 50¢. What is his per cent of profit?

18. A farmer sold 3 cows for $110.40, which was 8% less than they cost. What was the cost of each?

19. A boy was sick and missed 5% of the 180 school days of a year. How many days was he in school?

20. A man worked 320 days last year, at $3, and put 12% of his wages in the bank. How much did he put in the bank?

21. A dealer sold a suit of clothes for $4.50 more than it cost, thus making a profit of 25%. At what price did he sell the clothes?

22. A dealer sold an automobile for $90 less than the marked price, thereby throwing off 12%. What is the price at which he sold the automobile?

23. An agent charges a commission of $2\frac{1}{2}$% for selling some property, and thereby receives $36.25. At what price did he sell the property? What did he remit to his employer after deducting his commission?

24. A dealer bought a suit of clothes for $20 and sold it for $25. The suit cost the manufacturer $15. What was the manufacturer's rate of profit? What was the dealer's rate of gain on what he paid? on what the customer paid?

### ORAL EXERCISE

1. How many cubic feet in a room 20 ft. by 15 ft. by 10 ft.?

2. A rectangle is 6 ft. wide and $33\frac{1}{3}\%$ longer. What is the perimeter?

3. A dealer buys \$126 worth of books at $\frac{1}{6}$ discount. What does he pay?

4. A school buys \$12 worth of maps at 20% discount. What do the maps cost?

5. After spending 10% of his money a man has \$45. How much had he at first?

6. After gaining 5% on his money a man had \$210. How much had he at first?

7. After selling 25% of his papers a newsboy had 27. How many had he at first?

8. At a bargain sale a woman buys \$18 worth of dry goods at $\frac{1}{3}$ off. What does she pay?

9. The width of a parallelogram is $33\frac{1}{3}\%$ of the length. The length is 21 ft. What is the area?

10. John has gained 10% in height this year, and is now 4 ft. 7 in. tall. How tall was he a year ago?

11. The base of a triangle is 50% greater than the altitude. The altitude is 20 in. What is the area?

12. One of the two equal sides of an isosceles triangle is 20 in., and the base is 25% of the sum of the equal sides. What is the perimeter?

13. A square and an equilateral triangle have equal perimeters, 120 in. The side of the square is what per cent of the side of the triangle?

### WRITTEN EXERCISE

1. $3\frac{3}{3} \times 2\frac{7}{8}$.
2. $4\frac{1}{9} \times 3\frac{5}{7}$.
3. $2\frac{4}{5} \times 3\frac{7}{8}$.
4. $6\frac{3}{4} \times 5\frac{3}{8}$.
5. $4\frac{7}{9} \times 3\frac{2}{3}$.
6. $5\frac{3}{16} \times 2\frac{3}{8}$.
7. $16\frac{5}{8} \div 2\frac{3}{8}$.
8. $20\frac{4}{5} \div 3\frac{4}{5}$.
9. $16\frac{5}{7} \div 5\frac{4}{7}$.
10. $13\frac{11}{16} \div 4\frac{9}{16}$.
11. $22\frac{3}{5} \div 5\frac{13}{20}$.
12. $17\frac{1}{4} \div 2\frac{7}{8}$.

13. A bale of worsted weighing 75 lb. loses 8 oz. in reeling off. What is the per cent of loss?

14. It gains 0.45 lb. to the 75-lb. bale in dyeing. What is the per cent of gain?

15. It loses 12 oz. on the machines in being made into Astrakhan cloth. This is what per cent of the original weight of the bale?

16. This 75-lb. bale cost \$50.25, and it lost 4 oz. in the fulling mill. What was the value of the part lost?

17. The net loss (Exs. 13–16) is what per cent of the original weight? What is its value at 67¢ a pound?

18. A yard of this cloth contains 2 lb. of worsted at 67¢ a pound and $1\frac{1}{4}$ lb. of cotton at 18¢ a pound. Each is what per cent of the total cost of materials?

19. To the cost of materials in Exs. 16, 18, add 3¢ a yard for knitting the cloth, $2\frac{1}{2}$¢ a yard for dyes, $3\frac{1}{2}$¢ a yard for dyeing, and 20% of the cost of raw materials for factory expense, and find the total cost per yard.

20. A certain grade of this cloth costs \$1.48 a yard. It sells for \$2 less 10%. The salesman is allowed 5% on the list price. What profit will the factory make on 5000 yd.?

21. 75 lb. of worsted is made into Astrakhan cloth, at 2 lb. to the yard. In fulling, this cloth stretches 5 yd., and shrinks from 110 in. in width to 55 in. Does the piece gain or lose in area, and how much? What per cent?

### ORAL EXERCISE

1. At $31\frac{1}{2}$ gal. to the barrel, how many barrels in 126 gal.? in 315 gal.? in 630 gal.?

2. At $4\frac{1}{7}$ cu. ft. to the barrel, how many barrels in 29 cu. ft.? in $8\frac{2}{7}$ cu. ft.? in $20\frac{5}{7}$ cu. ft.?

3. At $24\frac{3}{4}$ cu. ft. to the perch, how many perches in 99 cu. ft.? in $49\frac{1}{2}$ cu. ft.? in 2375 cu. ft.?

4. At 231 cu. in. to the gallon, how many gallons in 924 cu. in.? in 462 cu. in.? in 693 cu. in.?

5. At $5.60 a ton, how much will 10 T. of coal cost? $\frac{1}{8}$ T.? $\frac{3}{8}$ T.? $10\frac{3}{8}$ T.? 10 T. 750 lb.? 2 T.? 20 T.?

6. At $31\frac{1}{2}$ gal. to the barrel, how many gallons in a 50-barrel water tank? in a 100-barrel water tank?

7. A vase contains 264 cu. in. of water. What fraction more than a gallon does it contain?

8. Allowing 35 cu. ft. to a ton of coal, how many tons will fill a bin containing 315 cu. ft.?

9. Allowing 22 bricks to the cubic foot, how many bricks in a wall containing 5000 cu. ft.?

10. An oat bin contains 50 cu. ft. How many bushels does it contain at $1\frac{1}{4}$ cu. ft. to the bushel?

11. A pile of wood is 8 ft. high, 4 ft. wide, and 16 ft. long. How many cords does it contain?

12. Allowing 500 cu. ft. of timothy hay to a ton, how many tons will fill a haymow containing 10,500 cu. ft.?

13. A cellar 21 ft. by 15 ft. is to be excavated to a depth of 6 ft. What will it cost at 50¢ a load (cubic yard)?

14. A wall is 60 ft. long, 5 ft. high, and 1 ft. thick. How many bricks does it contain at 22 to the cubic foot?

### WRITTEN EXERCISE

1. $42\%$ of $\frac{3}{14}$. 2. $24\%$ of $\frac{25}{36}$. 3. $36\%$ of $\frac{75}{81}$.
4. $56\%$ of $\frac{25}{48}$. 5. $64\%$ of $\frac{25}{32}$. 6. $84\%$ of $\frac{25}{49}$.

7. If $\frac{7}{8}$ yd. of velvet costs \$2.80, what do $6\frac{1}{4}$ yd. cost?

8. A dealer paid \$341.25 for 10,500 ft. of lumber. What was the price per M?

9. How many bushels in a bin 7 ft. 6 in. long, 6 ft. 6 in. wide, and 5 ft. deep? (Take 1 bu. = 2150 cu. in.)

10. What is the number which must be subtracted from $7\frac{3}{16}$ in order to leave a remainder of $4\frac{25}{48}$?

11. How much will $7\frac{1}{8}$ mi. of telegraph wire cost at $33\frac{1}{3}¢$ a pound, allowing 1 lb. of wire to 22 ft.?

12. A dealer sold 72 barrels of flour at \$8.50 a barrel, thereby gaining \$43.20. What did each barrel cost him?

13. A farmer sold $\frac{3}{8}$ of his 152-acre farm, and afterwards sold 19 A. The part that remained was what per cent of the farm?

14. A man spends $\frac{3}{5}$ of his income for board and clothes, $\frac{1}{8}$ of it for other expenses, and saves \$330 a year. What is his income?

15. A farmer weighed the hay from $4\frac{1}{2}$ A. of a 32-acre meadow and found he had 11 T. 500 lb. At this rate, how many tons did he get from the meadow?

16. In taking an inventory a dry goods merchant finds that he has $6\frac{1}{4}$ pieces of pongee, $48\frac{1}{2}$ yd. in each piece, which cost him $37\frac{1}{2}$ ct. a yard. What is the total value?

17. How much will it cost to dig a ditch for a water main extending 15 city blocks, the ditch being 6 ft. deep and 2 ft. wide, at $50¢$ a cubic yard, the blocks running 20 to the mile?

### ORAL EXERCISE

1. If 4 collars cost 90¢, what will 6 cost?

The pupil should think, "Half as much more." Many such examples admit of these short methods.

2. If 7 pencils cost 21¢, what will 8 cost?
3. If 4 arithmetics cost $2.40, what will 5 cost?
4. If 3 lamp chimneys cost 39¢, what will 2 cost?
5. If 4 lb. of butter cost 88¢, what will 9 lb. cost?
6. If 2 yd. of cloth cost 62¢, what will 10 yd. cost?
7. If 3 pairs of cuffs cost 90¢, what will 5 pairs cost?
8. If 5 reams of paper cost 90¢, what will 3 reams cost?
9. If chairs cost $18 per half dozen, what will 7 cost?
10. If 3 yd. of carpet cost $1.56, what will 20 yd. cost?
11. If 3 tablets cost 24¢, how many can be bought for 40¢?
12. If a 6-lb. roast of beef cost $1.08, what will a 9-lb. roast cost?
13. If the length of 2 meters is 2.2 yd., how many yards in 7 meters?
14. A dealer pays $45 for 3 suits of clothes. What is the price per dozen?
15. At $15 per half dozen, what will a dozen and a half of fountain pens cost?
16. If an express train makes 90 mi. in 2 hours, how long will it take it to make 135 mi.?
17. If 4 men can lay some stone pavement in 5 days, how long will it take 10 men to lay it?
18. If 8 men can do a piece of work in 10 days, how many men will it take to do it in 16 days?

### WRITTEN EXERCISE

1. A dealer sells for \$238 some goods that cost him \$59.50 less than this. What was his rate of gain on the cost?

\$59.50 is the required per cent of what number?

2. A dealer sells for \$2451 some goods that were marked \$129 more than this. What rate of discount did he allow on the marked price?

\$129 is the required per cent of what number?

3. A grocer sold some canned goods at a profit of 20%, making \$70. What was the selling price?

4. A merchant paid \$232.50 for some goods after a discount of 7% had been allowed. What was the marked price?

\$232.50 is 93% of what number?

5. The total cost of a house, including the architect's commission of 5%, was \$7644. How much was the commission?

6. After deducting his commission of 6% for selling some property, an auctioneer remitted \$258.50. How much was the commission?

7. The sum of the principal and the interest at 5% on a note for a year is \$246.75. What is the interest?

\$246.75 is 105% of what number?

8. A man's salary this year is \$935, an increase of 10%. What was it last year?

9. A number is increased 10%, and the result is decreased 20%. There remains 220. What is the number?

10. A man saved \$427.50 this year, which is 10% less than he saved last year. What did he save last year?

11. If a man pays \$196 for 28 weeks' board, what will he pay for 37 weeks?

12. If the interest on \$255 is \$7.65 a year, what is the interest on \$385 at the same rate?

13. If it costs \$140 to insure some property for \$10,000, what will it cost to insure it for \$7500?

14. If the area of a certain field is 90 A., what is the area of one of exactly the same shape, but half as long?

If it is half as long, what about the width? the area?

15. If the interest on a certain sum for 2 yr. is \$38, what is the interest on twice that sum for 6 mo. at the same rate?

16. If the freight on some boxes is \$2.80 for a distance of 150 mi., what is it on the same boxes for 250 mi. at the same rate?

17. If \$2.50 pays the freight on 750 lb. for 450 mi., for how many miles will it pay the freight on half a ton at the same rate?

18. An agent has \$3248 to invest after deducting his commission of $1\frac{1}{2}\%$ on the amount invested. What does he invest?

19. If a 3-ft. post casts a shadow 4 ft. long at the same time that a church spire casts a shadow 116 ft. 8 in., how high is the spire?

20. A building known to be 90 ft. high casts a shadow that just reaches across the street. At the same time a boy 4 ft. 6 in. high finds that his shadow is 6 ft. long. How wide is the street?

21. A dealer has a balance of \$2815.20 due him from his agent. Instead of taking the money he instructs the agent to invest it in certain goods, deducting his commission of 2% on the amount invested. What is the commission?

# TABLES FOR REFERENCE

## *Length*

**12 inches (in.) = 1 foot (ft.).**
**3 feet = 1 yard (yd.).**
**$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.).**
**320 rods, or 5280 feet = 1 mile (mi.).**
4 inches = 1 hand.
6 feet = 1 fathom.
120 fathoms = 1 cable length.
1.15 common miles = 1 knot (nautical mile).

## *Square Measure*

**144 square inches (sq. in.) = 1 square foot (sq. ft.).**
**9 square feet = 1 square yard (sq. yd.).**
**$30\frac{1}{4}$ square yards = 1 square rod (sq. rd).**
**160 square rods = 1 acre (A.).**
**640 acres = 1 square mile (sq. mi.).**

## *Cubic Measure*

**1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).**
**27 cubic feet = 1 cubic yard (cu. yd.).**
**128 cubic feet = 1 cord (cd.).**
$24\frac{3}{4}$ cubic feet = 1 perch (stone, etc.).

## *Weight*

**16 ounces (oz.) = 1 pound (lb.).**
**2000 pounds = 1 ton (T.).**
100 pounds = 1 hundredweight (cwt.).
112 pounds = 1 long hundredweight.
2240 pounds = 1 long ton.

## *Troy Weight*

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The avoirdupois pound contains 7000 gr., the Troy pound 5760 gr.

3.2 grains = 1 carat (diamond weight).

But "16 carats fine" means $\frac{16}{24}$ pure gold.

### *Apothecaries' Weight*

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).
12 ounces = 1 pound (lb.).

### *Liquid Measure*

**4 gills (gi.) = 1 pint (pt.).**
**2 pints = 1 quart (qt.).**
**4 quarts = 1 gallon (gal.).**
231 cubic inches = 1 gallon.
31.5 gallons = 1 barrel (bbl.) (varies).
2 barrels = 1 hogshead (varies).
16 fluid ounces = 1 pint (apothecaries').
57.75 cubic inches = 1 liquid quart.

### *Dry Measure*

**2 pints (pt.) = 1 quart (qt.).**
**8 quarts = 1 peck (pk.).**
**4 pecks = 1 bushel (bu.).**
2150.42 cubic inches = 1 bushel.
67.2 cubic inches = 1 dry quart.

### *Time*

**60 seconds (sec.) = 1 minute (min.).**
**60 minutes = 1 hour (hr.).**
**24 hours = 1 day (da.).**
**7 days = 1 week (wk.).**
**12 months (mo.) = 1 year (yr.).**

### *Value*

**10 mills = 1 cent (ct. or ¢).**
**10 cents = 1 dime (d.).**
**10 dimes = 1 dollar ($).**
**10 dollars = 1 eagle (E.).**

### *Angles and Arcs*

**60 seconds (60″) = 1 minute (1′).**
**60 minutes = 1 degree (1°).**
90 degrees = 1 right angle.
360 degrees = 1 circumference.

### *Counting*

**12 units = 1 dozen.**
**12 dozen, or 144 = 1 gross (gr.).**
12 gross, or 1728 = 1 great gross.
20 units = 1 score.

### *Paper*

**24 sheets = 1 quire.**
**20 quires = 480 sheets = 1 ream.**

Now, for convenience in counting, 500 sheets are more often called a *ream;* and the word *quire* is used only for folded note paper, other paper being usually sold by the pound.

# DEFINITIONS OF THE MOST COMMON TERMS USED IN ARITHMETIC

**Abstract number.** A number that does not refer to any particular kind of object or measure is called an *abstract number.* All *numbers* are, however, essentially abstract, whether the kind of thing numbered is mentioned or not.

**Addends.** Numbers to be added are called *addends.*

**Addition.** The process of finding a number that equals two or more numbers taken together is called *addition.*

**Aliquot part.** A number which is contained in a given number an integral number of times is called an *aliquot part* of that number. Thus $12\frac{1}{2}$ is an aliquot part of 100.

**Amount of note.** The sum of the principal (or face) and interest of a note is called the *amount.*

**Analysis.** In arithmetic the process of reasoning to find out a truth is called *analysis.*

**Antecedent.** The first term of a ratio is called the *antecedent.*

**Arc.** A portion of a circumference is called an *arc.*

**Area.** The number of square units contained in a surface is called its *area.*

**Arithmetic.** The science that treats of numbers and their applications is called *arithmetic.*

**Assets.** The property of an individual or corporation is called the *assets.*

**Average.** The sum of $n$ given numbers, divided by $n$, is called their *average.* Thus the average of three numbers is $\frac{1}{3}$ of their sum.

**Bank.** An establishment for the transaction of the business of loaning sums of money, receiving money on deposit, and exchange, is called a *bank.*

**Bank discount.** The interest paid in advance on money borrowed at a bank is called *bank discount.*

**Base.** The number of which some per cent is to be found is called the *base.* The line on which a figure appears to stand is called the *base.*

**Bill.** A written statement of goods sold or of work done is called a *bill.*

**Bill of exchange.** A foreign draft is often called a *bill of exchange.*

**Bond.** A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond.*

**Cancellation.** Dividing both terms of a fraction by the same factor is called *cancellation.* The factor is then said to be *canceled.*

**Capacity.** The measure of a receptacle, indicating the number of units it contains, is called the *capacity.*

**Capital.** The money contributed by the stockholders to maintain a corporation is called its *capital* or *capital stock.*

**Check.** An order on a bank to pay money is called a *check.* An operation that tends to verify another one is called a *check.*

**Circle.** A figure bounded by a curve, every point of which is equidistant from a point within called the *center,* is called a *circle.*

**Circumference.** The bounding line of a circle is called its *circumference.* In advanced mathematics *circle* is used for *circumference.*

**Commission.** The percentage charged by an agent for transacting business is called *commission.*

**Common denominator.** A denominator common to two or more fractions is called a *common denominator* of the fractions.

**Common divisor.** A factor that is common to two or more numbers is called a *common divisor* or *common factor.*

**Common factor.** *See* Common Divisor.

**Common fraction.** A fraction having both terms fully expressed by figures is called a *common fraction.*

**Common multiple.** A multiple of each of two or more numbers is called a *common multiple* of the numbers.

**Complex fraction.** A fraction that has a fraction in either numerator or denominator, or both, is called a *complex fraction.*

**Composite number.** A number not prime is called a *composite number.*

**Compound fraction.** An expressed multiplication of an integer or a fraction by a fraction is called a *compound fraction;* for example, $\frac{2}{3}$ of 7 or $\frac{2}{3}$ of $\frac{4}{5}$.

**Compound interest.** If interest is not paid when due, but is added to the principal to form a new principal, and interest is reckoned on this amount, the obligation is said to draw *compound interest.*

**Compound numbers.** Denominate numbers in which two or more units are expressed are called *compound numbers.*

**Concrete number.** A number that refers to some particular kind of object or measure is called a *concrete number.*

**Consequent.** The second term of a ratio is called the *consequent.*

**Consignee.** The person, firm, or corporation to which goods are sent (consigned) is called a *consignee.*

**Consignor.** The one who sends (consigns) goods to another is called a *consignor.*

**Corporation.** A number of individuals organized by law to act as one body is called a *corporation;* also called a *company.*

**Coupon.** A note attached to a bond, promising to pay the regular interest, to be cut off and presented for payment when due, is called a *coupon.*

**Creditor.** A person to whom another owes money is called his *creditor.*

**Cube.** A regular solid with six equal squares as faces is called a *cube.* The third power of a number is called its *cube.*

**Cube root.** One of the three equal factors of a number is called its *cube root.*

**Customhouse.** The place where government duties on imported goods are received is called a *customhouse.*

**Debtor.** A person who owes money to another is called his *debtor.*

**Decimal fraction.** A fraction whose denominator is not written, but is 10 or some power of 10, is called a *decimal fraction.*

**Decimal point.** A period written before tenths in a decimal fraction is called a *decimal point.*

**Decimal system.** A system of numbers based upon the powers of ten is called a *decimal system;* for example, our common system of numbers and the metric system.

**Denominate numbers.** Concrete numbers denoting measure (including weight) are called *denominate numbers.*

**Denominator.** The number which shows into how many equal parts a unit has been divided is called the *denominator* of the fraction, — in a common fraction, the number below the line.

**Difference.** The number which added to one number makes another is called the *difference* between them. It is the result in taking one number from another.

**Discount.** A deduction made on a price or amount is called a *discount.* The difference between the face or par value and a less market value is also called a *discount,* as when stocks sell at 10% discount. Interest paid in advance on a note is called *discount.*

**Dividend.** The number to be divided is called the *dividend.* It is the given product of the quotient and divisor.

**Dividends.** The shares of the earnings divided at intervals among the stockholders of a corporation are called *dividends*, a stockholder being said to receive a *dividend.*

**Division.** The operation by which, given the product of two numbers and one of them, the other is found, is called *division.*

**Divisor.** The number by which the dividend is divided is called the *divisor.*

**Draft.** A check drawn by one bank on another, or by one individual on another, is called a *draft.*

**Drawee.** A person to whose order a draft is payable is called the *drawee.*

**Drawer.** A person who signs a draft is called the *drawer.*

**Duties.** Government taxes on imported goods are called *duties.* Some governments levy taxes on exported goods, and they are also called duties.

**Equation.** The expressed equality of two quantities is called an *equation.*

**Equilateral triangle.** A triangle whose three sides are equal is called an *equilateral triangle.*

**Even number.** A number that contains the factor 2 is called an *even number.*

**Evolution.** The process of finding a root of a number is called *evolution.*

**Exact interest.** Interest reckoned on the basis of 365 days to the year is called *exact interest.*

**Exchange.** The payment of money by means of checks, money orders, or drafts, is called *exchange.*

**Exponent.** A small figure placed at the right and a little above a number is called an *exponent.* If it is an integer, it shows how many times the other number is taken as a factor; thus, $3^2 = 3 \times 3 = 9$. There are also fractional exponents, where the denominator indicates a root and the numerator a power; thus, $8^{\frac{2}{3}}$ means $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$.

**Extremes and means.** The first and last terms of a proportion are called the *extremes;* the second and third, the *means.*

**Face of note.** The principal mentioned in a note is called the *face of the note.*

**Factors.** The numbers which multiplied together make another number are called its *factors.* (This definition is limited to integers, except in speaking of roots.) In some lines of business agents are called *factors.*

**Fraction.** One or more of the equal parts of a unit is called a *fraction.* In general, any indicated division in the form $\frac{a}{b}$ is called a *fraction.*

**Greatest common divisor.** The greatest factor common to two or more numbers is called their *greatest common divisor.* It is also called *greatest common measure.*

**Improper fraction.** A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

**Index.** A figure written in a radical sign to indicate what root is to be extracted is called an *index* of the root; thus, $\sqrt[3]{8}$ means the cube root of 8. In square root the index is not written, as $\sqrt{4}$.

**Indorse.** To write one's name on the back of a note or other document is to *indorse* it.

**Insurance.** An agreement to compensate any one for some specified loss is called *insurance.*

**Integer.** *See* Number.

**Integral number.** An *integer* is also called an *integral* or *whole number.*

**Interest.** Money paid for the use of money is called *interest.*

**Involution.** The process of finding a power of a number is called *involution.*

**Isosceles triangle.** A triangle two of whose sides are equal is called an *isosceles triangle.*

**Latitude.** The degree measure of the distance of a place from the equator is called its *latitude.*

**Least common denominator.** The least denominator common to several fractions is called their *least common denominator.*

**Least common multiple.** The least of all the common multiples of two or more numbers is called their *least common multiple.*

**Length.** The number of linear units a figure contains is called its *length.*

**Liability.** An obligation to pay money is called a *liability.*

**Like numbers.** Numbers referring to the same unit are called *like numbers;* thus, 2 ft. and 3 ft.

**Longitude.** The degree measure of the distance of a place from a prime (or standard) meridian (usually the one through Greenwich, England) is called its *longitude.*

**Lowest terms.** If the terms of a fraction are prime to each other the fraction is said to be reduced to its *lowest terms.*

**Maker of a note.** The one who signs a note is called the *maker.*

**Maturity.** The date on which a note or bond is due is called the date of its *maturity.*

**Mean proportional.** If the two means of a proportion are equal, either is called the *mean proportional* between the extremes.

**Means.** *See* Extremes.

**Minuend.** The number from which we subtract is called the *minuend.*

**Mixed decimal.** A number composed of an integer and a decimal fraction is called a *mixed decimal.*

**Mixed number.** The sum of a whole number and a fraction is called a *mixed number.*

**Multiple.** The product of two abstract integers is called a *multiple* of either.

**Multiplicand.** The number multiplied is called the *multiplicand.*

**Multiplication.** The process of taking one number as many times as there are units in another is called *multiplication* (by an integer).

**Multiplier.** The number by which we multiply is called the *multiplier.*

**Mutually prime numbers.** Numbers that have no common factor except one are said to be *prime to each other*, or *mutually prime.*

**Net proceeds.** The money that remains after all discounts and charges are paid is called the *net proceeds* of any sum.

**Notation.** The writing of numbers by means of symbols is called *notation.*

**Note.** *See* Promissory Note.

**Number.** A unit or a collection of units is called a *whole number* or an *integer. See also* Fraction.

**Numeration.** The naming of numbers is called *numeration.*

**Numerator.** The number which shows how many parts have been taken to make a fraction is called the *numerator*, — in a common fraction, the number above the line.

**Obligation.** A sum which a person is obliged to pay is called an *obligation.*

**Odd number.** A number that is not even is called an *odd number.*

**Order.** In writing numbers the place occupied by a figure is called its *order.* Thus we speak of units of the second order, meaning the tens.

**Par value.** The face or nominal value of stock is called its *par value.* In railroad stocks this is usually $100 a share.

**Partial payment.** A part payment on a note is also called a *partial payment.*

**Payee.** The person to whom the sum mentioned in a note, check, draft, or other similar document is payable is called the *payee.*

**Per cent.** Another name for hundredths is *per cent.* It originally meant "by the hundred."

**Percentage.** The result found by taking a certain per cent of the base is called the *percentage.* That part of arithmetic that deals with per cent is often called *percentage.*

**Perimeter.** The length of the boundary of a plane figure is called the *perimeter.*

**Periods.** Groups of three figures each, marked off by a separatrix, are called *periods.*

**Perpendicular.** A line that makes a right angle with another is said to be *perpendicular* to it.

**Place value.** The value of a period or of a figure, depending upon its position, is called its *place value.*

**Policy.** A written contract of insurance is called a *policy.*

**Poll tax.** A tax on a person (usually limited to male citizens over twenty-one years of age), without regard to the amount of property owned, is called a *poll tax.* (The old name for head was "poll.")

**Power.** The product of two or more equal factors is called a *power.*

**Premium.** Money paid for insurance for a certain time is called a *premium.* The excess of the market value above the par value of stock or other evidences of value is called a *premium.*

**Present worth.** The present value of a debt due in the future is called the *present worth.*

**Prime factors.** Factors that are prime numbers are called *prime factors.*

**Prime number.** A number that has no factors except itself and one is called a *prime number.*

**Principal.** A sum of money that draws interest is called the *principal.*

**Proceeds.** The face (or amount, if it draws interest) of a note less the discount is called the *proceeds.*

**Product.** The result of multiplying two or more numbers together is called the *product.*

**Promissory note.** A paper signed by a debtor, agreeing to pay money, is called a *promissory note.*

**Proper fraction.** A fraction whose numerator is less than the denominator is called a *proper fraction.*

**Proportion.** An expression of equality of two ratios is called a *proportion.*

**Quotient.** The number found by division is called the *quotient.*

**Rate.** The number of hundredths of the base to be taken is called the *rate;* thus the rate of interest may be 6%, the *rate per cent* of interest being then 6. The amount of tax on a dollar is often called the *rate of taxation.* The premium on each $100 of fire insurance or each $1000 of life insurance is often called the *rate* of insurance.

**Ratio.** The relation of one quantity to another of the same kind, as expressed by division of the first by the second, is called their *ratio.*

**Reciprocal.** The *reciprocal* of a fraction is the fraction with its terms interchanged. Unity divided by any number is called the *reciprocal* of the latter.

**Reduction.** Changing the form of a number without changing the value is called *reducing* the number. (This process is used in dealing with fractions and compound numbers.)

**Reduction ascending.** Reduction of compound numbers to a higher denomination is called *reduction ascending.*

**Reduction descending.** Reduction of compound numbers to a lower denomination is called *reduction descending.*

**Remainder.** The part of the dividend remaining when the division is not exact is called the *remainder.* The difference in subtraction is also called the *remainder.*

**Root.** One of the equal factors of a number is called a *root.* *See also* Square Root.

**Scalene triangle.** A triangle whose three sides are unequal is called a *scalene triangle.*

**Separatrix.** The comma used in separating periods in large numbers is called a *separatrix.*

**Share.** One of a specified number of equal parts of the capital stock of a company is called a *share.*

**Similar fractions.** If several fractions have the same denominator, they are said to be *similar fractions.*

**Simple fraction.** A common fraction whose terms are integral is called a *simple fraction.*

**Solid.** A magnitude that has length, breadth, and thickness is called a *solid.*

**Square.** A plane figure with four equal sides and four equal angles is called a *square.*

**Square root.** One of the two equal factors of a number is called its *square root.*

**Stockholder.** One who has stock in a corporation or company is called a *stockholder.*

**Stocks.** The shares of corporations generally are called *stocks.*

**Subtraction.** The operation of finding the difference between two numbers is called *subtraction.*

**Subtrahend.** The number which is subtracted is called the *subtrahend.*

**Sum.** The result in addition is called the *sum.*.

**Surface.** A magnitude that has length and breadth, but not thickness, is called a *surface.*

**Tax.** Money assessed by a government for its support is called a *tax.*

**Terms of a fraction.** The numerator and denominator together are called the *terms of the fraction.*

**Triangle.** A plane figure formed by three straight lines is called a *triangle.*

**Unit.** Any one thing is called a *unit.* The number one is called *unity.*

**Unit fraction.** A common fraction whose numerator is 1 is called a *unit fraction.*

**Volume.** The number of cubic units a solid contains is called its *volume.*

**Whole number.** *See* Number.

# INDEX


| | PAGE |
|---|---|
| Abstract number | 14 |
| Acute-angled triangle | 152 |
| Addends | 6 |
| Addition | 6, 46, 80, 98 |
| Agriculture | 51, 83, 220, 226 |
| Aliquot parts | 18 |
| Altitude | 154 |
| America, extent, resources | 7 |
| historical facts | 12 |
| See Industries, Farms, Cities, etc. | |
| Amount of note | 214 |
| Analysis | 160, 164, 168 |
| Angle measure | 94 |
| Antecedent | 218 |
| Apothecaries' measure | 92 |
| Approximate answers | 88 |
| Arabic numerals | 2 |
| Arc | 94 |
| Area | 104, 154 |
| Avoirdupois weight | 92 |
| Bank discount | 215 |
| Banking | 204, 215 |
| Base | 137, 154 |
| Bills discounted | 193 |
| and receipts | 125 |
| of lumber | 235 |
| Birds, value of | 227 |
| Board measure | 234 |
| Buildings | 190, 236 |
| Business fractions | 50, 114 |
| Cancellation | 39, 58 |
| Capacity | 91–93, 207, 228 |
| Capital | 247 |
| Carat | 92 |
| Carpenter | 236 |
| Carpeting | 239 |
| Cattle | 28 |
| Centigrade | 231 |
| Checks | 6, 22, 204 |
| Cipher | 2 |
| Circular measure | 94 |
| Cities | 19, 142 |
| Clothing | 143 |
| Coal | 242 |
| Commission | 199 |
| Common divisor | 30 |
| fractions | 36, 37, 114 |
| multiple | 35 |
| Complex fraction | 70 |
| Compound fraction | 70 |
| numbers | 95 |
| Composite number | 31 |
| Concrete number | 14 |
| Consequent | 218 |
| Corn | 27 |
| Cotton | 116 |
| Credit, creditor | 125 |
| Cubic measure | 91, 207 |
| Debit, debtor | 125 |
| Decimal fractions | 76, 105 |
| Definitions | 268 |
| Denominate numbers | 90, 149, 206 |
| Denominator | 37 |
| Difference | 10 |
| Discount | 147, 191, 198, 215 |
| Dividend | 20 |
| Divisibility tests | 32, 33 |
| Division | 20, 62, 86, 102, 110 |
| Divisor | 20, 30 |
| Dozen | 149 |
| Dry goods | 197 |
| measure | 93 |
| Electricity | 253 |
| Equation | 176, 177 |
| Equilateral | 152 |
| Even number | 31 |
| Face of note | 214 |
| Factor | 15, 30–33 |

| | PAGE |
|---|---|
| Fahrenheit | 231 |
| Farm problems | 51, 83, 220, 226 |
| Fisheries | 145 |
| F.o.b. | 250 |
| Food supply | 26 |
| Forests | 232 |
| Fractions | 36, 76, 114 |
| Freight problems | 250 |
| Fruit | 60 |
| Grain | 26 |
| Greatest common divisor | 30, 34 |
| Grocer's problems | 196 |
| Grounds, laying out | 241 |
| Height | 154 |
| House building | 236 |
| Improper fraction | 37 |
| Indorse | 204, 216 |
| Interest | 202, 211 |
| Iron | 244 |
| Isosceles triangle | 152 |
| Kitchen problems | 65 |
| Land measure | 91 |
| Least common denominator | 44 |
| Least common multiple | 35 |
| Length | 91 |
| Liquid measure | 92 |
| Loss, profit | 194 |
| Lowest terms | 43 |
| Lumber | 234 |
| Magnitudes illustrated | 120 |
| Maker of note | 216 |
| Making change | 11 |
| Manufacturer | 143, 246, 247 |
| Measures | 91, 151, 157, 207, 228, 234 |
| Metals | 245 |
| Meteorology | 252 |
| Mexican measures | 207 |
| Mines | 242, 244 |
| Minuend | 10 |
| Miscellaneous problems | 254 |
| Mixed numbers | 37 |
| Multiple | 15, 23, 35 |
| Multiplicand, multiplier | 14 |
| Multiplication | 14, 52, 67, 84, 101, 106 |
| Names of numbers | 3 |
| Naught | 2 |
| Notation | 2 |
| Note | 214 |
| Numbers | 2, 4, 31, 77 |
| Numerals | 2, 31, 77 |
| Numeration | 2 |
| Numerator | 37 |
| Obtuse-angled triangle | 152 |
| Odd number | 31 |
| Oral analysis | 160 |
| Orders | 3 |
| Paper | 150 |
| Papering | 240 |
| Parallelogram | 151, 155 |
| Payee | 216 |
| Per cent | 129 |
| Percentage | 129, 137, 181 |
| Periods | 2 |
| Perpendicular lines | 94 |
| Pictures of magnitudes | 120 |
| Place value | 2 |
| Plastering | 238 |
| Polygon | 151 |
| Population | 142 |
| Potatoes | 83 |
| Poultry | 51 |
| Power | 15, 17, 85, 87 |
| Precipitation | 252 |
| Prime number | 31, 33 |
| Principal | 202 |
| Problem solving | 115, 159 |
| Problems in one step | 174 |
| Proceeds | 215 |
| Produce | 228, 250 |
| Product | 14 |
| Profit and loss | 194 |
| Promissory note | 214 |
| Proper fraction | 37 |
| Proportion | 223 |
| compound | 169 |
| Proportional quantities | 224 |
| sides | 224 |

| | PAGE |
|---|---|
| Quadrilateral | 151 |
| Quotient | 20 |
| Railways | 105 |
| Rainfall | 252 |
| Rate | 137, 147, 202 |
| of interest | 214 |
| Ratio | 217 |
| Receipts | 125, 205 |
| Reciprocal | 64 |
| Rectangle | 151 |
| Reduction | 95 |
| of fractions | 39, 78 |
| Remainder | 20, 22 |
| Right-angled triangle | 152 |
| Savings bank | 214 |
| Scalene triangle | 152 |
| School problems | 82 |
| Separatrix | 2 |
| Shipper | 250 |
| Shipping | 119, 249, 250 |
| Similar figures | 224 |
| fractions | 44 |
| Simple interest | 202, 211 |
| Solving equations | 177 |
| problems | 115, 159 |
| Square | 151 |
| measure | 91 |
| Stock yards | 29 |
| Subtraction | 10, 47, 48, 80, 98, 105 |
| Subtrahend | 10 |
| Sum | 6 |
| Tables | 91, 265 |
| Temperature | 231, 252 |
| Terms of a fraction | 37 |
| Thermometer | 231, 252 |
| Timber | 232 |
| Time | 93 |
| Trade discount | 191, 198 |
| Trapezoid | 151 |
| Triangle | 152, 154 |
| Troy weight | 92 |
| Unit | 36 |
| fraction | 37, 63 |
| Unitary analysis | 168 |
| Unknown quantity | 177 |
| Vara | 207 |
| Volume | 156 |
| Water, weight of | 229, 230 |
| Weather | 252 |
| Weight | 92, 228, 229 |
| Wheat | 26 |
| Wind | 252 |
| Wool | 118 |
| Written analysis | 164 |
| Zero | 2 |

# ANSWERS

## Page 3

1. 5,280,000 ft.
2. 60 sec., 3600 sec., 86,400 sec., 950,400 sec.
3. 8760 hr., 876,000 hr.
4. 4, 40, 40,000.

## Page 6

1. 26,776.
2. 18,947.
3. 269,162.
4. 2,383,868.
5. 4,429,515.

## Pages 7, 8

1. 76,303,387.
2. 6,429,474.
3. $4,289,626,071.
4. $2,272,400,000.
5. 3,756,884 sq. mi.
6. 1,740,032,459 bu.
7. 16,738,363.
8. 906,293,360 lb.
9. 293,298,516 T.
10. 13,157,000 bales.
11. $2,763,575,126.
12. 31,846 mi.
13. 3,753,986,804 bu.
14. $22,789,054.

## Page 9

1. 1,241,016.
2. 383,759.
3. 7,902,936.
4. 934,695.
5. 7,119,002.
6. 4707.
7. 29,556.
8. 1,125,624.
9. 337,887.
10. 4,336,886.

## Page 11

1. $16.27.
2. $88.75.
3. $190.75.
4. $186.25.
5. $191.75.
6. $126.70.
7. 307,890.
8. 254,648.

## Page 12

2. 117 yr.
5. 44 yr.
7. 128 yr.
8. 1775.

## Page 13

1. 14,338.
2. 4867.
3. 6789.
4. 4707.
5. 51,805.
6. 45,873.
7. 131,891.
8. 238,648.
9. 33,298.
10. 185,094.
11. $327.65.
12. $180.99.
13. $791.01.
14. $1832.63.
15. $49,355.25.
16. $22,725.42.
17. $16,393.25.
18. $8899.99.

### Page 16

1. 863,928.
2. 57,591.
3. 232,232.
4. 244,720.
5. 62,212.
6. 525,625.
7. $9780.99.
8. $28,890.05.
9. $125,555.50.
10. $366,713.10.
11. $30,258.24.
12. $317,343.75.
13. $247,247.
14. $37,622,000.
15. $260,257.25.
16. $6,017,511.50.
17. $24,621,931.50.
18. $16,323,216.

### Page 17

1. $72,660.
2. $134,720.
3. 745,600 ft.
4. 604,800 rd.
5. $41,280,000.
6. $35,475,000.
7. $107,940.
8. $195,750.
9. $81,500.
10. $85,305.
11. 964 mi.
12. $4425.

### Page 18

1. 3615, $12,500, $237, $3333.
2. 11,850, 21,250 ft., $3425, $49,370.
3. 3350, 20,050 ft., $1240, $9610.
4. 8000, 32,000, 363,000, $21,400, $62,200.

### Page 19

1. 76,303,388.
2. 1049.
3. $94,635,900, $221,106,010, $212,287,275.
4. 4,626,068.

### Page 21

1. $135, $540.
2. $28, $196.
3. $580.
4. $540, $945.
5. $295.50, $492.50, $9850.

### Page 22

1. $2.68, rem. $2.96.
2. $0.98.
3. $25.04.
4. $0.86, rem. $0.30.
5. $4.97, rem. $1.04.
6. $54.16⅔.
7. $3.96, rem. $0.90.
8. $84.12, rem. $0.06.
9. $13.81$\frac{9}{11}$.
10. $3.54$\frac{5}{12}$.
11. $5.49$\frac{1}{15}$.
12. $13.81¼.

### Page 23

1. 58.
2. 52.
3. 14.
4. 32.
5. 30.
6. 70.
7. 39$\frac{3}{7}$.
8. 271$\frac{2}{5}$.
9. 4$\frac{2}{3}\frac{7}{7}\frac{1}{5}$.
10. 7$\frac{4}{6}\frac{8}{3}\frac{1}{0}$.
11. 20 da.
12. 21 sec.
13. $21,133.69$\frac{6}{7}\frac{8}{3}$.
14. $1,313,013.69$\frac{6}{7}\frac{8}{3}$.

### Page 25

1. 44,528. 2. 34,342. 3. 302,133. 4. 578,154.
5. 254,347. 6. 66,717. 7. 201,501. 8. 200,802.
9. 478,813. 10. 144. 11. 408. 12. $2169\frac{12}{51}$.
13. 912. 14. 67,910. 15. 37,864. 16. 204,100.
17. 781,312. 18. 390,208. 19. 4,402,496. 20. 12,975.035.
21. 12,899,583.

### Page 26

1. 565,825,000 bu. 2. 6864 bu. 3. 517,200,000 bu.
4. $383,112,100. 5. $7400.

### Page 27

1. 84,204,096 A. 2. $701,700,800. 3. 695. 4. $3500
5. $978,810, Conn.; $52,410,800, Ind. 6. $37\frac{1}{8}$ bu.

### Page 28

1. $77,076,249. 2. $56,065,203. 3. $67,071,104.
4. $4,999,837.50. 5. $22. 6. $30.
7. 6,379,054, 26,147,604.

### Page 29

1. $50,000,000. 2. 644 lb., $53.02. 3. 12,300.
4. $500. 5. $15,471,616. 6. 10 ct.

### Page 31

1. 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97. Also odd numbers.
2. 2, 3, 3, 3; 2, 2, 2, 2, 3; 2, 2, 2, 2, 2, 2; 7, 11. 3. 12.

### Page 32

1. Those ending in even numbers. 2. Those ending in 0 or 5.
3. All but 4004, 7117, 475,633.

### Page 34

1. $5^2 \times 31$. 2. $2^2 \times 3 \times 5 \times 7$. 3. $5 \times 7 \times 11$.
4. $2^5 \times 11$. 5. $2^6 \times 5$. 6. $3^4 \times 11$.
7. $7^2 \times 3$. 8. $3^3 \times 11$. 9. $2^4 \times 11$.

10. $2^4 \times 3 \times 7$.
11. $2 \times 5 \times 13$.
12. $2 \times 3 \times 7 \times 11$.
13. $2 \times 3 \times 5 \times 13$.
14. $5 \times 3 \times 19$.
15. $3 \times 5^3$.
16. $2 \times 3^4 \times 5 \times 11$.
17. $2 \times 5^4$.
18. $2 \times 3^5 \times 5$.
19. $2^8 \times 5$.
20. $2^{10}$.
21. $2 \times 3^3 \times 5 \times 11$.
22. $5^3 \times 11$.
23. $2 \times 5 \times 7^2 \times 11$.
24. $2^5 \times 7^2$.
25. $11^3$.
26. $2^2 \times 5^2 \times 3^2 \times 7$.
27. $3 \times 5^4$.
28. $3^5 \times 5$.
29. $3^6 \times 5$.
30. $3^7 \times 5$.
31. 7.
32. 32.
33. 65.
34. 25.
35. 444.
36. 55.
37. 144.
38. 256.
39. 512.
40. 77.
41. 125.
42. 165.
43. 288.
44. 625.
45. 625.
46. 101.
47. 25.
48. 515.

## Page 35

1. 210.
2. 770.
3. 224.
4. 192.
5. 900.
6. 96.
7. 75.
8. 1056.
9. 50.
10. 64.
11. 3780.
12. 288.
13. 1785.
14. 756.
15. 29,304.

## Page 40

1. $\frac{3}{8}, \frac{5}{11}, \frac{6}{7}, \frac{1}{2}, \frac{2}{3}, \frac{4}{5}$.
2. $\frac{3}{15}, \frac{10}{15}, \frac{2}{15}, \frac{9}{15}, \frac{4}{15}, \frac{6}{15}, \frac{7}{15}, \frac{3}{15}$.
3. $\frac{12}{16}, \frac{14}{16}, \frac{8}{16}, \frac{5}{16}, \frac{7}{16}, \frac{8}{16}, \frac{3}{16}, \frac{5}{16}$.
4. $\frac{25}{50}, \frac{30}{50}, \frac{45}{50}, \frac{14}{50}, \frac{2}{50}, \frac{9}{50}, \frac{9}{50}, \frac{7}{50}$.
5. $\frac{8}{32}, \frac{20}{32}, \frac{6}{32}, \frac{33}{32}, \frac{14}{32}, \frac{9}{32}, \frac{9}{32}$.
6. $\frac{75}{100}, \frac{80}{100}, \frac{70}{100}, \frac{35}{100}, \frac{75}{100}, \frac{16}{100}, \frac{15}{100}, \frac{67}{100}$.
7. $\frac{84}{144}, \frac{96}{144}, \frac{8}{144}, \frac{132}{144}$.
8. $\frac{21}{24}, \frac{14}{24}; \frac{63}{72}, \frac{42}{72}; \frac{189}{216}, \frac{126}{216}$.

## Page 41

1. $\frac{42}{6}$.
2. $\frac{63}{7}$.
3. $\frac{60}{10}$.
4. $\frac{88}{11}$.
5. $\frac{66}{6}$.
6. $\frac{52}{4}$.
7. $\frac{18}{2}$.
8. $\frac{180}{20}$.
9. $\frac{45}{3}$.
10. $\frac{17}{3}$.
11. $\frac{55}{8}$.
12. $\frac{23}{3}$.
13. $\frac{89}{10}$.
14. $\frac{20}{3}$.
15. $\frac{31}{8}$.
16. $\frac{44}{9}$.
17. $\frac{58}{9}$.
18. $\frac{147}{16}$.
19. $\frac{81}{11}$.
20. $\frac{69}{8}$.
21. $\frac{45}{7}$.
22. $\frac{124}{15}$.
23. $\frac{101}{14}$.
24. $\frac{101}{12}$.
25. $\frac{50}{3}$.
26. $\frac{25}{2}$.
27. $\frac{100}{3}$.
28. $\frac{200}{3}$.
29. $\frac{43}{15}$.
30. $\frac{1000}{9}$.
31. $\frac{500}{3}$.
32. $\frac{2000}{3}$.
33. $\frac{251}{2}$.
34. $\frac{1403}{352}$.
35. $\frac{733}{5}$.
36. $\frac{2120}{9}$.
37. $\frac{5781}{7}$.
38. $\frac{8992}{9}$.
39. $\frac{2027}{3}$.

## Page 42

1. 11.
2. 6.
3. $10\frac{5}{7}$.
4. $9\frac{3}{8}$.
5. $10\frac{1}{8}$.
6. $7\frac{5}{6}$.
7. $15\frac{3}{5}$.
8. $9\frac{6}{7}$.
9. $7\frac{10}{11}$.
10. $7\frac{2}{3}$.

| | | | | |
|---|---|---|---|---|
| 11. 5. | 12. $7\frac{5}{33}$. | 13. $14\frac{1}{3}$. | 14. $1\frac{1}{92}$. | 15. 1. |
| 16. $18\frac{5}{8}$. | 17. $10\frac{23}{32}$. | 18. $15\frac{1}{3}$. | 19. $12\frac{3}{8}$. | 20. $17\frac{11}{42}$. |
| 21. $19\frac{3}{4}$. | 22. $26\frac{11}{16}$. | 23. $26\frac{5}{24}$. | 24. $5\frac{3}{16}$. | 25. $28\frac{1}{2}$. |
| 26. 2. | 27. $12\frac{1}{2}$. | 28. $20\frac{1}{11}$. | 29. 25. | 30. 7. |
| 31. $5\frac{3}{25}$. | 32. $1\frac{8}{17}$. | 33. $1\frac{9}{101}$. | 34. $1\frac{2}{3}$. | 35. 21. |
| 36. 5. | 37. $10\frac{1}{11}$. | 38. 13. | 39. 182. | 40. 12. |

## Page 43

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. $\frac{4}{11}$. | 2. $\frac{7}{9}$. | 3. $\frac{3}{11}$. | 4. $\frac{3}{4}$. | 5. $\frac{8}{9}$. | 6. $\frac{9}{11}$. | 7. $\frac{9}{11}$. |
| 8. $\frac{2}{3}$. | 9. $\frac{8}{18}$. | 10. $\frac{11}{13}$. | 11. $\frac{3}{5}$. | 12. $\frac{3}{13}$. | 13. $\frac{17}{31}$. | 14. $\frac{9}{19}$. |
| 15. $\frac{21}{23}$. | 16. $\frac{17}{19}$. | 17. $\frac{11}{13}$. | 18. $\frac{23}{31}$. | 19. $\frac{5}{7}$. | 20. $\frac{12}{13}$. | 21. $\frac{9}{13}$. |
| 22. $\frac{4}{7}$. | 23. $\frac{1}{2}$. | 24. $\frac{37}{41}$. | 25. $\frac{13}{14}$. | 26. $\frac{1}{12}$. | 27. $\frac{3}{5}$. | 28. $\frac{1}{18}$. |
| 29. $\frac{2}{3}$. | 30. $\frac{21}{50}$. | 31. $\frac{5}{7}$. | 32. $\frac{5}{6}$. | 33. $\frac{3}{4}$. | 34. $\frac{11}{20}$. | 35. $\frac{3}{4}$. |

## Page 45

| | | |
|---|---|---|
| 1. $\frac{21}{175}$, $\frac{30}{175}$. | 2. $\frac{18}{441}$, $\frac{35}{441}$. | 3. $\frac{32}{108}$, $\frac{15}{108}$. |
| 4. $\frac{66}{275}$, $\frac{85}{275}$. | 5. $\frac{9}{189}$, $\frac{8}{189}$. | 6. $\frac{10}{72}$, $\frac{7}{72}$. |
| 7. $\frac{8}{121}$, $\frac{11}{121}$. | 8. $\frac{144}{504}$, $\frac{448}{504}$, $\frac{189}{504}$. | 9. $\frac{42}{900}$, $\frac{55}{900}$. |
| 10. $\frac{4}{625}$, $\frac{30}{625}$. | 11. $\frac{1}{12}$, $\frac{1}{12}$. | 12. $\frac{133}{1430}$, $\frac{140}{1430}$. |
| 13. $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$. | 14. $\frac{350}{15540}$, $\frac{4074}{15540}$. | 15. $\frac{10}{16}$, $\frac{3}{16}$, $\frac{2}{16}$. |
| 16. $\frac{18}{150}$, $\frac{40}{150}$, $\frac{45}{150}$. | 17. $\frac{100}{225}$, $\frac{120}{225}$, $\frac{54}{225}$. | 18. $\frac{5}{32}$, $\frac{6}{32}$, $\frac{28}{32}$. |
| 19. $\frac{9}{60}$, $\frac{14}{60}$, $\frac{8}{60}$. | 20. $\frac{3}{50}$, $\frac{4}{50}$, $\frac{40}{50}$. | 21. $\frac{6}{81}$, $\frac{36}{81}$, $\frac{5}{81}$. |
| 22. $\frac{78}{180}$, $\frac{20}{180}$, $\frac{7}{180}$. | 23. $\frac{140}{210}$, $\frac{90}{210}$, $\frac{168}{210}$, $\frac{21}{210}$. | 24. $\frac{7}{112}$, $\frac{70}{112}$, $\frac{9}{112}$. |
| 25. $\frac{24}{222}$, $\frac{21}{222}$, $\frac{10}{222}$. | 26. $\frac{13}{156}$, $\frac{28}{156}$, $\frac{15}{156}$. | 27. $\frac{42}{441}$, $\frac{45}{441}$, $\frac{28}{441}$. |
| 28. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$. | 29. $\frac{16}{32}$, $\frac{24}{32}$, $\frac{28}{32}$, $\frac{10}{32}$, $\frac{3}{32}$. | 30. $\frac{4}{36}$, $\frac{24}{36}$, $\frac{30}{36}$, $\frac{21}{36}$, $\frac{5}{36}$. |
| 31. $31\frac{44}{297}$, $42\frac{21}{297}$. | 32. $29\frac{35}{77}$, $36\frac{13}{77}$. | 33. $42\frac{153}{162}$, $61\frac{50}{162}$. |
| 34. $23\frac{16}{84}$, $67\frac{27}{84}$. | 35. $32\frac{44}{693}$, $77\frac{54}{693}$. | 36. $81\frac{21}{96}$, $27\frac{14}{96}$. |
| 37. $71\frac{70}{150}$, $62\frac{33}{150}$. | 38. $68\frac{4}{74}$, $41\frac{1}{74}$. | 39. $18\frac{28}{260}$, $21\frac{55}{260}$. |

## Page 46

| | | | | |
|---|---|---|---|---|
| 1. $1\frac{1}{6}$. | 2. $\frac{7}{8}$. | 3. $1\frac{3}{10}$. | 4. $1\frac{3}{8}$. | 5. $\frac{7}{10}$. | 6. $1\frac{1}{16}$. |
| 7. $1\frac{5}{16}$. | 8. $\frac{7}{12}$. | 9. $1\frac{5}{8}$. | 10. $19\frac{1}{2}$. | 11. $18\frac{1}{2}$. | 12. $7\frac{5}{7}$. |

## Page 47

| | | | | | |
|---|---|---|---|---|---|
| 1. $\frac{1}{12}$. | 2. $\frac{13}{16}$. | 3. $\frac{1}{3}$. | 4. $\frac{1}{4}$. | 5. $\frac{9}{16}$. | 6. $\frac{5}{12}$. |
| 7. $\frac{2}{3}$. | 8. $\frac{1}{6}$. | 9. $\frac{7}{18}$. | 10. $\frac{1}{8}$. | 11. $\frac{7}{32}$. | 12. $\frac{44}{63}$. |
| 13. $\frac{4}{21}$. | 14. $\frac{13}{18}$. | 15. $\frac{3}{22}$. | 16. $\frac{1}{24}$. | 17. $\frac{3}{10}$. | 18. $\frac{17}{36}$. |

**19.** $\frac{1}{8}$. **20.** $\frac{43}{63}$. **21.** $\frac{14}{27}$. **22.** $\frac{4}{5}$. **23.** $\frac{17}{32}$. **24.** $\frac{16}{33}$.
**25.** $\frac{25}{42}$. **26.** $\frac{57}{64}$. **27.** $\frac{6}{25}$. **28.** $\frac{11}{14}$. **29.** $\frac{43}{51}$. **30.** $\frac{29}{39}$.
**31.** $\frac{19}{45}$. **32.** $\frac{2}{35}$. **33.** $\frac{1}{8}$ mi.

## Page 48

**1.** $365\frac{7}{9}$. **2.** $122\frac{5}{18}$. **3.** $162\frac{9}{10}$. **4.** $266\frac{11}{15}$.
**5.** $88\frac{17}{24}$. **6.** $97\frac{1}{21}$. **7.** $212\frac{9}{14}$. **8.** $382\frac{2}{3}$.
**9.** $\frac{67}{70}$. **10.** $1\frac{11}{40}$. **11.** $1\frac{11}{48}$. **12.** $2\frac{1}{4}$.
**13.** $1\frac{19}{30}$. **14.** $1\frac{1}{6}$. **15.** $\frac{59}{60}$. **16.** $1\frac{17}{30}$.
**17.** $\frac{139}{210}$. **18.** $2\frac{9}{32}$. **19.** $\frac{95}{144}$. **20.** $37\frac{23}{24}$.
**21.** $2\frac{5}{24}$. **22.** $69\frac{39}{40}$. **23.** $2\frac{5}{14}$. **24.** $\frac{13}{16}$ in.
**25.** $1\frac{5}{8}$ in. **26.** $69\frac{1}{4}$ in. **27.** $1\frac{3}{8}$ in. **28.** $380\frac{7}{24}$ rd.

## Page 49

**1.** 3 ft. $9\frac{1}{2}$ in. **2.** $10\frac{11}{12}$ yr. **3.** $3\frac{13}{16}$ in.
**4.** $78\frac{11}{12}$ lb. **5.** $196\frac{19}{24}$ yd. **6.** $36\frac{3}{8}$ in., $22\frac{1}{2}$ in.
**7.** 6 in. **8.** 26′ $9\frac{3}{8}$″. **9.** 2″, $1\frac{3}{8}$″.

## Page 50

**1.** $9\frac{49}{108}$. **2.** $3\frac{31}{120}$. **3.** $11\frac{13}{20}$. **4.** $7\frac{47}{144}$. **5.** $10\frac{121}{300}$. **6.** $14\frac{79}{144}$.
**7.** $17\frac{7}{8}$. **8.** $2\frac{29}{48}$. **9.** $4\frac{2}{5}$. **10.** $12\frac{7}{15}$. **11.** $9\frac{21}{32}$. **12.** $3\frac{23}{48}$.
**13.** $17\frac{23}{40}$. **14.** $22\frac{2}{35}$. **15.** $8\frac{21}{32}$. **16.** $84\frac{1}{2}$. **17.** $12\frac{7}{8}$. **18.** $12\frac{5}{8}$.
**19.** $\frac{1}{8}$ in., $\frac{1}{8}$ in., $\frac{1}{16}$ in., $\frac{1}{8}$ in., $\frac{1}{16}$ in.

## Page 51

**1.** 171, 3, 15. **2.** 2 bu., \$1.50. **3.** $1\frac{3}{8}$ bu., \$1.44.
**4.** $1\frac{1}{4}$ bu., 60 ct. **5.** $\frac{1}{3}$ bu., 20 ct. **6.** 264 lb., $5\frac{11}{16}$ bu., $4\frac{12}{19}$ lb.
**7.** \$5.70, \$0.10, \$0.00$\frac{3}{4}$. **8.** 12 ct., \$8.55. **9.** \$1.30, $2\frac{16}{57}$ ct.

## Page 54

**1.** $6\frac{3}{4}$. **2.** $4\frac{4}{5}$. **3.** $5\frac{1}{4}$. **4.** $3\frac{3}{8}$. **5.** 6. **6.** 44. **7.** $6\frac{3}{7}$.
**8.** $10\frac{1}{2}$. **9.** $1\frac{5}{9}$. **10.** $7\frac{1}{3}$. **11.** $7\frac{7}{8}$. **12.** $17\frac{7}{8}$. **13.** $1\frac{5}{8}$. **14.** $4\frac{10}{11}$.
**15.** $16\frac{1}{2}$. **16.** $10\frac{1}{12}$.

## Page 56

**1.** $\frac{8}{15}$. **2.** $\frac{5}{14}$. **3.** $\frac{9}{14}$. **4.** $\frac{2}{7}$. **5.** $\frac{1}{2}$. **6.** $\frac{2}{3}$.
**7.** $\frac{13}{35}$. **8.** $\frac{11}{21}$. **9.** $1\frac{1}{3}$. **10.** $1\frac{1}{8}$. **11.** $1\frac{1}{2}$. **12.** 9.
**13.** $\frac{4}{11}$. **14.** $\frac{2}{5}$. **15.** 1. **16.** \$0.46$\frac{7}{8}$. **17.** $\frac{21}{32}$ min. **18.** $\frac{1}{4}$.
**20.** 323 sq. ft., 12 sq. in., 7 sq. in.

### Page 57

1. $43\frac{1}{2}$. 2. 1122. 3. 4011. 4. 5067. 5. $1790\frac{2}{5}$.
6. 856. 7. 1701. 8. 1043. 9. $3391\frac{1}{2}$. 10. $487\frac{3}{5}$.
11. $3061\frac{1}{2}$. 12. $1669\frac{1}{2}$. 13. $1291\frac{1}{2}$. 14. $54\frac{9}{64}$. 15. $153\frac{11}{24}$.
16. $240\frac{1}{112}$. 17. $227\frac{41}{53}$. 18. $247\frac{4}{5}$. 19. $443\frac{11}{28}$. 20. $546\frac{11}{15}$.
21. $476\frac{4}{9}$. 22. \$4.12$\frac{1}{2}$.

### Page 58

1. $\frac{1}{5}$. 2. $\frac{8}{21}$. 3. $1\frac{1}{2}$. 4. 1. 5. $3\frac{7}{81}$. 6. $\frac{8}{5}$. 7. $1\frac{1}{84}$.

### Page 59

1. $11{,}139\frac{3}{16}$. 2. $8812\frac{1}{2}$. 3. 24,990. 4. $213{,}511\frac{1}{9}$. 5. $9887\frac{5}{8}$.
6. 1824. 7. $262\frac{1}{2}$. 8. $5002\frac{113}{180}$. 9. 435. 10. $2914\frac{92}{147}$.
11. 16. 12. $1\frac{7}{8}$. 13. $\frac{9}{10}$. 14. 4. 15. 2.

### Pages 60, 61

1. 140 cu. in.
2. $461\frac{1}{8}$ cu. in.
3. 3834 cu. in.
4. $70\frac{5}{8}$ cu. in.
5. $49\frac{1}{2}$ cu. in.
6. $159\frac{1}{2}$ cu. in. larger.
7. $489\frac{1}{2}$ cu. in. larger.
8. 9120 cu. in.
9. $370\frac{5}{32}$ cu. in. more in 1st.
10. $1235\frac{1}{4}$ cu. in.
11. $50\frac{55}{64}$ cu. in. more in 1st.
12. $869\frac{1}{2}$ cu. in., $427\frac{31}{32}$ cu. in., $1847\frac{11}{16}$ cu. in., $1059\frac{45}{64}$ cu. in.

### Page 62

1. $\frac{5}{36}$. 2. $\frac{3}{85}$. 3. $\frac{1}{63}$. 4. $\frac{7}{36}$. 5. $\frac{7}{16}$. 6. $\frac{1}{12}$.
7. $\frac{5}{24}$. 8. $34\frac{1}{18}$. 9. $45\frac{1}{15}$. 10. $29\frac{2}{3}$. 11. $136\frac{9}{16}$. 12. $165\frac{6}{25}$.

### Page 63

1. 648. 2. 144. 3. 161. 4. 192. 5. 216.
6. 87. 7. 240. 8. 441. 9. 243. 10. 204.
11. 1164. 12. 780. 13. 2000. 14. 4608. 15. 15,360.

### Page 64

1. $1\frac{5}{16}$. 2. $1\frac{1}{8}$. 3. $1\frac{1}{8}$. 4. $\frac{10}{21}$. 5. $\frac{9}{10}$. 6. $4\frac{2}{3}$.
7. $\frac{5}{14}$. 8. $1\frac{1}{5}$. 9. $\frac{5}{16}$. 10. $\frac{1}{4}$. 11. $\frac{12}{17}$. 12. $\frac{39}{50}$.
13. $1\frac{34}{333}$. 14. $\frac{42}{155}$. 15. $1\frac{1}{2}$. 16. $\frac{41}{111}$. 17. $4\frac{1}{18}$. 18. $\frac{21}{55}$.

### Page 65

1. $7\frac{3}{4}$ lb. 2. 1 qt. $1\frac{1}{4}$ pt. 3. 14. 4. \$1.11. 5. \$4.55.
6. 3 lb. 9 oz. 7. $11\frac{1}{4}$ ct. 8. 2 lb. 14 oz.; 1 lb. 8 oz. bread crumbs, 3 lb. blueberries, 1 lb. sugar, 4 oz. butter. 9. \$0.04$\frac{1}{6}$.

### Page 68

1. \$18.38. 2. \$18.83. 3. \$33.67. 4. \$220. 5. \$165.67.
6. \$126. 7. \$156. 8. \$6.

### Page 69

1. $140\frac{5}{6}$. 2. $155\frac{5}{12}$. 3. $76\frac{19}{20}$. 4. $97\frac{3}{15}$.
5. $102\frac{7}{15}$. 6. $107\frac{1}{2}$. 7. $96\frac{11}{18}$. 8. $112\frac{13}{14}$.
9. $117\frac{3}{10}$. 10. $68\frac{13}{20}$. 11. $150\frac{3}{10}$. 12. $55\frac{9}{14}$.
13. $57\frac{3}{10}$. 14. $50\frac{5}{6}$. 15. $24\frac{7}{8}$. 16. $54\frac{17}{20}$.
17. $10\frac{19}{24}$. 18. $13\frac{5}{8}$. 19. $16\frac{11}{40}$. 20. $24\frac{13}{24}$.
21. $25\frac{1}{8}$. 22. 396. 23. 2960. 24. $4984\frac{4}{5}$.
25. $1741\frac{1}{2}$. 26. 2074. 27. 3872. 28. 2754.
29. 4008. 30. 1. 31. $131\frac{1}{4}$. 32. $45\frac{3}{5}$.
33. $34\frac{17}{20}$. 34. $56\frac{7}{20}$. 35. $18{,}422\frac{1}{4}$. 36. $17{,}167\frac{1}{2}$.
37. 31,603. 38. $31{,}348\frac{2}{7}$. 39. $52{,}443\frac{3}{5}$. 40. $1\frac{1}{5}$.
41. $\frac{5}{6}$. 42. $\frac{9}{14}$. 43. $1\frac{5}{9}$. 44. $1\frac{3}{32}$.
45. $\frac{33}{35}$. 46. $1\frac{1}{7}$. 47. $\frac{7}{8}$. 48. $\frac{5}{22}$.
49. $\frac{25}{77}$. 50. $\frac{1}{7}$. 51. $4\frac{24}{40}$. 52. $2\frac{9}{10}$.
53. $3\frac{3}{35}$. 54. 5. 55. $87\frac{3}{5}$. 56. $1\frac{17}{70}$.
57. $\frac{4}{5}$. 58. $1\frac{53}{64}$. 59. 4. 60. $7\frac{11}{77}$.
61. $\frac{1}{4}$. 62. $\frac{2}{5}$ cu. ft. 63. $\frac{6}{25}$ cu. ft. 64. $\frac{1}{8}$ cu. in.
65. 20 sq. ft. 66. 432 sq. ft. 67. 56 sq. in. 68. 78 sq. in.

### Page 70

1. $\frac{1}{5}$. 2. $\frac{1}{5}$. 3. $\frac{2}{49}$. 4. $\frac{1}{24}$. 5. $\frac{5}{7}$.
6. 4. 7. $\frac{10}{13}$. 8. $\frac{3}{4}$. 9. $\frac{9}{10}$. 10. $15\frac{2}{75}$.
11. $\frac{26}{49}$. 12. $\frac{5}{9}$. 13. $\frac{9}{20}$. 14. $\frac{1}{2}$. 15. $\frac{4}{45}$.
16. $1\frac{1}{4}$. 17. 1. 18. $1\frac{8}{23}$. 19. $\frac{1}{2}$. 20. $\frac{9}{44}$.
21. $\frac{27}{100}$. 22. 18. 23. $\frac{1}{24}$. 24. 1. 25. $\frac{5}{32}$.

### Page 71

1. $\frac{4}{7}$. 2. $\frac{7}{9}$. 3. $\frac{16}{21}$. 4. $\frac{21}{121}$. 5. $\frac{1}{2}$.
6. $\frac{1}{2}$. 7. $\frac{1}{9}$. 8. $\frac{1}{8}$. 9. $\frac{1}{4}$. 10. $\frac{1}{9}$.

11. $\frac{3}{5}$. 12. $\frac{1}{6}$. 13. $\frac{1}{7}$. 14. $\frac{1}{3}$. 15. $\frac{9}{71}$.
16. $\frac{1}{2}$. 17. $\frac{1}{5}$. 18. $\frac{29}{27}$. 19. $\frac{856}{1675}$. 20. $\frac{8}{9}$.
21. $\frac{1}{7}$. 22. $\frac{1}{2}$. 23. $\frac{1}{3}$. 24. $\frac{1}{11}$. 25. $\frac{2}{3}$.
26. $\frac{1}{3}$. 27. $\frac{1}{2}$, $\frac{3}{4}$.

## Page 72

1. 69. 2. 57. 3. 72. 4. 200.
5. 50. 6. 250. 7. 259. 8. 160.
9. 256. 10. $250. 11. $3.60. 12. $18.45.
13. 153. 14. 1394. 15. $2250. 16. $535, $1605.

## Page 74

1. $51.06. 2. $0.54. 3. $1.92. 4. $4.40. 5. $0.62.
6. $0.07. 7. $2.43. 8. $7.94. 9. $1.57.

## Page 75

1. $1.56, change. 2. $1.82. 3. $6.82.
4. $8.65. 5. $1.68. 6. $0.25.

## Page 77

1. 0.3, 0.5, 0.9. 2. 4.4, 7.8. 3. 0.75, 0.33$\frac{1}{3}$, 300.03.
4. 0.144, 0.675, 0.0004. 5. 5.007, 8.042, 0.025, 0.000001.

## Page 78

1. $\frac{2}{5}$. 2. $\frac{3}{5}$. 3. $\frac{1}{5}$. 4. $\frac{4}{5}$.
5. $\frac{1}{4}$. 6. $\frac{3}{4}$. 7. $\frac{9}{20}$. 8. $\frac{11}{20}$.
9. $\frac{1}{8}$. 10. $\frac{3}{8}$. 11. $\frac{5}{8}$. 12. $\frac{7}{8}$.
13. $1\frac{1}{4}$. 14. $2\frac{3}{4}$. 15. $5\frac{1}{8}$. 16. $10\frac{3}{8}$.
17. $\frac{70}{100}$, $\frac{60}{100}$, $\frac{10}{100}$, $\frac{80}{100}$, $\frac{400}{100}$, $\frac{6300}{100}$. 18. $\frac{100}{1000}$, $\frac{250}{1000}$, $\frac{370}{1000}$, $\frac{90}{1000}$, $\frac{31000}{1000}$.
19. $\frac{47}{100}$, $\frac{68}{100}$, $\frac{10}{100}$, $\frac{4}{100}$, $\frac{780}{100}$. 20. $\frac{1}{8}$, $200.

## Page 79

1. 0.875. 2. 0.15. 3. 0.266+. 4. 0.833+.
5. 0.857+. 6. 0.555+. 7. 0.636+. 8. 0.107+.
9. 0.263+. 10. 0.542+. 11. 0.416+. 12. 0.806+.
13. 0.288+. 14. 0.610+. 15. 3.770+. 16. 3.075.
17. 4.052. 18. 6.754. 19. 4.005. 20. 7.256.

### Page 81

| | | | |
|---|---|---|---|
| 1. $47.96. | 2. $55.59. | 3. 177.63 ft. | 4. 436.58. |
| 5. 481.9. | 6. 258.68. | 7. 1072.44. | 8. 761.34. |
| 9. $0.18. | 10. $5.75. | 11. $6.85. | 12. $2.75. |
| 13. 57.225. | 14. 35.93. | 15. 41.084. | 16. 539.325. |
| 17. $121.36. | 18. $137.08. | 19. 27.884. | 20. 108.398. |
| 21. 68.9. | 22. 79.93. | 23. 60.44. | 24. 39.64. |
| 25. 108.9. | 26. 70.09. | 27. 533.46. | 28. 124.75. |
| 29. 9.08. | | | |

### Page 82

1. $3.80. 2. $9.75. 3. $17.50. 4. $10.15. 5. $0.47.

### Page 83

| | | | |
|---|---|---|---|
| 1. 333,070 bu. | 2. $4,305,184.83. | 3. 81.5 bu. | 4. $13,261,680. |
| 5. 173 bu. | 6. $1887.50. | 7. 204,960 lb. | |

### Page 84

| | | | |
|---|---|---|---|
| 1. 681.25. | 2. 2087.36. | 3. 968.40. | 4. 2287.34. |
| 5. 354.375. | 6. 644.025. | 7. 1895.625. | 8. 3416.25. |
| 9. 1500.375. | 10. 3570.07. | 11. 2614.85. | 12. 2558.409. |
| 13. 127.44 ft. | 14. 85.32 in. | 15. 44.16 in. | 16. 2440.94 in. |

### Page 85

| | | | |
|---|---|---|---|
| 1. 269.2. | 2. 146.1. | 3. 588.6. | 4. 7776. |
| 5. 2920. | 6. 43,771. | 7. 52,336. | 8. 33,372. |
| 9. 24,720. | 10. 1250. | 11. 13,450. | 12. 3,011,040. |
| 13. 622,062.5. | 14. 2,880,288. | 15. 3,319,180. | 16. 936,625. |
| | 17. 1,428,280. | 18. 43,750. | |

### Page 86

| | | | |
|---|---|---|---|
| 1. 42.75. | 2. 3.217. | 3. 21.11. | 4. 7.77. |
| 5. 20.02. | 6. 47.04. | 7. 70.11. | 8. 311.21. |
| 9. 212.75 in. | 10. 412.6$\frac{1}{3}$ in. | 11. 1.27 in. | |

### Page 87

| | | | |
|---|---|---|---|
| 1. 1.9. | 2. 0.18. | 3. 0.47. | 4. 7.49. |
| 5. 7.9. | 6. 0.63. | 7. 2.28. | 8. 0.746. |

| | | | |
|---|---|---|---|
| **9.** 0.816. | **10.** 0.025. | **11.** 0.031. | **12.** 0.042. |
| **13.** 1.44. | **14.** 0.0024. | **15.** $0.039\frac{7}{11}$. | **16.** 2.904. |
| | **17.** $0.012\frac{5}{6}$. | **18.** 0.031. | |

### Pages 88, 89

| | | | |
|---|---|---|---|
| **1.** 80¢. | **2.** $1.13. | **3.** $1.79. | **4.** $1.09. |
| **5.** $7. | **6.** $1.12\frac{1}{2}$. | **7.** $6.87\frac{1}{2}$. | **8.** 7.2 mi. |
| **9.** $1.20. | **10.** 23 gross. | **11.** $0.09\frac{1}{2}$. | **12.** $2.69. |
| **13.** $2.40. | **14.** $92.50. | **15.** $129\frac{1}{4}$ mi. | **16.** 23¢. |
| **17.** 28¢. | **18.** $40. | **19.** $21.60. | **20.** $1.12. |
| **21.** $4.50. | **22.** $339. | **23.** 24 da. 3 hr. | **24.** 20.95 mi. |
| | **25.** 32 yd. | **26.** $4035, $2690. | |

### Page 93

It is desirable at this point to give a great deal of oral and written drill upon the tables which have just been reviewed. Pupils should be required to write the tables rapidly, the tables themselves being the answers to this work.

### Page 96

| | | |
|---|---|---|
| **1.** 34 pk. | **2.** 1035 rd. | **3.** 168 ft. |
| **4.** 320 sq. rd. | **5.** $2896\frac{1}{2}$ ft. | **6.** 66 in. |
| **7.** 34 oz. | **8.** 16 ft. | **9.** 11 qt. |
| **10.** 440 sq. rd. | **11.** 1280 cu. ft. | **12.** 630 cu. ft. |
| **13.** 169 cu. ft., 292,032 cu. in. | | **14.** 1092 sq. ft., 157,248 sq. in. |

### Page 97

| | | |
|---|---|---|
| **1.** 80 ft. | **2.** 41 ft. | **3.** 30 lb. |
| **4.** 11 yd. | **5.** 11 lb. | **6.** 2 hr. |
| **7.** 60 hr. | **8.** 16 rd. 11 ft. | **9.** 68 bu. |
| **10.** 1 T. 1255 lb. | **11.** 8 bu. 7 qt. 1 pt. | **12.** 3 sq. ft. 68 sq. in. |
| **13.** 0.27 mi. | **14.** $0.062\frac{1}{2}$ ft., $0.0208\frac{1}{3}$ yd. | **15.** $\frac{5}{6}$ mi. |

### Page 98

| | | |
|---|---|---|
| **1.** 50 lb. | **2.** 100 rd. | **3.** 50 mi. |
| **4.** 83 yd. | **5.** 71 ft. 1 in. | **6.** 51 lb. 2 oz. |
| **7.** 71 rd. 2 ft. | **8.** 151 mi. 5 rd. | **9.** 51 yd. 2 in. |
| **10.** 1 yr. 3 mo. 13 da. | **11.** 6 wk. 4 da. | **12.** 62 bu. 1 pk. |
| **13.** 165 yd. | **14.** 206 gal. | **15.** 104 mi. 88 rd. |

### Page 99

1. 19 yd.
2. 2 ft. 8 in.
3. 9 lb. 12 oz.
4. 5 rd. 10 ft.
5. 9 mi. 300 rd.
6. 9 yd. 16 in.
7. $8\frac{3}{4}$ yd.
8. 9 ft. $7\frac{3}{4}$ in.
9. 8 lb. $14\frac{2}{3}$ oz.
10. 7 rd. $15\frac{1}{4}$ ft.
11. 46 mi. $319\frac{3}{4}$ rd.
12. 5 yd. $31\frac{3}{4}$ in.

### Page 100

1. 179 ft. 2 in.
2. 170 lb. 9 oz.
3. 229 rd. 11 ft.
4. 85 qt. 1 pt.
5. 229 yd. 9 in.
6. 182 bu. 1 pk.
7. 154 gal. 2 qt.
8. 179 mi. 262 rd.
9. 334 mi. 1657 ft.
10. 200 cu. ft.
11. 255 sq. ft. 132 sq. in.
12. 31 ft. 11 in.
13. 20 lb. 15 oz.
14. 5 rd. $15\frac{1}{2}$ ft.
15. 15 mi. 229 rd.
16. 17 yd. 32 in.
17. 16 ft. 9 in.
18. 48 lb. 14 oz.
19. 28 rd.
20. 19 mi. 240 rd.
21. 4 yr. 10 mo. 29 da.
22. 4 yr. 4 mo. 18 da.

### Page 101

1. 1187 ft. 6 in.
2. 2283 lb. 12 oz.
3. 4709 yd. 1 ft.
4. 3916 mi. 267 ft.
5. 5134 ft. 9 in.
6. 9560 ft. 10 in.
7. 11,155 lb.
8. 20,723 lb. 15 oz.
9. 27,273 yd. 2 ft.
10. 30,712 yd. 11 in.

### Page 103

1. 30 lb. $2\frac{3}{8}$ oz.
2. 157 yd. $1\frac{2}{3}$ ft.
3. 11 ft. $2\frac{10}{11}$ in.
4. 3 yd. 2 in.
5. 73 ft. 6 in.
6. 20 lb. 3 oz.
7. 20.
8. 4.
9. $3\frac{2}{25}$.
10. 10.
11. 3.
12. 3.
13. 11.
14. $15\frac{19}{20}\frac{38}{77}$.
15. 92 hr. 42 min.
16. 150 bu.
17. 60,237 mi. 1940 ft.
18. 39 T. 600 lb.
19. 393 min. 36 sec.
21. 29,492 mi. 5110 ft.
22. 920 ft. 10 in.
23. 1145 ft. 10 in.
24. 35 ft.
25. 4 ft. 8 in.
26. 8.
27. 600.

### Page 104

1. 45 mi.
2. 20 mi., 135 mi.
3. $16{,}666\frac{2}{3}$.
4. \$9750.
5. \$2148.30.
6. $2\frac{13}{16}$ T.
7. 280 ft.
8. 55 bu.
9. \$66.67.

### Page 105

1. 7932.73 mi. 2. 2600.97 mi. 3. 2264.69 mi.
4. 435.19 mi. 5. 6511.68 mi., 1637.29 mi. 6. 1195.03 mi.
7. 1763.25 mi., 6656.19 mi., 7602.7 mi., 6381 mi.

### Page 106

1. 1.58. 2. 2.88. 3. 3.12. 4. 6.51. 5. 6.56.
6. 6.03. 7. 3.32. 8. 1.89. 9. 5.25. 10. 12.10.
11. 43.38. 12. 73.84. 13. 139 in.

### Page 107

1. 67.2. 2. 59.2. 3. 28.8. 4. 75.6. 5. 7.84.
6. 3.43. 7. 4.23. 8. 2.97. 9. 100.5. 10. 105.
11. 174.8. 12. 164.5. 13. 9.18. 14. 4.94. 15. 16.92.
16. 65.61. 17. 110.236 in., 692.912 in.

### Page 108

1. 4259.52. 2. 4392.63. 3. 2782.35. 4. 2424.558.
5. 6348.861. 6. 154.8504. 7. 280.0027. 8. 1815.268.
9. 6912.121. 10. 372.6437. 11. 715.3543. 12. 3598.4167.
13. 1876.2345. 14. 0.3. 15. \$136.69. 16. \$1829.23.
17. \$12,144.50. 18. \$100. 19. 230. 20. 170.2.
21. 38.4. 22. 921.6. 23. 56.05. 24. 35.04.
25. 12.06. 26. 18.9. 27. 1563.25. 28. 1244.35.
29. 3238.17. 30. 2165.44. 31. 408.04. 32. 1744.6.
33. 5517.54. 34. 5431.69. 35. 52.5525. 36. 32.7096.
37. 227.151. 38. 453.22. 39. 15.76204. 40. 53.9352.
41. 14.63775. 42. 282.9. 43. 53.63205. 44. 0.170625.
45. 2.3672. 46. 23.21. 47. $0.069\frac{1}{8}$.

### Page 109

1. \$29.75. 2. 138.8 ft. 3. \$1925.25.
4. \$24.15. 5. \$9037.50. 6. \$13,100, \$1663.70, \$245.625.
7. \$496.40. 8. 15.836 gal. 9. 17.444 in.
10. 42.432 ft. 11. 3494.54 sq. ft., \$5241.81. 12. 132.795432 ft.
13. 9128 sq. rd., 57.05 A., \$4278.75. 14. 27.232 mi.

### Page 110

| | | | |
|---|---|---|---|
| 1. 25. | 2. 2500. | 3. 100. | 4. 10,000. |
| 5. 17. | 6. 2.1. | 7. 1700. | 8. 2.3. |
| 9. 2.2. | 10. 1.7. | 11. 5000. | 12. 3.4. |
| 13. 50. | 14. 7.6. | 15. 2.46. | 16. 7.6. |

### Page 111

| | | | |
|---|---|---|---|
| 1. 2666.9. | 2. 246.96. | 3. 238.59. | 4. 266.43. |
| 5. 19.809. | 6. 1.3263. | 7. 15.151. | 8. 1.0087. |
| 9. 85.46. | 10. 160.6. | 11. 5.281. | 12. 74.06. |
| 13. 0.159. | 14. 0.122. | 15. 79.572. | 16. 27.006. |
| 17. 57.529. | 18. 30.09. | 19. 10.06. | 20. 0.35. |
| 21. 9.296. | 22. 830.025. | 23. 225.024. | 24. 7099.488. |
| 25. 2546.2116. | 26. 329.4308. | 27. 317.2. | 28. 196.54. |
| 29. 74. | 30. 185. | 31. 235.68. | 32. 7.198. |

### Page 112

| | | | |
|---|---|---|---|
| 1. 0.0345. | 2. 0.01624. | 3. 1.168. | 4. 0.01031. |
| 5. 0.0001052. | 6. 0.00826. | 7. 0.00131. | 8. 0.0121. |
| 9. 0.0311. | 10. $7.605. | 11. $14.6755. | 12. $4.1567. |
| 13. 11.853. | 14. 270.565. | 15. 6.213−. | 16. 1568.889−. |
| 17. 45.351. | 18. 3.114. | 19. 7810.72. | 20. 11.326−. |

21. 66.104.

### Page 113

| | | | | | |
|---|---|---|---|---|---|
| 1. 0.21. | 2. 3. | 3. 56. | 4. $1\frac{1}{3}$. | 5. 3.5. | 6. 1. |
| 7. $\frac{6}{35}$. | 8. $3\frac{1}{3}$. | 9. 10. | 10. 0.1. | 11. 0.1. | 12. 0.3. |
| 13. 1. | 14. 7. | 15. 5. | 16. 9. | 17. 5. | 18. 2. |

### Pages 116, 117

1. 14 A., $203. 2. 3430 lb., 7 bales. 3. $8.75, $2.80.
4. $3.50. 5. 10 cu. ft., 17,280 cu. in. 6. $3.50.
7. $240.10. 8. 24 da., 6 da.
9. 2,675,363$\frac{1}{4}$ T. 10. 5,992,306 bales.
11. $92,196,000, Texas; $46,098,000, Ga.; $40,976,000, Miss.; $35,854,000, Ala.
12. $5,430,780. 13. 2,214,103 A., 1,107,051$\frac{1}{2}$ A. 14. $267,878,240.
15. $90,240,000, $60,160,000, $30,080,000, $15,040,000.

### Page 118

**1.** $39. **2.** $50.75. **3.** 261,740,817 lb. **4.** 103,989,126 lb.
**5.** 317,627,100 lb. **6.** 2,722,518,000 lb.
**7.** 9,627,000 lb., 7,701,600 lb., 6,931,440 lb., 6,931,440 lb.

### Page 119

**1.** $298.90. **2.** $2.25. **3.** $13,800. **4.** $1.26.
**5.** Better by car lot. $50.35, straight freight; $46, car lot.

### Page 122

**1.** 199,902,328 lb. **2.** 144,801 sq. mi. **3.** 1,625,240 sq. mi.
**4.** 12,224,102 T. **5.** 672,146 sq. mi. **6.** $19.36.
**7.** 8,750,000 bales. **8.** $68,225,214.

### Page 123

**1.** 1,098,861. **2.** 25,230,000. **3.** 2,442,549 oz.
**4.** 4,939,538 sq. mi. **5.** 65,500,000 lb. **6.** 54 sq. mi., 32,003,069.
**7.** 43,532,445.

### Page 124

| | | | |
|---|---|---|---|
| **1.** $985.03. | **2.** $1614.02. | **3.** $1535.04. | **4.** $928.70. |
| **5.** $1663.92. | **6.** $3553.04. | **7.** $1762.10. | **8.** $2330.81. |
| **9.** $341.71. | **10.** $572.88. | **11.** $775.14. | **12.** $701.25. |
| **13.** $2447.13. | **14.** $397.38. | **15.** $18,155.34. | **16.** $18,969.21. |
| **17.** $4790.02. | **18.** $12,841.09. | **19.** 526.26. | **20.** 464.69. |
| **21.** 20,672. | **22.** 32,985. | **23.** 87,896.4. | **24.** 26,015.9. |
| **25.** $4.06. | **26.** $1.19. | **27.** $6.85. | **28.** $1.05. |
| **29.** $7.11. | **30.** $1.25. | **31.** $2\frac{1}{161}$. | **32.** 1. |
| **33.** 5. | **34.** 3.75. | **35.** 7. | **36.** 2. |

### Page 126

**1.** $58.75. **2.** $6.90. **3.** $3.83. **4.** $4.42. **5.** $19.65.
**6.** $4.37. **7.** $20.26. **8.** $4.63. **9.** $6.86.

### Page 127

**1.** 4.91. **2.** 2. **3.** 3.57. **4.** 2.708. **5.** 1.01. **6.** 8.34.
**7.** 1.01. **8.** 3.94. **9.** 24.88. **10.** 7 wk. **11.** 5 yr.

### Page 128

1. $33\frac{1}{3}$, 50. 2. 10, 15, 25. 3. 1, 2. 4. 1, 6, 7.
5. 896.5. 6. 2496.9375. 7. 2153.4. 8. 10.8106.
9. 724.098. 10. 525.9375. 11. 10.5. 12. 115.41.
13. 91.4. 14. 3.16. 15. 1.46. 16. 27,500.
17. 3. 18. $2\frac{4}{8}\frac{6}{7}$. 19. 2. 20. 42.
21. 2. 22. 60.

### Page 129

1. 9%, 0.09. 2. 17%, 0.17. 3. 0.28, $\frac{28}{100}$. 4. 37%, 0.37.
18. 1, 100 per cent.

### Page 131

1. 316. 2. 148. 3. 166.5. 4. \$854.
5. \$2458.50. 6. 108 mi. 7. 489 bu. 8. 24 ft. 2 in.
9. \$11.41. 10. \$9.21. 11. 9 lb. 1 oz. 12. 1 lb. 1 oz.
13. \$2.70. 14. \$0.98. 15. 19, 133. 16. 88, 1320.

### Page 132

1. 297. 2. 474. 3. 84. 4. \$6.85.
5. \$1.30. 6. \$13.31. 7. 498 rd. 8. 1629 bu.
9. 74 ft. 3 in. 10. 159 men. 11. 121 bu. 1 qt. 12. 94 lb. 10 oz.

### Page 133

1. 0.25. 2. $0.33\frac{1}{3}$. 3. 0.01. 4. 0.025. 5. 0.075.
6. 1.25. 7. 0.005. 8. 0.0075. 9. 2.00. 10. 0.003.
11. $\frac{8}{25}$. 12. $\frac{6}{25}$. 13. $\frac{9}{25}$. 14. $\frac{17}{100}$. 15. $\frac{33}{40}$.
16. $\frac{17}{25}$. 17. $\frac{1}{16}$. 18. $1\frac{1}{4}$. 19. $3\frac{3}{4}$. 20. $\frac{1}{3}$.
21. 50%. 22. 80%. 23. $87\frac{1}{2}$%. 24. $33\frac{1}{3}$%. 25. 140%.

### Page 134

1. $14\frac{2}{7}$%, $28\frac{4}{7}$%, $42\frac{6}{7}$%, $57\frac{1}{7}$%, $71\frac{3}{7}$%, $85\frac{5}{7}$%, 1.
2. $11\frac{1}{9}$%, $22\frac{2}{9}$%, $33\frac{1}{3}$%, $44\frac{4}{9}$%, $55\frac{5}{9}$%, $66\frac{2}{3}$%, $77\frac{7}{9}$%, $88\frac{8}{9}$%, 1.
3. $8\frac{1}{3}$%, $16\frac{2}{3}$%, 25%, $33\frac{1}{3}$%, $41\frac{2}{3}$%, 50%, $58\frac{1}{3}$%, $66\frac{2}{3}$%, 75%, $83\frac{1}{3}$%, $91\frac{2}{3}$%, 1.
4. $6\frac{1}{4}$%, $12\frac{1}{2}$%, $18\frac{3}{4}$%, 25%, $31\frac{1}{4}$%, $37\frac{1}{2}$%, $43\frac{3}{4}$%, 50%, $56\frac{1}{4}$%, $62\frac{1}{2}$%, $68\frac{3}{4}$%, 75%, $81\frac{1}{4}$%, $87\frac{1}{2}$%, $93\frac{3}{4}$%, 1. 5. $1\frac{1}{5}$ or 1.2.
6. $1\frac{1}{3}$ or $1.33\frac{1}{3}$. 7. $\frac{1}{4}$ or 0.25. 8. $\frac{1}{2}$ or 0.5. 9. $1\frac{3}{5}$ or 1.6.
10. $\frac{1}{10}$ or 0.1. 11. $\frac{1}{5}$ or 0.2.

### Page 135

1. 1204.5. 2. $5980. 3. $149.76. 4. 4089 ft.
5. $593.75. 6. $2754. 7. $248.40. 8. $700.
9. 1848 ft. 10. 1525.2 lb. 11. 2170 mi. 12. $12.60.

### Page 136

1. 123, 764.5, $22.22.
2. 243, $307.74, 1760 ft.
3. 880 ft., $205.76, 288 cu. in.
4. $14.
5. $74.70.
6. $64.
7. $595.
8. $466.50 gain.

### Page 137

1. 50.75. 2. $216. 3. 187 da. 4. $980.
5. 2250 lb. 6. 191.1 lb. 7. $1500. 8. $85,808,000.

### Page 138

1. 5%, $3\frac{1}{3}$%.
2. 3%, 20%.
3. 4%, 5%.
4. 6%, $16\frac{2}{3}$%.
5. $33\frac{1}{3}$%, $6\frac{2}{3}$%.
6. 25%, 1%.
7. $266\frac{2}{3}$%, 150%.
8. $33\frac{1}{3}$%, 20%, 10%.
9. 11%.
10. 12%.
11. $275, 11%.

### Page 139

1. 1180. 2. 40. 3. $190. 4. 220.

### Page 140

1. 20. 2. 200. 3. 100. 4. 220.
5. 1000. 6. 2200. 7. 172. 8. 250.
9. 33. 10. 7000. 11. $35. 12. $65.

### Page 141

1. $0.48. 2. $210. 3. $27. 4. $9, $8.
5. $0.204, $0.17.

### Page 142

1. 1,120,000 dark, 2,080,000 light.
2. 483,000 dark, 1,817,000 light.
3. 735,000 dark, 765,000 light.
4. 3.9%.
5. 36.8%.
6. 1.3%.
7. 18.6%.

### Pages 143, 144

1. $310,500,000, $103,500,000. 2. 360,000, 9,054,612.
3. 10,003. 4. 209,700, 1,922,700. 5. $259,776,000. 6. $5150.
7. $5370. 8. $6⅔. 9. $6.60. 10. $1.58. 11. $1.20, $60.
12. $10,734,319⅓, $18,240,660; $26,181,266⅔, $64,855,680; $2,356,314, $12,160,440; $6,080,220.

### Pages 145, 146

1. 111,367,809 lb. 2. 65%. 3. 42%.
4. 37%. 5. 36,808. 6. 248 bbl., $12,152.
7. 79 bbl., $2765. 8. 6700 lb., $18,090. 9. 11,180 men, 250%.
10. $315,900. 11. $549,900. 12. $364,000.
13. 21,000 sq. ft. 14. 2333⅓ sq. ft., ⅓. 15. 187 lb. 8 oz., $3.75.

### Pages 147, 148

1. $11.20. 2. $1.05. 3. $2.20. 4. $5.60.
5. $64.50. 6. $56.20. 7. $42.55. 8. $209.95.
9. $329. 10. $14.42. 11. $10.66. 12. $396.
13. $39.09. 14. $220. 15. $546.25. 16. $448.14.
17. $568.75. 18. $374. 19. $275.20. 20. $10.20.
21. $5333.72. 22. $44.03⅓. 23. $13.16. 24. $3303.30.
25. $9.62. 26. $14.88. 27. $1.53. 28. $0.51.
29. $23.25. 30. $1.08, $0.96, $0.90. 31. $0.81, $0.72.
32. $125. 33. $650.25.

### Page 149

1. 12,096, 1392. 2. $27. 3. $0.60. 4. 132.

### Page 150

1. 408, 8. 2. 36,240. 3. $1.25.
4. $1.19, $0.81, $1.50. 5. $0.32.

### Page 152

1. 827 ft. 2. 117 ft. 8 in. 3. 129.5 ft. 4. 2 right angles.

### Page 153

**1.** 37′ 9″. **2.** 42⅓ yd. **3.** 29′ 5″. **4.** 27′ 7″.
**5.** 15 yd. 1¼ ft. **6.** 29′ 8″. **7.** 10′ 6⅞″. **8.** 2′ 10″.
**9.** 56 mi. 250 rd. **10.** 33½′. **11.** 17.6′, 35.2′, 35.2′. **12.** 9′, 8′, 7′.

### Page 155

**1.** 331.8 sq. in. **2.** 242.55 sq. ft. **3.** 4662.08 sq. ft.
**4.** 8.75 sq. in. **5.** 28½ sq. ft. **6.** 136.125 sq. ft.
**7.** 425.88 sq. in. **8.** 118.925 sq. in. **9.** 195 sq. ft.
**10.** 142.725 sq. ft.

### Page 156

**1.** 1292 cu. in. **2.** 1830 cu. ft. **3.** $83.20.
**4.** 168 sq. ft., 468 sq. ft., 168 sq. ft., 1512 cu. ft.

### Pages 157, 158

**1.** $134\frac{2}{8}$ bu. **2.** 108 bu. **3.** $7\frac{1}{5}$ bu. **4.** 900 gal.
**5.** 6 gal. **6.** $382\frac{13}{16}$ gal.; using 231 cu. in. = 1 gal., $381\frac{9}{11}$ gal.
**7.** $43\frac{13}{29}$ bbl. **8.** $59\frac{8}{29}$ bbl. **9.** $2\frac{31}{58}$ bbl. **10.** $65\frac{13}{35}$ T.
**11.** $24\frac{2}{7}$ T. **12.** 5.544 T. = $5\frac{68}{125}$ T. **13.** 14.88 T. **14.** $7\frac{7}{64}$ cords.
**15.** $8\frac{15}{16}$ cords. **16.** 30. **17.** 11,000 cu. ft.
**18.** $28\frac{1}{2}$ gal. **19.** $7\frac{367}{512}$ gal. **20.** $1.80.
**21.** $85\frac{3}{99}$ perches. **22.** $8662\frac{1}{2}$, or, say, 8700 bricks. **23.** $12.50, $11.67.

### Pages 164-167

**1.** $5.81. **2.** 111. **3.** 17 cars. **4.** 19 cars.
**5.** $4166.67. **6.** 354 bu. **7.** 625 bu. **8.** 125 chests.
**9.** 8064 bricks. **10.** 4 mi. **11.** 54,912 ties. **12.** 6 min. 25 sec.
**13.** 7. **14.** $337\frac{1}{2}$ T. **15.** $13\frac{1}{5}$ min. **16.** $3,000,000.
**17.** $15,625,000, $12,625,000. **18.** 14.67 ft., 117.33 ft., 10 mi., 80 mi.
**19.** $408. **20.** $1.52. **21.** $1.26. **22.** 56 yd.
**23.** 90 pieces. **24.** 29 yd. **25.** $4.75. **26.** $2665.
**27.** $125, $175. **28.** $5760. **29.** $270. **30.** $164.25.
**31.** $0.48. **32.** $9. **33.** $3.76. **34.** $202.50 gain.

### Page 168

**1.** $518.42. **2.** $3250. **3.** $2100.
**4.** 16 bbl., 170 bbl., 1860 bbl. **5.** $7.77, $4.41, $5.18, $9.03, $18.06.
**6.** $0.56, $0.91, $1.16, $1.59. **7.** 12,300 lb., 11,300 lb., 77,000 lb.

### Pages 169, 170

**1.** 64 da. **2.** 8 da., 32 da., $10\frac{2}{3}$ da., $6\frac{2}{5}$ da. **3.** 40 da. **4.** $7\frac{1}{5}$ da.
**5.** 189 bu., 189 bu. **6.** 15 da. **7.** 3159 bricks. **8.** $960.
**9.** 15 da., 3 men, 5 men, 2 men. **10.** 75 men. **11.** 10 da.
**12.** 30 A. **13.** $14\frac{2}{5}$ lb. **14.** 22 A. **15.** 394.2 A. **16.** 177 A.

### Page 171

**1.** $66. **2.** $5400. **3.** $504, $36, $198, $720.
**4.** $30, $75, $112.50, $8.25. **5.** 1470 lb., 3430 lb., 4410 lb., 17,150 lb.
**6.** $105, $300, $4200. **7.** 294 mi., $367\frac{1}{2}$ mi., $477\frac{3}{4}$ mi.
**8.** 195 lb., 455 lb., 585 lb.

### Pages 172, 173

**1.** $47.50. **2.** $11,500. **3.** 7840 ft., 10,640 ft., 117,600 ft.
**4.** 1948 lb., 3409 lb., 7792 lb., 12,175 lb. **5.** 54 ft. **6.** 15,000 bricks.
**7.** 24 ft. **8.** 55 ft. **9.** $4\frac{1}{11}$ hr. **10.** $9.60, $9, $9.60.
**11.** 8 men. **12.** $9\frac{3}{5}$ T. **13.** 900, 800. **14.** 1,950,000,000.

### Page 174

**1.** $362,700. **2.** $2\frac{161}{000}$ mi. **3.** $3\frac{2}{11}$ mi. **4.** 376,320. **5.** $13.20.

### Page 175

**1.** 124.8 lb. **2.** $1704. **3.** $6.30. **4.** 213.6 mi. **5.** 122. **6.** 284 A.

### Page 176

**1.** 548. **2.** 222.5. **3.** $169\frac{1}{4}$. **4.** 276.25. **5.** 41.83.
**6.** 634.11. **7.** 129. **8.** 383.7. **9.** 23.5. **10.** 16.

### Page 178

**1.** 64. **2.** 7. **3.** 30. **4.** 10. **5.** 30. **6.** 20.

### Page 179

**1.** $350. **2.** 625. **3.** 30. **4.** 119. **5.** $337. **6.** 13 yr.

### Page 180

**1.** $2812.50. **2.** $90. **3.** $6250. **4.** $76.50. **5.** 3420. **6.** $62.

### Page 181

**1.** \$61.50. **2.** \$2.10. **3.** \$925. **4.** \$48.20. **5.** \$12.10.
**6.** \$69.55. **7.** \$5025. **8.** \$3971. **9.** \$3267. **10.** \$760.80.
**11.** \$9205. **12.** \$381.88.

### Page 182

**1.** \$1071.40. **2.** \$22.68. **3.** \$30,776.80. **4.** \$35,850. **5.** \$15.
**6.** \$4936.25. **7.** \$591.10. **8.** \$21.04. **9.** \$780. **10.** 535.
**11.** 114 bbl.

### Page 183

**1.** $3\frac{1}{3}\%$. **2.** 20%. **3.** 25%. **4.** 60%. **5.** $13\frac{1}{3}\%$.
**6.** $33\frac{1}{3}\%$. **7.** 5%. **8.** $16\frac{2}{3}\%$. **9.** $11\frac{1}{9}\%$. **10.** $12\frac{1}{2}\%$.
**11.** 58%. **12.** 48%, 52%. **13.** 75%. **14.** $66\frac{2}{3}\%$, 50%, $58\frac{1}{3}\%$. **15.** 2, 4.

### Page 184

**1.** \$7. **2.** \$6.50. **3.** 28 ft. 4 in. **4.** $1\frac{1}{4}$ yd.
**5.** 181 lb. 4 oz. **6.** 250 ft. **7.** \$2110. **8.** 145.
**9.** Yes. **10.** 120%, \$50. **11.** 140 girls, 259 pupils. **12.** \$451.

### Page 185

**1.** \$1250. **2.** 150 A. **3.** 55 lb. **4.** 61,000. **5.** 7134.

### Page 186

**1.** 12. **2.** 75. **3.** \$900. **4.** 390. **5.** 52 ft. **6.** 650 ft., 65 ft.

### Pages 187-189

**1.** 132. **2.** 371. **3.** \$125.
**4.** 19.18 mi. **5.** 54. **6.** \$450, \$900.
**7.** 84. **8.** 76. **9.** $0.8\frac{3}{121}\%$.
**10.** \$1512. **11.** \$280.80. **12.** $78\frac{246}{333}\%$.
**13.** $60\frac{5}{17}\%$. **14.** $23\frac{169}{597}\%$, $26\frac{3823}{5433}\%$. **15.** 50,309,000 sq. mi.
**16.** 90%, 71 lb. **17.** $2755\frac{10}{27}\%$. **18.** $19\frac{3554}{5097}\%$, $12\frac{2296}{4017}\%$.
**19.** $12\frac{552}{809}\%$, $27\frac{2641}{4017}\%$. **20.** 5%. **21.** 4.21 lb.
**22.** \$3915. **23.** 35.3675 lb. **24.** 32%.
**25.** $9\frac{1}{2}\%$. **26.** 20 cu. ft., 25 cu. ft. **27.** \$1515.63.
**28.** 40%. **29.** \$100, \$1600, \$500, \$300, \$320.

### Page 190

1. $51\frac{40}{741}\%$. 2. $57\frac{193}{501}\%$. 3. $317\frac{1}{7}\%$. 4. 15%. 5. 0.75%.

### Page 191

1. \$596.90. 2. \$297.50. 3. \$233.75. 4. \$276.25.
5. \$19.55. 6. \$36.40. 7. \$360.24. 8. \$140.40.
9. \$180.60. 10. \$878.13. 11. \$1466.40. 12. \$186.
13. \$22.22. 14. \$2654.31.

### Page 192

1. \$112. 2. \$432.25. 3. \$318.75. 4. \$431.20.
5. \$526.50. 6. \$252. 7. \$229.50. 8. \$222.75.
9. \$140. 10. \$450. 11. \$50. 12. No difference.

### Page 193

1. \$79.63. 2. \$15.31. 3. \$147.05. 4. \$2.43.

### Page 195

1. 20%. 2. $22\frac{22}{49}\%$. 3. $2\frac{7}{49}\%$. 4. $11\frac{1}{9}\%$ gain.

### Page 196

1. $15\frac{1}{3}\%$. 2. \$0.10. 3. \$0.20. 4. 25%. 5. 20%. 6. \$7.50 gain.

### Page 197

1. $33\frac{1}{3}\%$. 2. \$0.24. 3. $29\frac{3}{5}\%$. 4. \$72, 25%. 5. $6\frac{1}{4}\%$.
6. \$2.52 gain.

### Page 198

1. 50% above. 2. $66\frac{2}{3}\%$ above. 3. $85\frac{5}{7}\%$ above. 4. $78\frac{4}{7}\%$ above.
5. 100% above. 6. 120% above. 7. $16\frac{2}{3}\%$ above. 8. $29\frac{1}{31}\%$ above.
9. 30%, 80%, above. 10. 50% above ; at cost.

### Page 199

1. \$21.25. 2. \$66. 3. \$32.10. 4. \$29.20, \$262.80. 5. \$196.

### Page 200

1. \$11.44. 2. \$17.38. 3. \$784. 4. \$2.10, \$2.95. 5. \$253.20.

### Page 201

**1.** \$473.60. **2.** \$21.48. **3.** \$168. **4.** $5\frac{1}{3}\%$.
**5.** \$4.88. **6.** \$66.70. **7.** \$787.50. **8.** \$25.53.

### Page 202

**1.** \$22.50, \$15. **2.** \$47.50, \$28.50. **3.** \$9.40, \$11.75.
**4.** \$0.81. **5.** \$503.50.

### Page 203

**1.** \$26.08. **2.** \$18.75. **3.** \$0.95. **4.** \$9.75. **5.** \$13.
**6.** \$1.89. **7.** \$11. **8.** \$18.01. **9.** \$1.88. **10.** \$43.50.
**11.** \$13.76. **12.** \$3.78. **13.** \$26.75. **14.** \$31.26. **15.** \$2.40.
**16.** \$2.05. **17.** \$3.50. **18.** \$4.38. **19.** \$2.25. **20.** \$9.38.
**21.** \$11.25. **22.** \$0.90. **23.** \$13.50. **24.** \$10.13. **25.** \$0.35.
**26.** \$4.88. **27.** \$14.40.

### Page 204

**2.** \$4.12. **3.** \$47.96. **4.** \$212.75.

### Page 205

**1.** \$64.75. **2.** \$132.50. **3.** \$68.25. **4.** \$145.60, \$445.20, \$393.75.
**5.** \$3.45. **6.** \$257.50, \$307.50, \$153, \$386.25. **7.** \$1753.10.

### Page 206

**1.** 2880 cu. ft., $5420\frac{5}{8}$ cu. ft. **2.** 288 sq. ft., $551\frac{1}{7}$ sq. ft.
**3.** 680 sq. ft., $924\frac{1}{3}$ sq. ft. **4.** $5\frac{5}{32}$ cd., $18\frac{3}{64}$ cd.
**5.** $1781\frac{9}{11}$ gal., $2025\frac{2}{77}$ gal.

### Pages 207, 208

**1.** 3,537,440.9 cu. in., $5,899,318.86\frac{2}{3}$ cu. in. **2.** 1,015,000 cu. ft.
**3.** \$18,000. **4.** $21\frac{12}{35}\%$, $17\frac{5}{7}\%$. **5.** 45 lb. **6.** $50\frac{35}{77}\%$, $53\frac{41}{77}\%$.
**7.** 2%, $2\frac{2}{49}\%$. **8.** 25%, 20%. **9.** $525\frac{5}{77}$ gal. **10.** $1\frac{3}{4}$ ft.
**11.** 72 ft. **12.** Any two factors of 1155 give the number of inches.
**13.** \$13.96. **14.** $1520\frac{7}{8}$ cu. in., $132\frac{1}{4}$ sq. in.

### Pages 209, 210

**1.** 1234′ 9″. **2.** 919′ 5″. **3.** 1043′ 6″.
**4.** 759 yd. 9 in. **5.** 617 gal. 2 qt. **6.** 1014 bu. 1 pk.
**7.** 282′ 4″. **8.** 10 lb. 1 oz. **9.** 123 lb. 9 oz.

**10.** 4′ 4″. **11.** 74 yd. 29 in. **12.** 9 rd. $3\frac{1}{8}$ yd.
**13.** 275 bu. 2 pk. **14.** 8 bu. 1 pk. **15.** 48 gal. 2 qt.
**16.** 6 mi. $303\frac{1}{3}$ rd. **17.** 4286 lb. 10 oz. **18.** 1704 lb. 6 oz.
**19.** 7972 lb. **20.** 1257 lb. **21.** 18,237′ 6″.
**22.** 31,640′. **23.** 4456 yd. 2 ft. **24.** 25,146 yd. 2 ft.
**25.** 9900 gal. **26.** 4552 bu. **27.** 21 lb. 14 oz.
**28.** 31 lb. $11\frac{1}{4}$ oz. **29.** 162. **30.** 109.
**31.** 151 lb. 4 oz. **32.** 107 lb. $1\frac{1}{2}$ oz. **33.** 25′ 2″.
**34.** 17′ $\frac{1}{5}$″. **35.** 11 lb. $\frac{3}{5}$ oz. **36.** 25 lb. $\frac{1}{5}$ oz.
**37.** 476. **38.** $1061\frac{1}{4}$. **39.** 370.
**40.** 740. **41.** 2 yr. 25 da. **42.** 1 yr. 9 mo. 27 da.
**43.** 4 yr. 10 mo. 8 da. **44.** 143 yr. 5 mo. 27 da. **45.** 1 yr. 6 mo. 28 da.
**46.** 2 yr. 2 mo. 25 da. **47.** 1 yr. 6 mo. 28 da. **48.** 1 yr. 5 mo. 29 da.
**49.** 6 min. 43 sec. **50.** Monday. **51.** 38 hr.
**52.** 11 hr. 51 min. 13 sec. A.M. **53.** $2\frac{5}{11}$.
**54.** $63\frac{7}{11}$ min. **55.** 90 da., 230 da., 100 da.
**56.** $99\frac{2}{3}$ sq. yd.

## Page 211

**1.** $26.16, $13.08, $8.72, $17.44, $21.80.
**2.** $18, $9, $15, $1.50, $13.50.
**3.** $23.46, $46.92, $70.38.
**4.** $3.04, $15.20, $30.40.
**5.** $21.60, $1.80, $0.90, $0.30, $0.06, $0.42.
**6.** $7.02, $3.51, $2.34, $1.76, $0.59, $0.20.

## Page 212

**1.** $7.50. **2.** $11. **3.** $1127.50. **4.** $87.50. **5.** $3.15.
**6.** $4.10. **7.** $87.50. **8.** $11.33. **9.** $47.25. **10.** $286.88.

## Page 213

**1.** $27.31. **2.** $24.75. **3.** $42.33. **4.** $9.79. **5.** $22.78.
**6.** $100.94. **7.** $99.45. **8.** $244.38. **9.** $42.50. **10.** $5.69.
**11.** $19.53. **12.** $109.50. **13.** $53.70. **14.** $357.58.

## Page 214

**1.** $76.88. **2.** $26.50. **3.** $260.42. **4.** $424.

### Page 215

**1.** \$0.38, \$74.62. **2.** \$0.45, \$89.55. **3.** \$3.13, \$246.87.
**4.** \$3.44, \$271.56. **5.** \$7.08, \$417.92. **6.** \$4.50, \$295.50.
**7.** \$11.25, \$363.75. **8.** \$6, \$394. **9.** \$1.13, \$148.87.
**10.** \$1.83, \$398.17. **11.** \$2, \$198. **12.** \$1.31, \$173.69.
**13.** \$5.50, \$294.50. **14.** \$2.92, \$497.08. **15.** \$26.03, \$2948.97.
**16.** \$65.88, \$4184.12. **17.** \$2.08, \$1247.92. **18.** \$6.25, \$2493.75.
**19.** \$5, \$1495. **20.** \$7.81, \$1242.19. **21.** \$12.60, \$2737.40.
**22.** \$39.60, \$3480.40.

### Page 216

**1.** \$371. **2.** \$386.25. **3.** \$825. **4.** \$78.75. **5.** \$293.75.
**6.** \$133.75. **7.** \$282.08. **8.** \$369.69. **9.** \$311.25.

### Page 218

**1.** 20. **2.** 9. **3.** 8. **4.** 15. **5.** 10. **6.** 5. **7.** 3. **8.** 4.
**9.** 4. **10.** 6. **11.** 50. **12.** 10. **13.** 30. **14.** 25. **15.** 2 ft. **16.** 4 ft.

### Page 219

**1.** \$80, \$20; \$70, \$30; \$10, \$90. **2.** 33 : 40. **3.** 18, 12.
**4.** 1,008,000 sq. mi.

### Pages 220-222

**1.** $\frac{9}{10}$. **2.** \$650. **3.** \$550. **4.** \$300.
**5.** 16 lb., 16 lb., 8 lb. **6.** 8 lb., 4 lb., 9 lb., 3 lb., 8 lb.
**7.** 850%, 770%, 1200%, 840%. **8.** $6\frac{1}{4}$ gal.
**9.** $357\frac{1}{7}$ lb., $257\frac{1}{7}$ lb. **10.** 9 lb., 3.3 lb., 2.7 lb.
**11.** $12\frac{1}{2}$%, $12\frac{1}{2}$%, 75%. **12.** $11\frac{1}{9}$%. **13.** \$13.33.
**14.** \$13.25, first. **15.** \$20.25. **16.** 4760 bu.
**17.** 1 : 6. **18.** \$626.25. **19.** 50%.
**20.** \$0.02 per lb., ducks; \$0.$04\frac{2}{3}$ per lb., chickens.

### Page 223

**1.** 36. **2.** 18. **3.** 82. **4.** 175. **5.** 4. **6.** 56.
**7.** 160. **8.** 455. **9.** 114. **10.** 13. **11.** 5. **12.** 15 boys.

### Page 225

**1.** 17,000,000. **2.** 27 ft. **3.** 1 : 160 = 50,000 : 8,000,000.
**4.** 3 in. **5.** $12\frac{1}{2}$ in. **6.** 27 ft.

### Page 226

1. 1350 lb. 2. $12\frac{1}{2}$ lb. 3. 3.675 lb. 4. 2.28 lb.
5. 0.18 lb. 6. 1034.8 lb. 7. $167\frac{2}{3}$ hhd. 8. 300 lb.
9. 2180. 10. 5.655 gal. 11. 10.85 ft.

### Page 227

1. 19%. 2. $87\frac{1}{2}$%. 3. 93%. 4. 110.4 oz.
5. 90%. 6. 4%. 7. 875 T.

### Page 228

1. 30 T., 16 T. 2. $33\frac{1}{3}$ bu. 3. 11. 4. 80%.
5. $6\frac{2}{3}$%, $6\frac{1}{4}$%. 6. $16\frac{2}{3}$%, $24\frac{4}{9}$%, 75%. 7. $33\frac{1}{3}$%, 25%.
8. 70 : 56 = 60 : 48. 9. 33 : 26, 26 : 33, 1, 11 : 12.

### Page 229

1. $\frac{125}{216}$ oz. 2. 125 lb. 3. 15 lb. 4. 1206.25 lb.
5. $5250. 6. 70 lb. 7. 15 lb. 8. 4975 lb.

### Page 230

1. 261.45 lb.
2. 8.549 lb., 2.13725 lb., 1.068625 lb.
3. 6.557 lb., 206.5455 lb.
4. 5.81 lb., 183.015 lb.
5. 559.375 lb., 74.285 lb.
6. 8.217 lb., 258.8355 lb.
7. 864 gal.
8. 10.292 lb.

### Page 231

1. 34°. 2. $2\frac{1}{2}$°. 3. $45\frac{1}{2}$°, 35°, 12°.

### Page 232

1. 1,050,000 sq. mi. 2. 94.42%. 3. $410.
4. $2000. 5. $19,828,200. 6. 2500 trees.

### Page 233

1. 29 ft. 2. 43 ft. 2 in. 3. 55 ft., 51 ft. 4. 66 ft. 5. 40 ft.

### Page 234

1. $13\frac{1}{3}$ bd. ft. 2. $5\frac{5}{6}$ bd. ft. 3. $9\frac{1}{3}$ bd. ft. 4. $16\frac{1}{2}$ bd. ft.
5. $19\frac{1}{6}$ bd. ft. 6. $18\frac{5}{12}$ bd. ft. 7. $9\frac{3}{4}$ bd. ft. 8. 12 bd ft.

**9.** $11\frac{2}{3}$ bd. ft. **10.** $32\frac{2}{3}$ bd. ft. **11.** 21 bd. ft. **12.** $14\frac{2}{3}$ bd. ft.
**13.** 1 bd. ft. **14.** $2\frac{2}{3}$ bd. ft. **15.** 4 bd. ft. **16.** $5\frac{1}{3}$ bd. ft.
**17.** $8\frac{1}{3}$ bd. ft. **18.** 10 bd. ft. **19.** 6 bd. ft. **20.** 28 bd. ft.
**21.** 15 bd. ft. **22.** 9 bd. ft. **23.** 30 bd. ft. **24.** 15 bd. ft.

### Page 235

**1.** $8.64. **2.** $17.14. **3.** $12.96.
**4.** $71.68. **5.** 2700 bd. ft. **6.** $46.51+

### Pages 236, 237

**1.** 900 bd. ft. **2.** $15.19. **3.** $283\frac{1}{2}$ bd. ft. **4.** 3375.
**5.** 188 bd. ft., $5.64. **6.** 735 bd. ft. **7.** 2552 bd. ft. **8.** 4560 bd. ft.
**9.** $149.09. **10.** $184.58. **11.** $973.10. **12.** $1381.77.

### Page 238

**1.** $93\frac{5}{9}$ sq. yd. **2.** $852\frac{3}{9}$ sq. yd. **3.** $41.81. **4.** $667.80.

### Page 239

**1.** 20 yd. **2.** $32\frac{5}{6}$ yd. **3.** $55\frac{1}{12}$ yd. **4.** $46\frac{1}{8}$ yd.
**5.** $24\frac{1}{3}$ yd. **6.** $48.93. **7.** $56.93. **8.** $46.53.
**9.** $39.67. **10.** $29.47. **11.** $19.07. Answers 6–11 to nearest cent.

### Page 240

**1.** $7.20. **2.** 21 rolls, $7.35. **3.** $4.50.

### Page 241

**1.** 3200 sq. ft., 4800 sq. ft. **2.** 32. **3.** 30. **4.** 100.
**5.** 144 sq. ft., 108 sq. ft., 108 sq. ft., 216 sq. ft., 216 sq. ft.
**6.** 9800 sq. ft., 8865 sq. ft. without house.

### Page 243

**1.** 1521 lb., 46.4 lb. **2.** 91.4 lb., 91.4%. **3.** $6\frac{2}{3}$%.
**4.** $20,439,000. **5.** $80,000,000. **6.** $351,204,000.
**7.** $23,618\frac{2}{3}$ loads. **8.** $36\frac{2}{3}$%. **9.** 12,806,000 T.

### Page 244

**1.** 12%. **2.** $1.357. **3.** 842,600,000 lb.
**4.** $34,090\frac{10}{11}$ mi. **5.** $15.84. **6.** 363 T., 265 T., $36\frac{52}{53}$%.
**7.** 15,186,400 T.

### Page 245

1. \$26.25. 2. \$400. 3. \$0.066 per lb. 4. $53\frac{1}{2}$ ct., or $53\frac{12}{57}$ ct.
5. $109\frac{5}{7}$. 6. \$0.60. 7. \$36, $66\frac{2}{3}\%$.

### Page 246

1. 51,171,360 gr. 2. \$15,072,000. 3. 100%.
4. \$2,231,075. 5. \$5,550,000, 555. 6. \$3,333,333$\frac{1}{3}$, \$0.03$\frac{1}{3}$.
7. $85\frac{1}{2}\%$.

### Pages 247, 248

1. \$1,050,000. 2. \$63,000, 21%. 3. \$1950.
4. \$2.50. 5. \$0.15, $6\frac{8}{47}\%$. 6. Rate is greater on second.
7. 1.8%. 8. 4%. 9. \$5580.
10. \$7155. 11. \$1,027,000. Yes. 12. \$2107.50, \$0.75, \$210.75.
13. \$15,000. 14. \$13.44, \$13.09, \$8.68, \$11.20.

### Page 249

1. 20,880 T. 2. $5\frac{16}{21}\%$. 3. 1340. 4. 20,000 ft. 5. $118\frac{3}{4}\%$.

### Pages 250, 251

1. \$66.406. 2. \$67.50. 3. \$12.50 loss. 4. \$138 gain. 5. \$524.
6. $x$=\$50.04, \$56.77, \$18.14; $y$=\$142.03; $z$=\$25.54, \$57.77, \$58.72.
7. \$129.92, \$28.57, \$75.56, \$25.79.
8. \$105.60, \$21.72, \$47.21, \$27.07, \$7.20, \$2.40.

### Page 252

1. 36°. 2. 29.7°, 39.6°. 3. $-11°$.
4. 114.519°, 109.74°. 5. 9 mi., 7 mi. 6. 70,848 lb.
7. 50 in., 3 in., 30 in.

### Page 253

1. 1700%. 2. \$85,500,000. 3. \$20,000,000.
4. 6¢ gain is $31\frac{11}{19}\%$ of 19¢, $26\frac{54}{71}\%$. 5. 2,855,395,000. 6. 1 : 130.

## Page 255

| | | | | |
|---|---|---|---|---|
| **1.** $1\frac{66}{161}$. | **2.** $1\frac{83}{104}$. | **3.** $1\frac{32}{231}$. | **4.** $1\frac{121}{340}$. | **5.** $1\frac{48}{183}$. |
| **6.** $\frac{127}{460}$. | **7.** $\frac{19}{77}$. | **8.** $\frac{47}{105}$. | **9.** $\frac{223}{420}$. | **10.** \$2.01. |
| **11.** \$1.40. | **12.** \$2.25. | **13.** \$35.10. | **14.** \$35.65. | **15.** \$3.54. |
| **16.** \$1.60. | **17.** \$0.12$\frac{1}{2}$. | **18.** \$0.32. | **19.** 42.6 mi. | **20.** \$18. |
| | **21.** \$0.03$\frac{3}{8}$. | **22.** \$137.50. | **23.** \$0.02$\frac{1}{2}$. | |

## Page 257

| | | | | |
|---|---|---|---|---|
| **1.** 1. | **2.** $1\frac{2}{25}$. | **3.** $\frac{6}{7}$. | **4.** $1\frac{2}{7}$. | **5.** $1\frac{7}{9}$. |
| **6.** $\frac{11}{36}$. | **7.** 1. | **8.** $\frac{13}{35}$. | **9.** $\frac{5}{7}$. | **10.** $\frac{4}{5}$. |
| **11.** $1\frac{1}{2}$. | **12.** 6. | **13.** $1\frac{21}{44}$. | **14.** $1\frac{29}{35}$. | **15.** $2\frac{13}{72}$. |
| **16.** 504 bbl. | **17.** 20%. | **18.** \$40. | **19.** 171 da. | **20.** \$115.20. |
| | **21.** \$22.50. | | **22.** \$660. | |
| | **23.** \$1450, \$1413.75. | | **24.** $33\frac{1}{3}$%, 25%, 20%. | |

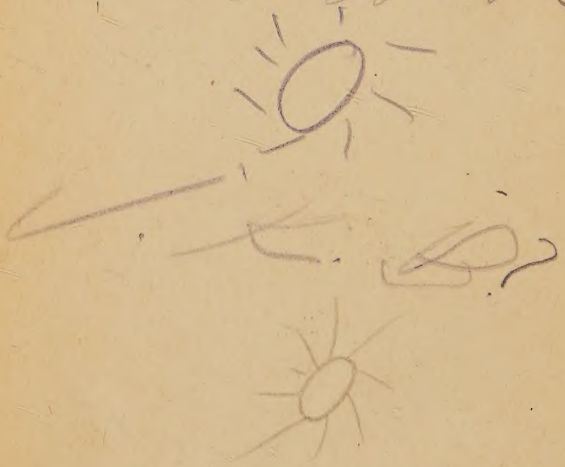
## Page 259

| | | | | |
|---|---|---|---|---|
| **1.** $10\frac{13}{24}$. | **2.** $15\frac{17}{63}$. | **3.** $10\frac{17}{20}$. | **4.** $36\frac{9}{32}$. | **5.** $17\frac{14}{27}$. |
| **6.** $12\frac{41}{128}$. | **7.** 7. | **8.** $5\frac{9}{19}$. | **9.** 3. | **10.** 3. |
| **11.** 4. | **12.** 6. | **13.** $\frac{2}{3}$%. | **14.** $\frac{3}{5}$%. | **15.** 1%. |
| **16.** \$0.16$\frac{3}{4}$. | **17.** 1.4%, \$0.7035. | | **18.** $85\frac{19}{31}\frac{5}{3}$%, $14\frac{11}{31}\frac{8}{3}$%. | |
| **19.** \$1.968. | **20.** \$1100. | **21.** Loss, $49\frac{7}{12}$ sq. yd., $43\frac{1}{3}$%. | | |

## Page 261

| | | | | |
|---|---|---|---|---|
| **1.** 0.09. | **2.** 0.16$\frac{2}{3}$. | **3.** 0.33$\frac{1}{3}$. | **4.** $0.30\frac{10}{23}$. | **5.** 0.5. |
| **6.** $0.42\frac{8}{7}$. | **7.** \$20. | **8.** \$32.50. | **9.** $195\frac{33}{43}$ bu. | **10.** $2\frac{3}{5}$. |
| **11.** \$570. | **12.** \$7.90. | **13.** 50%. | **14.** \$1200. | **15.** 80 T. |
| | **16.** \$113.67$\frac{3}{16}$. | | **17.** \$880. | |

## Pages 263, 264

| | | | | |
|---|---|---|---|---|
| 1. $33\frac{1}{3}\%$. | 2. 5%. | 3. \$420. | 4. \$250. | 5. \$364. |
| 6. \$16.50. | 7. \$11.75. | 8. \$850. | 9. \$250. | 10. \$475. |
| 11. \$259. | 12. \$11.55. | 13. \$105. | 14. $22\frac{1}{2}$ A. | 15. \$19. |
| 16. \$4.67. | 17. $337\frac{1}{2}$ mi. | 18. \$3200. | 19. $87\frac{1}{2}$ ft. | 20. 120 ft. |
| | | 21. \$55.20. | | |

Kenneth Brooke